Title: Breaking the Area-Space Barrier

Brief Overview:

Students will discover the relationship between the area and volume of prisms and cylinders.

NCTM Content Standard/National Science Education Standard:

Standard 1 Knowledge of Algebra, Patterns or Functions - Describe how a change in one variable in a linear function affects the other variable in a table of values.

Standard 3 Knowledge of Measurement – Estimate and determine the surface area and volume of rectangular prism and triangular prism. Estimate and determine the circumference and area of a circle. Estimate and determine the surface area and volume of a cylinder. Estimate pi using physical models.

Grade/Level:

Grades 6-8

Duration/Length:

Three forty minute lessons.

Student Outcomes:

Students will:

- Identify the relationship between the area of a rectangle and the area of a triangle.
- Calculate the area of rectangles and triangles.
- Construct a prism and a triangular prism.
- Calculate the surface area and volume of a prism and triangular prism.
- Explain the definition and use of pi.
- Calculate the circumference and area of a circle.
- Construct a cylinder and calculate its surface area and volume.
- Describe the relationship between increases in area to corresponding increases in the volume of prisms and cylinders.
- Construct a function for volume based on an increase in size of one variable.
Materials and Resources:

Lesson 1
Grid paper with 1 cm grid
Scissors
Scotch tape
Large triangular prism shipping box
Rectangular prism examples in room (file cabinet, box, CPU)
Metric ruler

Lesson 2
Various length strips of paper
Circular lids of varying sizes (peanut butter jars, coffee can lids, margarine tub lids, yogurt container lids, Pringles chip lids)
Rulers

Lesson 3
Cylinders of various sizes
Paper towel
Colored pencils
Scissors/Tape

Lesson 4
Graph paper
Scissors/Tape

Development/Procedures:

Lesson 1

Preassessment – How is area calculated for a square? How are two equal triangles created from a square? How is area calculated for a triangle?

Launch – Use series of rapid square area translations with visible or imagined examples to tax the reasonable student into calculating unreasonable areas. See puzzle sheet W1.C.

Teacher Facilitation – The student can now see the need for the intuitive level of understanding rather than immediate calculation. The problem must be understood before solving. The lesson goes through the evaluation of the puzzle sheet W1.C in order to find and correct the quick answers, which stemmed from faulty intuitive reasoning. Use Student Resource 1D to correctly find answers while teaching fundamental concepts. Each pair of students will use the templates of centimeter grid paper (Student Resource
1A and 1B) to construct the rectangular prism and the triangular prism and calculate the surface area of each prism. Each pair to explore how to determine the amount of volume and class discussion will derive the complete formula.

**Student Application**  Student completes Puzzle Worksheet in timed (5 minutes) challenge. Student then uses the Student Resource 1 D to calculate correct answers during instruction. Students in pairs then construct a rectangular prism and triangular prism using Student Resources 1A and 1B. Students determine volume of each prism. Student recruits measure real examples for the class to practice calculation of volume for shipping box, file cabinet, CPU, etc.

**Embedded Assessment** – Practical application of concepts will be demonstrated through completion of puzzle sheet W1C after answers and work are checked. Further assessment will be demonstrated through student involvement in solutions and contribution to discussion of volume formula. Practice examples of volume calculations can be graded for accuracy and completion.

**Reteaching/Extension** –
- For those who have not completely understood the lesson, review concept of area using the geoboard or other area manipulatives such as cut squares of the centimeter grid.
- For those who have understood the lesson, take them to the lesson two on the area of a circle.
- Students design new puzzle problems for the next year’s students. Another group will evaluate entries at a later date. Each entry to include explanation and answer.

**Lesson 2**

**Preassessment** – What measurements are needed to find the perimeter and area of a rectangle? (Length x Width) Why can’t these same measures be used for a circle?

**Launch** – What could you use to find the area of the circle? (perimeter and diameter)

**Teacher Facilitation** – The perimeter or distance of the outside edge (circumference) and the distance across the widest point of the circle (diameter) are used to calculate the area of a circle. One half of the diameter, or the distance to the midpoint is known as the radius.
We use a ratio to calculate the circumference and diameter of every circle: \[ \frac{\text{Circumference}}{\text{Diameter}} = \frac{C}{D} \]

**Student Application** – The student will measure the circumference of a series of lids by wrapping a strip of paper around the outer perimeter, marking the distance and measuring his result against a ruler. He will record his results in the table on Student Resource Sheet 2A. The student will then measure the diameter of each lid with his ruler, recording his results in the appropriate column of the worksheet. After completing measurements for five lids, the student will divide the circumference of each by the diameter and average the results.

**Embedded Assessment** – What happened in column C? When the worksheet is complete, the student should be able to explain the average result of column C is approximately equal to 3.14 no matter what size circle is measured.

The solution is always approximately 3.14159265.

This ratio is known as \( \pi \). We will round and use 3.14 in each of our formulas to measure the approximate circumference and area of circles.

**Reteaching/Extension** – To measure the approximate circumference of any circle we multiply \( \pi \times D \). Since the radius is one-half the diameter, we use the formula:

\[ \text{Circumference} \approx \pi \times 2 \times r \]

The formula used to measure the area of any circle is:

\[ \text{Area} \approx \pi \times r^2 \]

Use overhead fractional manipulatives to demonstrate a model for the calculation of area for the circle. Any uniform fractional division 1/8 or greater will suffice. Take the fractional pieces out of the circle and align them to form a “lumpy parallelogram”. Demonstrate how the area of the parallelogram can be related to the area of the circle by using the radius as height and one half the circumference as a side.
Area = b \times h
Area = (1/2 \text{ circumference}) \times h
Area = 1/2(2\pi r) \times r
Area = \pi r \times r
Area = \pi r^2

Complete Student Resource Sheet 2B to identify the formulas used for circumference and area of several circles. Solve each problem for circumference and area.

Lesson 3

Preassessment – How could you find the volume of a can of potato chips? A circular fish tank? Why would a calculation of these cylinders be different from a rectangular fish tank?

Launch – How did we use the area to find the volume of the prism? (Area \times Height) How can we use this knowledge to find the volume of a cylinder?

Teacher Facilitation – Wrap a paper towel around a canister. Unroll it to prove a cylinder is made up of a rectangle and two circles. Give careful explanation of difference between area and volume using student valued examples.

Demonstrate the surface area of the Pringles can and ask “But who wants an empty can? What is important is the full can! The contents or space inside is where the chips are. That is volume. The space for the chips!
Area is the can. We don’t want the can we want what is in it. That explains the difference between surface area and volume. In this lesson it will be important to know are we now talking about the can or the chips.” Periodically refer back to the example to clarify the difference between area and volume.

**Student Application** – Students will find the area of the identical circles on Student Resource Sheet 3A. Cut out both circles to use as end caps for a cylinder. Instruct the students to find the area of Student Resource Sheet 3B. Construct a cylinder from the end cap circles and Student Resource Sheet 3B. Tape in place.

Ask: What happened to the length measurement of our paper? (It became the circumference of the circle ends.) What happened to our width measurement? (It became our height measurement.)

Calculate the surface area and volume of the cylinders created.

**Embedded Assessment** – Ask class to brainstorm for characteristics of the cylinder created. Write these characteristics on the board or overhead. Construct a working definition of a cylinder from these characteristics. Students should be able to describe and estimate the volumes of other examples (drum, tanker, silo, tube, pipe)

**Reteaching/Extension** – Review the use of area of circles to find the surface area and volume.

\[
\text{Surface area} = 2 \text{ (Area)} + (\text{Circumference} \times \text{Height})
\]

\[
\text{Surface Area} \approx 2(\pi \times r^2) + (\pi \times 2r \times H).
\]

\[
\text{Volume} \approx (\pi \times r^2) \times (\pi \times 2r) \times H.
\]

The answers should be expressed in cubic measurements.

**Summative Assessment**:

The student will write a summary paragraph explaining the use of area to find the volume of a cylinder or prism. The student will create three geometric solids to illustrate the concept (a cylinder, prism and a triangular prism). Students will be invited to present their cylinders and findings to the class.

**Authors**:

Peter Dubyoski
Lamb of God School

Rhonda Tully
Cardinal Gibbons School

Archdiocese of Baltimore
1) On the grid draw three rectangles side by side 3 blocks by 5 blocks each and be sure to leave four blocks empty above your pictures
2) Above one of your rectangles and below one of your rectangles draw an equilateral triangle with sides three blocks long
3) Label each long block A, B or C and label each end triangle D, E. Then cut the figure out without separating each the pieces.
4) Calculate the area of the pieces and then the total area of the cut out figure
5) Fold the figure into a triangular tube by using the triangles as the ends.
6) How can you figure out how much space is in the container?
Name:_____________________________________________________

1. A checkerboard has black and red squares, and each square has sides 2.5 cm across. The board is a rectangle with 10 squares front to back, and 8 blocks side to side. What is the area of the board?
What is the area of one black square?
1. __________________

2. This classroom has square tiles, measuring 30 cm across one side. This room is a rectangle 19 tiles front to back, 22 tiles from one side to the other. What is the area of the classroom?
2. ________________
3. A $20 bill has sides that measure 6cm and 15 cm. If you cover the floor of the room in question number 2, with $20 bills, exactly adjacent, then how much money do you have?

3. ____________

4. My safety deposit box has dimensions of 60 cm, 18 cm, and 8 cm. How many $20 bill can I fit in it? Is this where my riches are if I only have $1 bills in it? Calculate the value in dollars for each question.

4. ____________

5. Cut the checkerboard (from question #1) from one corner to the other. What is the area of each piece?

5. ____________

6. Cut the room (from question #2) from one corner to the other. Draw each piece and put measurement on two sides. Tell the area of each piece

6. ____________
**CIRCLE ROUND**

**DIRECTIONS:**
1. Measure the **circumference** of a series of lids by wrapping a piece of string around the outer perimeter and measuring the result against a ruler. Record the results in **column 1** on the table below.
2. Measure the **diameter** of each lid with your ruler. Write the measurement in **column 2** on the table below.
3. **Divide** the circumference for a lid by the diameter and write your answer in **column 3** below.
4. **Find** the **average** of your answers in column 3.

<table>
<thead>
<tr>
<th>Lids</th>
<th>Circumference (in inches)</th>
<th>Diameter (in inches)</th>
<th>Circumference/Diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lid 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lid 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lid 3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lid 4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lid 5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Average:**

1. Write a sentence to explain what happened to your answers for lids 1 through 5.

   ___________________________________________________________________

   How did it affect your average? _______________________________________

2. **Write an equation** we could use to find the **circumference** if it were unknown.

   ___________________________________________________________________
Write the following formulas:

Circumference = _________________    Area = _________________

Directions: Measure the radius. Find the area and circumference for each circle.

Circle A

Circumference = _________________

Area = _________________

Circle B

Circumference = _________________

Area = _________________
CIRCLE ROUND

DIRECTIONS:
5. Measure the **circumference** of a series of lids by wrapping a piece of paper around the outer perimeter and measuring the result against a ruler. Record the results in **column 1** on the table below.
6. Measure the **diameter** of each lid with your ruler. Write the measurement in **column 2** on the table below.
7. **Divide** the circumference for a lid by the diameter and write your answer in **column 3** below.
8. **Find** the average of your answers in column 3.

<table>
<thead>
<tr>
<th>Lids</th>
<th>Circumference (in inches)</th>
<th>Diameter (in inches)</th>
<th>Circumference Diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lid 1</td>
<td></td>
<td></td>
<td>3.14</td>
</tr>
<tr>
<td>Lid 2</td>
<td></td>
<td></td>
<td>3.14</td>
</tr>
<tr>
<td>Lid 3</td>
<td></td>
<td></td>
<td>3.14</td>
</tr>
<tr>
<td>Lid 4</td>
<td></td>
<td></td>
<td>3.14</td>
</tr>
<tr>
<td>Lid 5</td>
<td></td>
<td></td>
<td>3.14</td>
</tr>
</tbody>
</table>

**Average:** __3.14__

1. Write a sentence to explain what happened to your answers for lids 1 through 5.
   _______ The answer for column 3 was always 3.14 ____________________________

How did it affect your average? _______ The average was 3.14 also _______

2. **Write an equation** we could use to find the **circumference** if it were unknown.
   __________ C = 3.14 x D ________________
Write the following formulas:

\[ \text{Circumference} = 2 \pi r \quad \text{Area} = \pi r^2 \]

Directions: Measure the radius. Find the area and circumference for each circle.

Circle A

Circumference = 19 cm
Area = 28 cm²

Circle B

Circumference = 31 cm
Area = 79 cm²