Title: Solving Equations: A Kinesthetically Engaging Approach

Brief Overview:

This Concept Development Unit walks students through the algebraic process for solving increasingly complex algebraic equations. It does so by engaging students in methods that are kinesthetically pleasing, yet, which constantly emphasize the vocabulary and format of the algebraic process.

NCTM Content Standard/National Science Education Standard:

Number and Operations
Algebra

Grade/Level:

Grade 7/8, all learning levels.

Duration/Length:

Five 50-minute blocks.

Student Outcomes:

Students will:

- Solve equations using inverse operations and proper order of operations in order to solve problems
- Write one-step and two-step algebraic equations in order to represent and solve problems.

Materials and Resources:

- Number and operation cards (Student Resource Sheet 1A) copied and cut into cards for student use
- Four sheets of lined loose leaf paper
- Scissors, stapler, colored pencils
- Other Teacher and Student Resource Sheets, copied onto overhead slides or on paper as appropriate.
- Algebra tiles
- Pennies
Development/Procedures:

Lesson 1  

Preassessment – $8 + n = 17$. Teacher will ask students what the $n$ represents in the equation. She will ask students to explain how they figured out the answer.

Launch – The teacher will ask the students to rewrite the Preassessment equation as an equation using a different operation. Two possible answers are $17 - n = 8$ or $17 - 8 = n$. The teacher will then give students cards with additional numbers and symbols on them, and ask them to use them to rearrange additional equations in a similar manner (Student Resource Sheet 1A). Students may give the teacher the results of the exercise to put on the overhead projector to summarize results (Teacher Resource Sheet 1A).

Teacher Facilitation – Teacher will guide students in noticing that all equations that begin with addition can be rewritten with subtraction, and all equations beginning with multiplication can be rewritten using division. She will introduce the term “inverse operations” to describe this relationship. She will then show students how to apply inverse operations to solve one-step equations using addition and subtraction, using proper algebraic form, emphasizing the need to keep equations "balanced" - any operation performed on one side of the equal sign must be performed on the other side of the equal sign.

Student Application – Pairs of students will make up four of their own equations using addition or subtraction, then trade equations with another pair of students. They will solve their neighbor’s equations using the previously demonstrated method. Students will begin to create their own sample book of problems to demonstrate how one-step equations might be, using a two-part foldable booklet (see Teacher Resource 5C for a sample illustration of such a project). This booklet is part of a summative assessment to be turned in at the end of the unit.

Embedded Assessment – When they finish, the authors of the equations and the solvers of the equations will compare answers to determine if they are correct. The teacher will invite a representative from each of the groups of four to demonstrate one of their examples on the board or the overhead.

Reteaching/Extension – Students will, with teacher guidance, write their own equations from word sentences, and then solve to reinforce solving equations while introducing the idea of representing ideas as equations. (Teacher Resource Sheet 1B)
Lesson 2  

Preassessment – Students will find the solution to the equation.

\[ 4x = 32 \text{ and } \frac{x}{3} = 8 \]. The teacher will ask students how they found each answer, and if it had anything to do with inverse operations.

Launch – Students will again rewrite equations using number cards to create similar equations with inverse operations (Student Resource Sheet 1A and Teacher Resource Sheet 2A). Again, the teacher can record student responses on the overhead projector.

Teacher Facilitation – The teacher will demonstrate proper algebraic form for solving algebraic equations with division and multiplication. She will reinforce the vocabulary of inverse operations, and remind students to keep equations balanced while solving. The teacher will also introduce/remind students how to find the reciprocal of a number, and how using reciprocals can help solve equations with fraction coefficients. (Teacher Resource Sheet 2B).

Student Application and Embedded Assessment – Students will practice newly learned skills (Student Resource Sheet 2A). When ready, they will add examples of equations they have learned to solve to their sample booklet, due at the end of the unit.

Reteaching/Extension – Teacher will review vocabulary that would indicate multiplication or division, and will assist students in writing equations based on real-world problems. (Teacher Resource Sheet 2C)

Lesson 3  

Preassessment – Teacher will ask students to solve the following problem:

Ms. Jordan has three apples and two bananas. She goes to the store and buys another four bananas and an apple. How much of each fruit does she have in the refrigerator after her shopping trip?

Launch – Teacher will ask students why they can’t add the apples and bananas together, and discuss the fact that apples and bananas are different, and have to be added separately. Similarly, we can only add “like terms” together in algebra. She will review the definition of like terms, and give a few examples of how to add like terms together (Teacher Resource Sheet 3A).
Teacher Facilitation – Teacher will help students model equations using themselves as variables, and have pennies or another small object they can use to complete their equation. Example: $3x + 2x = 30$. The teacher will pick three students to be the $3x$, two more students to be the $2x$. They will stand in the front of the class in groups on the left, with a plus in between them. The teacher will set up 30 pennies on the right side of the room to finish the equation. As students “solve” their equation, they will join into one group of $x$’s. The teacher will model how the grouping looks algebraically and remind them that they need to use their steps for solving a one-step equation after all like terms are combined. This can be repeated as needed.

Student Application – Students will practice solving equations by combining like terms in pairs. They can share answers after working on this for a while (Teacher Resource Sheet 3B).

Embedded Assessment – Students will come to the overhead to demonstrate their work. Teacher will inspect work as students are doing it as well. When they are ready, they can add examples of this sort to their sample booklet, due at the end of the unit.

Reteaching/Extension – Teacher will give students real-world situations that can be modeled by an equation with like terms. She will ask them to write the equations and solve them for additional practice and remediation. (Teacher Resource Sheet 3C)

Lesson 4

Preassessment- The students will be asked if they can describe the difference between a one step equation and a two-step equation.

Launch- The teacher will remind the students that the inverse operation as well as balancing the equations is important when solving a two-step equations. The teacher will then hand out algebra tiles along with a tile mat (Student Resource Sheet 4A). The students will use the tiles so that the students can visually create and solve equations. The teacher will present to the class several equations on an overhead for them to practice solving equations using the algebra tiles. (Teacher Resource Sheets 4A and 4B)

Teacher Facilitation- The teacher will give the students examples of a two-step equation. Ex: $2x – 9 = 11$. The teacher will then give the steps for solving this equation as well as other two-step equations. Step 1: The variable $x$ needs to be alone so that you can get the
value of the variable. So you need to get the 9 on the other side of the problem. To isolate the variable, you have to add 9 to each side. \(2x - 9 + 9 = 11 + 9\). The problem then looks like this \((2x = 20)\).

Step 2: Solve for the variable. You need to divide each side by 2. The answer would be \(x = 10\).

Student Application - Students will work on solving the equations given to them on the overhead. The class will then be divided into groups of three. One person will create a two-step problem. One person will complete the first step in the algebraic equation. The last person would solve for the variable. The students would rotate their roles until each of them has performed one of the steps.

Embedded Assessment - Students will be given the opportunity to demonstrate their knowledge of the material by completing the prior exercise. Also students will be asked to complete a worksheet (Student Resource Sheet 4B) in which they need to show and explain their steps. Worksheet will be collected so that the teacher can determine students’ progress on the lesson. Students will also add examples of two-step problems in their sample booklet.

Reteaching/Extension - The teacher will be able to determine if the students need to be re-taught the lesson based on the information obtained from the students on the worksheets. One option for reteaching would be to introduce students to the Do/Undo method of organizing algebraic steps. (Teacher Resource Sheet 4C)

Lesson 5

Preassessment – The teacher will present the following problem to the students:

Three circles and two triangles balance two circles and seven triangles. How much is a circle worth?

A strategy for solving this problem might be to get all the circles on one side of the balance, and the triangles on the other side. How can we group the circles on the left?

Launch – The teacher will hand out algebra tiles and help students’ model equations with variables on both sides of the equation using the overhead tiles to assist in modeling. She will solicit ideas from students on how to collect all the variables on one side of the equation. Similarly, she will solicit ideas on how to collect the constants on the opposite side of the equation from the variables.
In both cases, students can take like numbers of like tiles from both sides, adding zero pairs as needed.

Teacher Facilitation – The class will work through several examples of solving equations using algebra tiles, both with tiles, and at the same time demonstrating how to translate the algebra tile movement with algebraic steps (Teacher Resource Sheet 5A).

Student Application – Students will work through additional problems on their own with examples from their textbooks or other sources of the teacher’s choice. They may continue to use algebra tiles as needed, but must demonstrate the algebraic steps on paper at the same time.

Embedded Assessment – Students will hand in their algebra work from the abovementioned problems to confirm they were able to solve them using the algebraic process, with or without the tiles. They will add examples to their final project booklet when they are prepared to do so, either in class or as homework.

Reteaching/Extension – Extension is inappropriate here, because the difficulty of the objective for the day will require the entire class period. Reteaching, as necessary, can be completed on another day with additional practice. Adding real-world problems would be appropriate for an extension, however, getting to them at this stage is unlikely.

**Summative Assessment:**

The summative assessment has two parts – a subjective sample booklet that students make on their own throughout the unit and an objective test. The test is included in the attachments (Teacher Resource 5B). A picture of the sample booklet (Teacher Resource Sheet 5C) is also attached. The sample booklet can take many forms; the book *Dinah Zike’s Teaching Mathematics with Foldables*, by Dinah Zike offers many different options. You can access information about this source by going to [www.dinah.com](http://www.dinah.com), or the publisher of the book, www.glencoe.com.

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<tr>
<th>Original Equation</th>
<th>Rewritten Equation</th>
<th>n=?</th>
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<td>$n + 5 = -9$</td>
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<td>$-3 + n = 6$</td>
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Ms. Sanders had a balance of $405.87 in her savings account. She deposited $d$ dollars, and her new balance was $552.25$.

The temperature was 54 degrees Fahrenheit. The temperature dropped $d$ degrees. The new temperature was 35 degrees.

A plane at BWI airport is 52 feet above sea level. On takeoff it rises $f$ feet to level off at 28,000 feet.

The bottom of the Dead Sea is 1300 feet below sea level. Mount Everest is 29,035 feet above sea level. The difference between the two heights is $f$ feet.
Ms. Sanders had a balance of $405.87 in her savings account. She deposited $d$ dollars, and her new balance was $552.25.

\[ 405.87 + d = 552.25, \quad d = $146.38 \]

The temperature was 54 degrees Fahrenheit. The temperature dropped $d$ degrees. The new temperature was 35 degrees.

\[ 54 - d = 35, \quad d = 19 \text{ degrees Fahrenheit} \]

A plane at BWI airport is 52 feet above sea level. On takeoff it rises $f$ feet to level off at 28,000 feet.

\[ 52 + f = 28000, \quad f = 27,948 \text{ feet} \]

The bottom of the Dead Sea is 1300 feet below sea level. Mount Everest is 29,035 feet above sea level. The difference between the two heights is $f$ feet.

\[ 29,035 - (-1300) = f \quad \text{or} \quad 29,035 - f = -1300, \]
\[ f = 30,335 \text{ feet} \]
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<td>$5n = -7$</td>
<td>$-7/5 = n$</td>
<td>$-\frac{7}{5}$</td>
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<td>$\frac{n}{6}=5$</td>
<td>$5\cdot 6 = n$</td>
<td>30</td>
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<tr>
<td>$\frac{n}{-3}=-9$</td>
<td>$-9\cdot -3 = n$</td>
<td>27</td>
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<tr>
<td>$12=\frac{n}{6}$</td>
<td>$6\cdot 12 = n$</td>
<td>72</td>
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<tr>
<td>$\frac{3}{4}n=6$</td>
<td>$6\cdot \frac{4}{3} = n$</td>
<td>8</td>
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Reciprocal: A number related to another in such a way that when multiplied together their product is one.

\[ \frac{m}{n} \cdot \frac{n}{m} = 1 \]
How to use the definition of reciprocal when solving equations with fraction coefficients:

\[
\frac{2}{3}x = 8
\]

\[
\frac{3 \cdot 2}{2} \cdot \frac{x}{3} = 8 \cdot \frac{3}{2}
\]

\[
\frac{6}{6}x = 12
\]

\[
x = 12
\]
Pizza’s here! The Domino’s man has brought 8 pizzas for Ms. Jordan’s class to share. There are 72 slices of pizza total, and 24 students in her class.

- Write an equation modeling the situation.
  Let \( p = \)______

- Solve the equation.

Ms. Sanders has parent-teacher conferences with one fourth of her students. She has 30 conferences. How many students does she have?

- Write an equation modeling the situation.
  Let \( s = \)______

- Solve the equation.
2/3 of the world’s surface area is covered in water. The total water surface area is 130,000,000 square miles. What is the total surface area of the world?

- Write an equation modeling the situation.
  Let \( a = \) ______

- Solve the equation.

The Smith family took a five-day trip from Baltimore to Arizona. The total mileage for the trip was 2345 miles. How many miles did they travel per day?

- Write an equation modeling the situation.
  Let \( m = \) ______

- Solve the equation.
Pizza’s here! The Domino’s man has brought 8 pizzas for Ms. Jordan’s class to share. There are 72 slices of pizza total, and 24 students in her class.

• Write an equation modeling the situation.
Let \( p = \frac{\text{# pizza slices per child}}{24} \)
\[ 24p = 72 \]

• Solve the equation.
Each child can have 3 slices

Ms. Sanders has parent-teacher conferences with one fourth of her students. She has 30 conferences. How many students does she have?

• Write an equation modeling the situation.
Let \( s = \frac{\text{total # of students}}{4} \)
\[ S/4 = 30 \]
• Solve the equation.
\( S = 120 \) students
2/3 of the world’s surface area is covered in water. The total water surface area is 130,000,000 square miles. What is the total surface area of the world?

• Write an equation modeling the situation.
  Let \( a = \text{surface area of earth} \)
  \[
  \frac{2}{3}a = 130,000,000
  \]

• Solve the equation.
  \[195,000,000 \text{ mi}^2\]

The Smith family took a five-day trip from Baltimore to Arizona. The total mileage for the trip was 2345 miles. How many miles did they travel per day?

• Write an equation modeling the situation.
  Let \( m = \text{# miles per day} \)
  \[
  5m = 2345
  \]

• Solve the equation.
  \( M = 469 \text{ miles per day} \)
Name: _____________________

Show all work when solving the equations algebraically:

1. \( \frac{9}{5} = 3x \)  
2. \( 4y = -18 \)  
3. \( -54 = -9z \)  

4. \( \frac{a}{6} = -2 \)  
5. \( -36 = \frac{x}{2} \)  
6. \( \frac{2}{3}b = -12 \)  

7. \( -7c = 7 \)  
8. \( 4m = -2 \)  
9. \( \frac{-x}{5} = \frac{2}{7} \)  

10. \( \frac{1}{3} = -\frac{4}{3}h \)  
11. \( \frac{2}{3}w = \frac{3}{4} \)  
12. \( 7x = -\frac{14}{15} \)
Name: _____________________

Show all work when solving the equations algebraically:

1. \( \frac{9}{5} = 3x \)  
   \( x = \frac{3}{5} \)

2. \( 4y = -18 \)  
   \( y = -\frac{9}{2} \)

3. \(-54 = -9z \)  
   \( z = 6 \)

4. \( \frac{a}{6} = -2 \)  
   \( a = -12 \)

5. \(-36 = \frac{x}{2} \)  
   \( x = -72 \)

6. \( \frac{2}{3} b = -12 \)  
   \( b = -18 \)

7. \(-7c = 7 \)  
   \( c = -1 \)

8. \( 4m = -2 \)  
   \( m = -\frac{1}{2} \)

9. \( -\frac{x}{5} = \frac{2}{7} \)  
   \( x = -\frac{10}{7} \)

10. \( \frac{4}{3} h = -\frac{4}{3} \)  
    \( h = -1/4 \)

11. \( \frac{2}{3} w = \frac{3}{4} \)  
    \( w = 9/8 \)

12. \( 7x = -\frac{14}{15} \)  
    \( x = -\frac{2}{15} \)
Like Terms: Terms that have a) the same variable and b) the same exponent.

Like or unlike??

3a + 9a = 12a + 12b =

2c^2 + 6c^2 = -7d^2 + 9d^3 =

4e – 9e = 6f – 8g =
Like Terms: Terms that have a) the same variable and b) the same exponent.

Like or unlike??

\[3a + 9a = \]  
like, 12a

\[12a + 12b = \]  
unlike, different variables

\[2c^2 + 6c^2 = \]  
like, 8c^2

\[-7d^2 + 9d^3 = \]  
unlike, different exponents

\[4e - 9e = \]  
like, -5e

\[6f - 8g = \]  
unlike, different variables
Solve each equation. Show all work.

1. \(4x - 9x = -50\)  
2. \(y + 7y = 56\)

3. \(6a - 3a = 18\)  
4. \(-3g - g = 24\)

5. \(37f - 24f = -26\)  
6. \(-47h + 12h = 28\)

7. \(-9t + 15t = 36\)  
8. \(6u - 6u = 9\)
Solve each equation. Show all work.

1. \(4x - 9x = -50\)
   \[x = 10\]

2. \(y + 7y = 56\)
   \[y = 7\]

3. \(6a - 3a = 18\)
   \[a = 6\]

4. \(-3g - g = 24\)
   \[g = -6\]

5. \(37f - 24f = -26\)
   \[f = -2\]

6. \(-47h + 12h = 28\)
   \[h = -\frac{4}{5}\]

7. \(-9t + 15t = 36\)
   \[t = 6\]

8. \(6u - 6u = 9\)
   no solution
Jaden bought three computer games on Monday and four more computer games on Tuesday. She spent $77 total.

- Write an equation to model the situation.

- How much was each computer game?

Maria has two packs of gum. One pack has five pieces of gum. The other pack has 17 pieces of gum. She spent $1.10 on gum total.

- Write an equation to model the situation.

- How much was each piece of gum?
The ice cream truck comes to your neighborhood! Some kids bought bomb pops, which cost $1.10 each. The same number of kids bought Strawberry Shortcakes for $1.50 each. All together, they spent $15.60. How many kids bought ice cream?

Jeff buys two lollipops on Monday, two on Tuesday, and three on Wednesday. He had one lollipop before he started his candy shopping spree. The lollipops have a combined calorie count of 480.

The Johnson family is going on a family trip to New York, 270 miles away. They drive for 3 hours, then stop for lunch, then drive for another 1 ½ hours before arriving. What is their average speed?
Jaden bought three computer games on Monday and four more computer games on Tuesday. She spent $77 total.

- Write an equation to model the situation.
  Let \( c \) = # computer games
  \( 3c + 4c = 77 \)

- How much was each computer game?
  Each computer game was $11.

Maria has two packs of gum. One pack has five pieces of gum. The other pack has 17 pieces of gum. She spent $1.10 on gum total.

- Write an equation to model the situation.
  Let \( g \) = # of sticks of gum
  \( 5g + 17g = 1.10 \)

- How much was each piece of gum?
  Each piece of gum was $0.05.
The ice cream truck comes to your neighborhood! Some kids bought bomb pops, which cost $1.10 each. The same number of kids bought Strawberry Shortcakes for $1.50 each. All together, they spent $15.60. How many kids bought ice cream?

Let \( k \) = \# of kids who bought bomb pops

\[
1.10k + 1.50k = 15.60
\]

\( k = 6 \), so six kids bought bomb pops and another 6 kids bought strawberry shortcakes, making 12 kids total.

Jeff buys two lollipops on Monday, two on Tuesday, and three on Wednesday. He had one lollipop before he started his candy shopping spree. The lollipops have a combined calorie count of 480.

Let \( c \) = calorie count of each lollipop

\[
2c + 2c + 3c + c = 480
\]

Each lollipop has 60 calories.

Let \( s \) = avg. speed

\[
3s + 1.5s = 270
\]

Their avg. speed was 60 mph

The Johnson family is going on a family trip to New York, 270 miles away. They drive for 3 hours, then stop for lunch, then drive for another 1 ½ hours before arriving. What is their average speed?
1. $3b + 3 = 9$ 

2. $2a - 8 = 6$

3. $-4a + 5 = -3$ 

4. $-4 = 3x + 8$

5. $-5 + 6y = 1$ 

6. $2 - 3z = 5$

7. $6 = 9 - x$
1. $3b + 3 = 9$
   
   $b = 2$

2. $2a - 8 = 6$
   
   $a = 7$

3. $-4a + 5 = -3$
   
   $a = 2$

4. $-4 = 3x + 8$
   
   $x = -4$

5. $-5 + 6y = 1$
   
   $y = 1$

6. $2 - 3z = 5$
   
   $z = -1$

7. $6 = 9 - x$
   
   $x = 3$
Using Algebra Tiles in Your Classroom

There are two algebra tiles you will use to solve two-step equations, the □ and the □.

The rectangles represent one variable. Hence, □ would represent 2x.

The squares represent one unit. Hence, □ would represent 3.

To make either the variables or the units negative, turn them to the red side □.

To model an equation, for example 2x – 5 = -3, you would place the following tiles:

\[
\begin{align*}
2x & \quad - 5 & = & \quad -3 \\
\text{方块} & \quad \text{方块} & & \text{方块} \\
\text{方块} & \quad \text{方块} & & \text{方块} \\
\text{方块} & & & \text{方块}
\end{align*}
\]

To isolate the variable term (2x), you will add five positive unit tiles to both sides.

\[
\begin{align*}
2x - 5 + 5 & = -3 + 5 \\
\text{方块} & \quad \text{方块} & & \text{方块} \\
\text{方块} & \quad \text{方块} & & \text{方块} \\
\text{方块} & \quad \text{方块} & & \text{方块} \\
\text{方块} & \quad \text{方块} & & \text{方块}
\end{align*}
\]
The positive and negative units will form “zero pairs” (because one positive plus one negative equals zero). Zero pairs leave the board.

\[
\begin{array}{c|c}
2x & + 0 \\
\hline
\end{array} = 2
\]

The remaining tiles on the board will look like this:

\[
\begin{array}{c|c}
2x & = 2 \\
\hline
\end{array}
\]

The final step is to divide any unit tiles evenly between variable tiles.

\[
\begin{array}{c|c}
x & = 1 \\
\hline
\end{array}
\]
Students sometimes find it helpful to list the steps taken when solving an equation. One method of organizing this is to use a Do/Undo chart. An example is illustrated below.

\[-3x + 6 = -21\]

The instructor will ask the students, “What is being DONE to the \(x\)?” The answer would be, “The \(x\) is multiplied by \(-3\) then \(6\) is added.” Remind students that they must use the order of operations to establish what is DONE. Students will then put this information into the DO side of the chart.

<table>
<thead>
<tr>
<th>DO</th>
<th>UNDO</th>
</tr>
</thead>
<tbody>
<tr>
<td>•((-3))</td>
<td></td>
</tr>
<tr>
<td>+ 6</td>
<td></td>
</tr>
</tbody>
</table>

To fill in the UNDO side of the chart, students will reverse the order of the operations, and change original operations to reciprocal operations.

<table>
<thead>
<tr>
<th>DO</th>
<th>UNDO</th>
</tr>
</thead>
<tbody>
<tr>
<td>•((-3))</td>
<td>÷((-3))</td>
</tr>
<tr>
<td>+ 6</td>
<td>- 6</td>
</tr>
</tbody>
</table>

Students will then apply the UNDO column to their algebra equation using proper algebra form:

\[-3x + 6 = -21\]
\[-3x + 6 - 6 = -21 - 6\]
\[-3x = -27\]
\[\frac{-3x}{-3} = \frac{-27}{-3}\]
\[x = 9\]
Solve each equation. Use the two-step method to solve each equation.

1. $6x + 21 = 57$  
2. $4x - 17 = -1$  
3. $2a - 15 = 31$  
4. $5d + 10 = 25$  
5. $6c \cdot 12 = 48$  
6. $3f \cdot 15 = 45$  
7. $\frac{3}{4}p - 12 = 27$  
8. $\frac{1}{2}r + 18 = 36$
1. $6x + 21 = 57$  
   $X = 6$

2. $4x - 17 = -1$  
   $X = 4$

3. $2a - 15 = 31$  
   $A = 23$

4. $5d + 10 = 25$  
   $D = 3$

5. $6c \cdot 12 = 48$  
   $C = \frac{2}{3}$

6. $3f \cdot 15 = 45$  
   $F = 1$

8. $\frac{3}{4}p - 12 = 27$  
   $P = 52$

8. $\frac{1}{2}r + 18 = 36$  
   $R = 36$
Equations with Variables on Both Sides of the Equation

\[3u - 10 = u + 4\] \[-6x - 3 = 2x + 5\]

\[w + 7 = -3w + 14\] \[6j + 9 = -6j - 81\]

\[12 - 3p = 2p + 72\] \[8 - d = 16 - 2d\]
Equations with Variables on Both Sides of the Equation

\[3u - 10 = u + 4\] \hspace{1cm} \[-6x - 3 = 2x + 5\]

\[u = 7\] \hspace{1cm} \[x = -1\]

\[w + 7 = -3w + 14\] \hspace{1cm} \[6j + 9 = -6j - 81\]

\[w = 21/4\] \hspace{1cm} \[j = 15/2\]

\[12 - 3p = 2p + 72\] \hspace{1cm} \[8 - d = 16 - 2d\]

\[p = -12\] \hspace{1cm} \[d = 8\]
Unit Assessment – Solving Equations in One Variable

Solve each equation, showing all work algebraically. You may use algebra tiles to help model the problem if you wish.

1. \( \frac{3}{5}a = -6 \)  
2. \(-g = 7\)  
3. \(7h = -84\)

4. \(-13 = -\frac{b}{9}\)  
5. \(4 - 6x = -2\)  
6. \(15 + \frac{1}{3}y = -6\)

7. \(4c - 19 = -3c + 2\)  
8. \(3f + 6f = -108\)

For the following problems, write an equation to model the situation, then solve the equation to solve the problem.

9. A bag contains 84 cookies. There are 7 children at Miss Sally’s daycare. How many cookies could each child have?

10. Mr. Ted had $497.65 in his checking account. He deposited his paycheck into the account. His new balance is $991.25. How much was his paycheck?
Unit Assessment – Solving Equations in One Variable

Solve each equation, showing all work algebraically. You may use algebra tiles to help model the problem if you wish.

1. \[ \frac{3}{5}a = -6 \]
   \[ a = -10 \]

2. \[ -g = 7 \]
   \[ g = -7 \]

3. \[ 7h = -84 \]
   \[ h = -12 \]

4. \[ -13 = -\frac{b}{9} \]
   \[ b = 117 \]

5. \[ 4 - 6x = -2 \]
   \[ x = 1 \]

6. \[ 15 + \frac{1}{3}y = -6 \]
   \[ y = -63 \]

7. \[ 4c - 19 = -3c + 2 \]
   \[ c = 3 \]

8. \[ 3f + 6f = -108 \]
   \[ f = -12 \]

For the following problems, write an equation to model the situation, then solve the equation to solve the problem.

9. A bag contains 84 cookies. There are 6 children at Miss Sally’s daycare. How many cookies could each child have?

   Let \( c \) = # cookies per child, \( 6c = 84 \), \( c = 14 \) cookies apiece

10. Mr. Ted had $497.65 in his checking account. He deposited his paycheck into the account. His new balance is $991.25. How much was his paycheck?

    Let \( p \) = amount of paycheck, \( 497.65 + p = 991.25 \), \( p = $493.60 \)
Sample Final Project Cover:

My Equation Book

Sample Final Project Inside First Page:

1

2

STEP

STEP
Sample Final Project Additional Pages:

Please refer to Dinah Zike’s Teaching Mathematics with Foldables (Glencoe) for additional booklet-making ideas.