Title: Probability in the 6th Grade

Brief Overview:

In this unit, students will learn about probability. The topics covered include theoretical probability, experimental probability, and the law of large numbers. Students will use spinners, colored cubes, coins, and dice.

NCTM 2000 Principles for School Mathematics:

- **Equity:** *Excellence in mathematics education requires equity - high expectations and strong support for all students.*
- **Curriculum:** *A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.*
- **Teaching:** *Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.*
- **Learning:** *Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.*
- **Assessment:** *Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.*
- **Technology:** *Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.*

Links to NCTM 2000 Standards:

- **Content Standards**

  <u>Data Analysis and Probability</u>
  The students will learn about finding outcomes with and without replacement, using theoretical as well as experimental probability.

- **Process Standards**

  <u>Problem Solving</u>
  The students will determine how many cubes are in the bag by creating a systematic way to find the outcomes, by making decisions whether or not to change a playing board, and by creating their own carnival game. All of the activities in this unit are done in pairs or in groups. Students will examine different perspectives on solving problems. Students will examine different perspectives on solving problems.

Communication

Students will justify their answers, as well as write journals on the differences between experimental and theoretical probability.

Grade/Level:

Grade 6, all ability levels

Duration/Length:

12 days/55 minute periods

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Fractions (adding and subtracting)
- Some computer skills

Student Outcomes:

Students will:

- demonstrate the ability to find, list, and predict outcomes with replacement.
- demonstrate the ability to find, list, and predict outcomes without replacement.

Materials/Resources/Printed Materials:

- Computer lab equipped with the Mathtools software program, or equivalent software which can simulate a spinner and tabulate the results of a specified number of spins.
- Worksheets on probability - labeled
- 20 paper bags with colored cubes (blue, yellow, red, green, at least 3 of each)
- 25 large coins
- 50 6-sided dice

Development/Procedures:

Days 1, 2, and 3 - Computer Lab Activity (Worksheet Packet 1)

(Modify the activity as necessary if software other than Mathtools is used.)

- Have students open Mathtools on their computer. Direct step-by-step what should be done on the monitor. Steps are listed on the worksheets.
- Answers to give students for questions 8, 11, 12, and 17:
    8. Experimental Probability is how likely an event is based on the data collected.

11. A spreadsheet shows data electronically that is collected.
12. Step 1 - Click on the spinner; Step 2 - Click on the link button on the far left column. It looks like a paper clip; Step 3 - Click and HOLD on the upper left part of the spinner; and Step 4 - Move the cursor to the upper left box of the spreadsheet and let go of the button.
17. Step 1 - Click on the spinner; Step 2 - Click on the spinner name under the Mathtools menu bar; Step 3 - Scroll down and click on reset.

Days 4 and 5 - Using the Bags and Colored Cubes (Worksheet Packet 2)

- Give students a paper bag (with 3 red and 2 blue cubes inside) for each group of two students. Do not let students look in the bags. Tell them to draw a cube 20 times, one cube at a time. Once they draw a cube, they write down the color and put the cube back in the bag. After drawing 20 times, the students then make a prediction on the color of cubes and how many of each color are in the bag. They report to the rest of the class. Have the class vote on which group is correct. The students can then open the bags and look at the cubes.
- Give out Worksheet Packet 2 to each student. Have them work through the experimental probabilities.

Day 6 - Coins (Worksheet 3)

- Give students definitions for theoretical and experimental probabilities. Have partners flip a coin 20 times and keep track of the flips. They then combine their data with another group of students. The last step is to combine their data as an entire class. Take the students' data and record it on the chalkboard. At the end of the day, combine all the classes together and give the results the next day for discussion.

Day 7 - 1 Die (Worksheet Packet 4) for each group of 3-4 students

- What would the probability be of rolling a 1? 2? 3? 4? 5? 6?
- In groups of 3-4, have the students roll a 6-sided die 24 times and record their findings on the chart. Combine data with another group, and then as a whole class. What would the experimental probability be?

Day 8 - 2 Dice (Worksheets 5 and 6) for each pair of students

- Have students play the Even/Odd game in pairs by using Worksheet 5 and 2 dice. They place 6 cubes on each side on odd and even. A student rolls the dice. If the product is even, that students remove a cube from the even side. If the product is odd, the students remove a cube from the odd side. Invariably, the even student will probably win the game. The students then get the opportunity to change the number of cubes for the second game to make it a fair game. After they play the second time, have them make a chart like the one on Worksheet 6 that shows all possible outcomes of rolling a product that is even or odd. Have the students find the theoretical probability of even and odd products.
- After students have finished creating their product chart, they should then create a chart that shows the sum of the dice. What would the theoretical probability be for rolling each sum?

Day 9 - Theoretical Probability (Worksheet Packet 7)

- Hand out Worksheet Packet 7 to students. Have them explain Theoretical Probability to you. Together, create a formula that defines what Theoretical Probability is. Work through each example on the worksheet.

Performance Assessment: (Days 10, 11, and 12)

Students will create a carnival game in groups of 3 or 4 for their assessment. It needs to include the theoretical probabilities, as well as their test of the game, with experimental probabilities. See Assessment (Welcome to the Carnival) for the instructions given to the students, as well as the rubric used to grade students.

Extension/Follow Up:

After a geometry unit, have students create a landing pad for a mini-parachute man. This landing pad needs to cover at least a 12 in. by 12 in. area and consist of various geometric shapes where the parachute man could land. The students then determine the theoretical probabilities of landing in each of the these shapes. After performing the trial runs, they would then show the experimental probabilities.

Author:

Toni Smith
Meade Middle School
Anne Arundel County, MD

Mathtools and Introduction to Probability

Name _____

Part I. Spinner

Directions: Using Mathtools, answer the following questions. Some questions are fill in the blank, and some require complete sentences. Those that require complete sentences, remember to give a RARE response.

1. How many sections does the spinner have? _____

2. If you spin the spinner, what are the outcomes? (Where could the spinner land?)

_____

3. Predict where the spinner arrow will land the first time you spin it. _____

4. Click the spin button once with the left mouse button and record your result in the table below.

5. If you spin the spinner 10 times, predict how many times it will land in Section 1 of the spinner _____ and Section 2 of the spinner _____. (The total for Section 1 and Section 2 should add up to 10.)

6. Explain your prediction. _____

_____

_____

Section 1	Section 2

7. Now click the spin button 9 more times and record your results in the table above. **DO NOT CHANGE YOUR PREDICTION! Remember: the total number of tallies in the table should equal 10 and the total number of spins in your spin box should equal 10.**

8. Put your notes on experimental probability here. _____

_____

_____

_____

9. Compare your prediction from question #5 to the results in the table above. Were they the same? _____

10. If they were not the same, explain why you think there is a difference? If they were the same, explain why they are the same. _____

_____

_____

_____

Part II. Spreadsheet

11. Explain what a spreadsheet is. _____

_____

_____

12. Now wait for directions to setup your own spreadsheet in Mathtools. In the space below, list in sequential order the steps you must take to link your spinner box with the spreadsheet. _____

_____

_____

_____

13. What two things occur to let you know that you have successfully linked your spinner box to your spreadsheet?

_____

_____

14. Now spin the spinner one more time. As you spin the spinner, observe the spreadsheet.
    a. Analyze what column A represents_____
    b. Analyze what column B represents _____

15. When you click on the spreadsheet to activate it, observe what change occurs on the Mathtools menu bar. _____

_____

16. When you click on the spinner box to activate it, observe what change occurs on the Mathtools menu bar. _____

_____

17. List the steps to reset the spinner. _____

_____

_____

_____

18. Record the number that appears in row 1 column A. _____

19. Record the number that appears in row 1 column B. _____

20. Record the number that appears in row 2 column A. _____

21. Record the number that appears in row 2 column B. _____

22. Record the number that appears in Total Spins in the spinner box. _____

23. Analyze the relationship between resetting the spinner and the spreadsheet/spinner box. _____

_____

_____

24. Change the Spins to Do box to 100 **STOP HERE!**! Proceed to question 25.

25. Predict the outcomes of these 100 spins. (What will show up in Row 1 column B and Row 2 column B?)

	A	B
1		
2		

26. Click the spin button **ONCE**. Observe and record what happens in the spreadsheet.

	A	B
1		
2		

27. Reset the spinner. Change the spins to do 1000. **STOP HERE!!**

28. Predict the outcomes of these 1000 spins.

	A	B
1		
2		

29. Click the spin button ONCE. Observe and record what happens in the spreadsheet.

	A	B
1		
2		

30. What do you notice happens as the spin numbers get bigger? _____

_____

_____

_____

_____

Experimental Probability

Name _____

- A game is fair if each player has the same chance of winning. Players are equally likely to win in a fair game.
- Play these three games with a partner. For each game, place the given numbers of colored cubes in a bag. Then draw cubes without looking. Answer Question 1 **BEFORE** each game. Answer Question 2 and 3 after each game.

Game 1

Use 1 red and 1 blue cube. Draw 1 cube at a time. If red is drawn, player A wins, if blue is drawn, player B wins.

1. Before playing, decide whether the game seems fair or unfair. Which player seems more likely to win? _____

_____

2. Play the game 20 times. Record your results in the table below.

Number of times A won	
Number of times B won	
Number of games played	20

3. Does the game seem fair or unfair? Why? _____

_____

_____

Game 2

Use 2 red and 2 blue cubes. Draw 1 cube and then a second cube. If the 2nd cube is the same color as the 1st was, A wins. If the 2nd cube is a different color that the 1st was, B wins.

4. Before playing, decide whether the game seems fair or unfair. Which player seems more likely to win? _____

_____

5. Play the game 20 times. Record your results in the table below.

Number of times A won	
Number of times B won	
Number of games played	20

6. Does the game seem fair or unfair? Why? _____

_____

_____

Game 3

Use 3 red and 1 blue cube. Draw 1 cube and then a second cube. If the 2nd cube is the same color as the first was, A wins. If the 2nd color is different than the first was, B wins.

7. Before playing, decide whether the game seems fair or unfair. Which player seems more likely to win? _____

_____

8. Play the game 20 times. Record your results in the table below.

Number of times A won	
Number of times B won	
Number of games played	20

9. Does the game seem fair or unfair? Why? _____

_____

_____

You can write a ratio to show the fraction of games that a player wins. This ratio is the experimental probability of winning.

Experimental Probability

$$\text{Probability (A wins)} = \frac{\text{Number of games A won}}{\text{Total number of games played}}$$

$$\text{Probability (B wins)} = \frac{\text{Number of games B won}}{\text{Total number of games played}}$$

In 20 rounds of Game 2, Player A won 8 and player B won 12. What is the experimental probability the A wins?

$$\text{Probability (A wins)} = \frac{8 \text{ - games won by A}}{20 \text{ - total games played}}$$

Reduce the fraction to $\frac{2}{5}$

10. What is the probability that B wins? _____

_____

11. If you play the games tomorrow, will the experimental probabilities be the same as today? Explain. _____

_____

12. Ki-Jana and Dwayne played game 1. The first 3 cubes drawn were blue. Is the fourth cube more likely to be red or blue? Why? _____

_____

13. Suppose probability (A wins) equals probability (B wins). What can you say about the fairness of the game. _____

_____

14. Si and Karen played a fair game. Karen won 7 times and Si won 13 times. How can this happen if the game is fair? _____

_____

Experimental and Theoretical Probabilities - Coin Tossing

Experimental Probability - _____

_____

_____

Theoretical Probability - _____

_____

_____

Toss a coin 20 times. Record your data in the chart below.

Heads	Tails

Share your results with the class.

Combine your results with one other group.

Heads	Tails

What do you notice? _____

_____

Write your entire class results below.

Heads	Tails

What do you notice? _____

_____

Theoretical and Experimental Probability - 1 Die

What would the Probability (1) be ? _____

Probability (2) _____ Probability (3) _____

Probability (4) _____ Probability (5) _____

Probability (6) _____

In a group, roll a die 24 times. Record your results below using tally marks

1	
2	
3	
4	
5	
6	

What are the experimental probabilities for each?

Probability (1) _____ Probability (2) _____

Probability (3) _____ Probability (4) _____

Probability (5) _____ Probability (6) _____

Combine your data with another group.

1	
2	
3	
4	
5	
6	

What are the experimental probabilities for each?

Probability (1) _____ Probability (2) _____

Probability (3) _____ Probability (4) _____

Probability (5) _____ Probability (6) _____

Combine the whole class data.

1	
2	
3	
4	
5	
6	

What are the experimental probabilities for each?

Probability (1) _____ Probability (2) _____

Probability (3) _____ Probability (4) _____

Probability (5) _____ Probability (6) _____

What do you notice happens as the rolls increase? _____

_____

_____

_____

Even	Odd

To Play the Game:

- Put 6 markers on even and 6 markers on odd. Roll 2 dice. Multiply the two numbers together to find the product. If the product is even, the even person takes a marker off. If the product is odd, the odd person takes a marker off.

- The first with all the markers off is the winner.

Worksheet 6

For students on their own on scratch paper:

X	1	2	3	4	5	6
	1 (O)	2 (E)	3 (O)	4 (E)	5 (O)	6 (E)
	2 (E)	4 (E)	6 (E)	8 (E)	10 (E)	12 (E)
	3 (O)	6 (E)	9 (O)	12 (E)	15 (O)	18 (E)
	4 (E)	8 (E)	12 (E)	16 (E)	20 (E)	24 (E)
	5 (O)	10 (E)	15 (O)	20 (E)	25 (O)	30 (E)
	6 (E)	12 (E)	18 (E)	24 (E)	30 (E)	36 (E)

What would the probability of rolling an odd product be?

What would the probability of rolling an even product be?

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

What would the probability of rolling a sum of 5 be?

What would the probability of rolling a sum of 7 be?

What would the probability of rolling a sum of 10 be?

What would the probability of rolling a sum of 2 be?

What would the probability of rolling a sum of 8 be?

List all of the possible outcomes and probabilities.

Theoretical Probability

- You can find experimental probability by doing simulations. You can find theoretical probability without a simulation. When all outcomes are equally likely, the theoretical probability of an event is the ratio below.

  Probability (event) = $\frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$

- Suppose you had 2 red cubes, 2 blue cubes, and 1 yellow cube in your bag. What is the probability that you draw a red cube?

  Probability (red) = $\frac{\text{red cubes}}{\text{all cubes}} = $

1. How many yellow cubes should you add to the bag to make Probability (yellow) equal Probability (red)? _____

2. After you draw yellow cubes to the bag, what is the probability of drawing yellow? Drawing red? _____

- A bag contains 3 red cubes and 1 blue cube. Suppose you draw one cube.

3. Are all cubes equally likely to be drawn? _____

   Why? _____

   _____

4. How many possible outcomes are there? How many outcomes are favorable for red? _____

   _____

5. Find Probability (red). Write it as a fraction. _____

- Find the theoretical probability for one roll of a six sided die. Write each as a fraction.

6. Probability (4) _____

7. Probability (9) _____

8. Probability (not 5) _____

9. Probability (2 or 5) _____

- Find the theoretical probability of each event.

10. A spinner has equal sections of red, pink, green, and yellow. You spin blue or pink. _____

11. You toss two coins. Both tosses are heads. _____

Journal Entry - Summarize the difference between theoretical probability and experimental probability. You need to have a proper, complete paragraph.

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

Welcome to the Carnival

- *Your Goal: Create a carnival game in a group of 4 to 5.*

- *Your game does not have to be fair.*

- *You must include in your game:*

  - Instructions
  - How you win
  - A List of prizes and how they are to be awarded

- *Once your game is finished, you must find the theoretical probabilities for your game.*

- *Then, play your game and find your experimental probabilities. Would you have won a prize?*

- *How points are awarded:*

Creativity	*10 points*
Neatness	*30 points*
Originality	*15 points*
Instructions are easy to understand	*15 points*
Theoretical Probabilities	*15 points*
Experimental Probabilities	*15 points*