Title: Probability and Simulation

Brief Overview:

This learning unit will review the basic concepts of theoretical and experimental probability and simulations. Students will compare and contrast the two types of probability as a writing component. This writing component will be based on an in-class activity in which students will collect and analyze data. Two classroom activities will involve students in performing two different simulations, one using a student created spinner and one using the randINT function on the TI-83 calculator. This learning unit is structured in three separate modules that may be used independently.

NCTM 2000 Principles for School Mathematics:

- **Equity:** Excellence in mathematics education requires equity - high expectations and strong support for all students.

- **Curriculum:** A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.

- **Teaching:** Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.

- **Learning:** Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.

- **Assessment:** Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.

- **Technology:** Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students’ learning.

Links to NCTM 2000 Standards:

- **Content Standards**

  **Data Analysis and Probability**
  Students will demonstrate an understanding of and will apply basic concepts of probability. They will select and use appropriate statistical methods to analyze data. Students will develop and evaluate inferences and predictions that are based on data.
• Process Standards

**Mathematics as Problem Solving, Reasoning and Proof, Communication, Connections, and Representation**
These five process standards are threads that integrate throughout the unit, although they may not be specifically addressed in the unit. They emphasize the need to help students develop the processes that are the major means for doing mathematics, thinking about mathematics, understanding mathematics, and communicating mathematics.

**Links to Maryland High School Mathematics Core Learning Units:**

**Data Analysis and Probability**

- **3.1.1**
  Students will design and/or conduct an investigation that uses statistical methods to analyze data and communicate results.

- **3.1.3**
  Students will use simulations to make statistical inferences from data to estimate the probability of an event.

- **3.2.1**
  Students will make informed decisions and predictions based upon the results of simulations and data from research.

**Grade/Level:**

Grades 8-12; Algebra I

**Duration/Length:**

Three or four class periods or two block schedule periods. (variable)

**Prerequisite Knowledge:**

Students should have working knowledge of the following skills:

- Ability to create a pie chart
- Understand and work with the basic concepts/formula for theoretical and experimental probability
- Calculate percentages from a table of data
**Student Outcomes:**

Students will be able to:

- state the experimental probability of an event.
- collect data and find the experimental probability of an event.
- compare and contrast theoretical and experimental probability based on a given event and data collected during a classroom activity.
- collect and use data to construct a tool (a spinner) to simulate an event.
- generate and interpret random data using the spinner.
- generate and interpret random data using the TI-83 (Plus) calculator’s randInt feature.
- use results from a simulation to determine probabilities and apply them to a random sampling.

**Materials/Resources/Printed Materials:**

- Student simulation worksheets
- Heavy cardstock and large paper clips for the spinner
- TI-83 overhead projector
- TI-83 (Plus) calculators

**Development/Procedures:**

This activity will consolidate the student’s knowledge about probability and culminate with two simulation activities.

Day One: Use Teacher Instruction Sheet.
Day Two: Use Teacher Instruction Sheet. (End of day one in block scheduling)
Day Three: Use Teacher Instruction Sheet.
Day Four: Assessment. (End of day two in block scheduling)

**Assessment:**

Students’ progress will be determined by:

- In-class activities, worksheets, and discussions
- A quiz following each module or a consolidated quiz following the unit
- Individually completed project following the unit

**Extension/Follow Up:**

Possibilities include:

- Investigations of games of chance and the spread of legalized gambling
- Investigation of state lotteries
- Actuarial studies
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STIMULATING SIMULATIONS
Simulation and Probability
MODULE 1
Theoretical vs. Experimental Probability

Objectives:

Students will be able to:

• state the definition and formula for theoretical and experimental probability.
• state the theoretical probability of an event.
• perform an experiment to determine the experimental probability of an event.
• compare and contrast theoretical and experimental probability based on the results of an event.
• make inferences from an experiment.

Materials:

• Four (could vary) trash cans of equal dimensions
• Scrap paper to make “basketballs” (Alternative: Nerf/foam balls)
• Data collection worksheets
• Lesson worksheets

Lesson:

Introduction:

Advise students that they:

• will be studying both theoretical and experimental probability.
• must be able to state the formula for both types of probability and use them to calculate probability for an event.
• must actively participate in the collection of data.
• must be prepared to write a paper comparing and contrasting theoretical and experimental probability as it relates to the event covered in the lesson.

Procedures:

• Divide the room into four sections and have students pair up within their area.
• Hand out student worksheets.
• Work in pairs, have students complete Questions 1 – 3 on the worksheet.
• As a class, discuss Question 3. Each student needs a clear understanding of the differences between theoretical and experimental probability.
• Have students complete Question 4. Stress with reasoning. Discuss as a class.
• Collect experimental data. (Teacher guidelines/class handout included)
• Have students work with their partner to complete Questions 6 – 7. Discuss.
• Have students begin their compare/contrast paragraph. Complete as homework.
Data Collection Procedures
MODULE 1

Option 1:

• Set four trashcans (baskets) in the four corners of the classroom. (For adequate space, use a hallway or go outside for the data collection. If you go outside, your students could feel free to cheer their teammates on.)
• Place a piece of masking tape on the floor (foul line) at an equal distance from each basket to establish a consistent shooting position. Make the distance far enough to make the shot challenging, but not too difficult. Try ten feet if space allows. The paper will have to be balled up tightly to make it closely simulate a basketball.
• Make sure each group has a “basketball” and have them begin collecting data. One student in each group should be designated as the data recorder.

  ➢ Allow each student one practice shot.
  ➢ Have each student take five shots from the “foul line.”
  ➢ Have the recorder write the student’s name in the appropriate box and place a check mark in the box for each made shot. Ex.: If shot one is made, place a checkmark in column 1. If the shot is missed, leave the box blank.
  ➢ Record how many shots made in the # Made column.
  ➢ After all students have attempted five shots, record how many shots the group made and how many shots the group attempted.
  ➢ Have each group’s recorder write down his/her group’s data on the Class Data Table (preferably on an overhead transparency).
  ➢ Have each group add up the totals for the class and the teacher will record the accurate totals on the transparency.

Option 2:

Collect your data by taking the class to the gym or an outdoor basketball area and have each student attempt real foul shots. This would require prearranging access to facilities and equipment ahead of time.

Possible discussion topics when comparing theoretical and experimental probability:
  ❖ How would changing the diameter of the basket opening affect the outcomes.
  ❖ How would changing the “ball” affect the outcomes.
  ❖ How would changing the distance of the shot affect the outcomes.
  ❖ What other factors might change the outcome?
Classroom Worksheet
Theoretical vs. Experimental Probability

1.) Write in words or in formula format the formula for theoretical probability.

2.) Write in words or in formula format the formula for experimental probability.

3.) What is the basic difference between theoretical probability and experimental probability?

4.) State the theoretical probability P (making a free throw). Explain your reasoning.

5.) Follow the teacher instructions for collecting the experimental data for your group. Use the attached worksheet to record and total your group’s data. Be prepared to combine your data with the rest of the class.

6.) What is the experimental probability of making the next free throw based on your group’s data? Based on the class’s data?

7.) How did the experimental probabilities calculated in Step 6 compare to the experimental probability you recorded in Step 4? Explain why they are different or the same. List at least two factors that might make the experimental probability either higher or lower than the theoretical probability?

8.) Write a compare/contrast paragraph on the two types of probability. Include at least two similarities and two differences in your paragraph.
Classroom Worksheet (KEY)
Theoretical vs. Experimental Probability

1.) Write in words or in formula format the formula for theoretical probability.

\[ P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} \]

2.) Write in words or in formula format the formula for experimental probability.

\[ P(\text{event}) = \frac{\text{number of times an event happens}}{\text{number of times the experiment is done}} \]

3.) What is the basic difference between theoretical probability and experimental probability?

Theoretical probability is based on what might happen while experimental probability is based on what has happened in an experiment.

4.) State the theoretical probability \( P \) (making a free throw). Explain your reasoning.

\[ \frac{1}{2} \quad \text{A free throw attempt only has two possible outcomes, you either make it or you don’t. Therefore, there is one favorable outcome, making it, and two possible outcomes.} \]

5.) Follow the teacher instructions for collecting the experimental data for your group. Use the attached worksheet to record and total your group’s data. Be prepared to combine your data with the rest of the class.

6.) What is the experimental probability of making the next free throw based on your group’s data? Based on the class’s data?

This will vary by group and for the overall class depending on the data collected.

7.) How did the experimental probabilities calculated in Step 6 compare to the experimental probability you recorded in Step 4? Explain why they are different or the same. List at least two factors that might make the experimental probability either higher or lower than the theoretical probability?

Students should say the two probabilities were close or they differed in some way. The answer should state which probability was higher. Students should state why they think the experimental probability was higher or lower. Examples: “The experimental probability was higher because the basket’s opening was so big.” “The experimental probability was lower because we were shooting from 25 feet away.”
8.) Write a compare/contrast paragraph on the two types of probability. Include at least two similarities and two differences in your paragraph.

This paragraph should include an introductory sentence, three or more supporting sentences, and a concluding sentence. The support sentences must include data from the experiment. The types of information from the experiment might be numerical or factual, i.e. “None of the people in my group have ever played basketball before.” or “The experimental probability for the class was 60% while the theoretical probability was 50%, a difference of 10%.”
### MODULE 1 Data Table
Free Throws
(6 Players)

<table>
<thead>
<tr>
<th>Shooter</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th># Made</th>
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</table>

**Total Made**

**Total Attempts**

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### MODULE 1 Data Table
Free Throws
(6 Players)

<table>
<thead>
<tr>
<th>Shooter</th>
<th>1</th>
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</tbody>
</table>

**Total Made**

**Total Attempts**
### MODULE 1 Class Data Table
#### Free Throws

<table>
<thead>
<tr>
<th>Group #</th>
<th>Shots Made</th>
<th>Shots Tried</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>2</td>
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<tr>
<td>Class</td>
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</tbody>
</table>

\[
P(\text{Made free throws}) = \frac{\text{Number of free throws made}}{\text{Number of free throws tried}}
\]
Module 1 Quiz Questions

1.) What is the basic difference between theoretical probability and experimental probability?

2.) The theoretical probability of making a shot in basketball is $\frac{1}{2}$ or 50%. In other words, the shot will either go in or it won’t. As compared to this, what will the experimental probability of making a shot in basketball be if we shoot only lay-ups? What if we only shoot 3-point shots (from about 20 feet away from the basket)?

3.) Three step question:
   - Design a situation in which you can identify the theoretical probability.
   - What is the theoretical probability in your example?
   - Explain the steps you would use if you were to find the experimental probability for your example.

4.) In your example in Question 3, describe two factors that would affect your experimental probability, either making the percentage higher or lower than the theoretical probability.
Module 1 Quiz Questions

1.) What is the basic difference between theoretical probability and experimental probability?

In experimental probability you perform an experiment or simulate results to determine probability while in theoretical probability you base it on possible outcomes without ever actually performing a task.

2.) The theoretical probability of making a shot in basketball is ½ or 50%. In other words, the shot will either go in or it won’t. As compared to this, what will the experimental probability of making a shot in basketball be if we shoot only lay-ups? What if we only shoot 3-point shots (from about 20 feet away from the basket)?

It will probably be much higher for lay-ups and lower for 3-point shots.

3.) Three step question:
   • Design a situation in which you can identify the theoretical probability.
   • What is the theoretical probability in your example?
   • Explain the steps you would use if you were to find the experimental probability for your example.

Student responses will vary. Grading might be based on creativity and completeness.

4.) In your example in Question 3, describe two factors that would affect your experimental probability, either making the percentage higher or lower than the theoretical probability.

Again, student responses will vary. Stress the need for at least two factors and possibly give extra credit for others.
Simulation and Probability
MODULE 2
Zodiac Simulation Module

Objective:

Students will be able to:

- gather data for a Probability Simulation.
- analyze data to determine the Probability percentage.
- compute and interpret the expected value of random variables.
- understand the concepts of sample space and probability distribution.
- conduct a simulation inferring that probability on a Random Population.

Materials:

- Signs of Zodiac Transparency
- Zodiac Data Collection Worksheet
- Zodiac Data Collection Transparency
- Birth Date Data Collection Worksheet
- Spinner Black Line Master
- Heavy Card Stock for Spinner
- Paperclips
- Pencils
- Thumbtacks/push pins (optional)
- Heavy card stock/Styrofoam board (optional)

Lesson:

Preparatory Activity:

Students should complete Birth Date Collection Worksheet for homework the night BEFORE this activity.

Procedures:

1. Students will collect birth date data on his/her family. Each student should get the birth date(s) for each member of his/her immediate family.

2. The next day in class, the students will assign the appropriate Zodiac symbol to each family member. A Signs of Zodiac master has been included.
3. Then the class completes the Zodiac Data Collection Worksheet while the teacher completes the Zodiac Data Collection Transparency as a model.

4. Based on the class’s data, have the students determine probability percentage for each Zodiac Sign. Use the Zodiac Data Collection Worksheet.

5. Based upon that probability, have the student (or groups) build a spinner on Black Line Master. (The spinner should be a pie chart graphically displaying the resultant probability.)

Note: The black line master has been divided into 5% increments for your students. This should assist in the construction of the spinner. Once the backing (pie chart) has been completed, have a student hold an open paper clip on the center of the spinner with a sharpened pencil. It is recommended that you glue the spinner Black Line Master sheet to a piece of cardboard or Styrofoam-board, then attach the paperclip with a thumbtack or push pin.

6. Using the spinner, determine the Zodiac sign for Ten (10) trials of Five (5) random people. Have students complete the Zodiac Data Collection Worksheet and answer the questions.
BIRTH DATE COLLECTION HOMEWORK

Instructions: Collect your immediate family’s birthdays in the space provided. If you can not get that family member’s birth date, just put N/A. We will fill in the Zodiac signs tomorrow.

My Birthday:___________________
My Zodiac Sign:__________________

My Father’s/ Stepfather’s/ Guardian’s Birthday:___________________
My Father’s/ Stepfather’s/ Guardian’s Zodiac Sign:___________________

My Mother’s/ Stepmother’s/ Guardian’s Birthday:__________________
My Mother’s/ Stepmother’s/ Guardian’s Zodiac Sign:_________________

My Brother’s/ Sister’s Birthday:________________
My Brother’s/ Sister’s Zodiac Sign:________________

My Brother’s/ Sister’s Birthday:________________
My Brother’s/ Sister’s Zodiac Sign:________________

My Brother’s/ Sister’s Birthday:________________
My Brother’s/ Sister’s Zodiac Sign:________________

My Brother’s/ Sister’s Birthday:________________
My Brother’s/ Sister’s Zodiac Sign:________________
# Signs of the Zodiac

<table>
<thead>
<tr>
<th>DATES</th>
<th>ZODIAC SIGN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan 21 – Feb 18</td>
<td>Aquarius</td>
</tr>
<tr>
<td>Feb 19 – Mar 20</td>
<td>Pisces</td>
</tr>
<tr>
<td>Mar 21 – Apr 20</td>
<td>Aries</td>
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<tr>
<td>Apr 21 – May 20</td>
<td>Taurus</td>
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<td>May 21 – Jun 21</td>
<td>Gemini</td>
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<td>Jun 22 – Jul 22</td>
<td>Cancer</td>
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<td>Jul 23 – Aug 22</td>
<td>Leo</td>
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<td>Aug 23 – Sept 22</td>
<td>Virgo</td>
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<td>Sept 23 – Oct 23</td>
<td>Libra</td>
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<td>Oct 24 – Nov 21</td>
<td>Scorpio</td>
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<tr>
<td>Nov 22 – Dec 22</td>
<td>Sagittarius</td>
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<tr>
<td>Dec 23 – Jan 20</td>
<td>Capricorn</td>
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</tbody>
</table>
**Zodiac Data Collection Worksheet**

**Instructions:** Record class data for each Zodiac Sign from Birth Date Collection Worksheet. Based on those results, determine the percentage of the data that represents each of the 12 Zodiac Signs. Use the attached worksheet for calculations.

<table>
<thead>
<tr>
<th>Zodiac Sign</th>
<th>Dates</th>
<th>Class Results</th>
<th>Total</th>
<th>Percentage</th>
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</thead>
<tbody>
<tr>
<td>Aquarius</td>
<td>Jan 21 - Feb 18</td>
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<tr>
<td>Pisces</td>
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<td>Aries</td>
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<td>Taurus</td>
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<td>Gemini</td>
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<td>Capricorn</td>
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<tr>
<td>Total</td>
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<td>100%</td>
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</table>
Percentage Calculations

Example: \[
\frac{\# \text{ of Aries data}}{\# \text{ of birth dates collected}} = \text{PERCENTAGE OF ARIES in class}
\]

Following the above example, use the space provided to calculate the percentage of each zodiac sign and record those results in your chart.

Aquarius: ______________________ =

Pisces: ______________________ =

Aries: ______________________ =

Taurus: ______________________ =

Gemini: ______________________ =

Cancer: ______________________ =

Leo: ______________________ =

Virgo: ______________________ =

Libra: ______________________ =

Scorpio: ______________________ =

Capricorn: ______________________ =

\[\text{100}\%\]
Simulation

Using the probability percentages in your chart, answer the following questions.

What are the possible outcomes of the simulation?

What are the probability percentages for these outcomes?

Construct a spinner. Graph each outcome based on their respective probability percentages, just as you would on a pie chart. For example, if Aries is 23 percent probable then 23 percent of the spinner should represented Aries.

Theoretical Probability \[=\ \frac{\text{# of favorable outcomes}}{\text{# of possible outcomes}}\]
Use your spinner to conduct ten (10) trials to determine the zodiac signs of five (5) random people. Record your results below.

<table>
<thead>
<tr>
<th>Trial #</th>
<th>Person 1</th>
<th>Person 2</th>
<th>Person 3</th>
<th>Person 4</th>
<th>Person 5</th>
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<tbody>
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</table>

In any of the trials above, did one Zodiac sign have a frequency greater than one? In which trial did it occur? What sign was it? What is the significance of this? What occupation would find this type of information interesting, i.e., birth month?

Out of your ten trials, what was the most frequently occurring Zodiac sign? Which occurred the least?
Note: The Black Line Master has been divided into 5% increments for your students. This should assist in the construction of the spinner. Once the backing (pie chart) has been completed, have a student hold an open paper clip on the center of the spinner with a sharpened pencil. It is recommended that you glue the spinner Black Line Master sheet to a piece of cardboard or Styrofoam-board, then attach the paperclip with a thumbtack or push pin.
MODULE 2 – Assessment

A person has a 32% probability of being born as a Male.

1. What would be the probability percentage of being born as a female?

2. Explain how you would conduct a simulation, using a spinner, with these probabilities.
MODULE 2 – Assessment

A person has a 32% probability of being born as a male.

1. What would be the probability percentage of being born as a female?
   
   ➢ 68% probability for being born as a female

2. Explain how you would conduct a simulation, using a spinner, with these probabilities.

Answer Rubric

4 points - Student explains that the simulation has more than eight trials with multiple random candidates, the spinner is divided according to the outcome percentages.

3 points - Student explains that the simulation has more than five trials with multiple random candidates, the spinner is divided into equal portions for each outcome.

2 points - Student explains that the simulation has more than one trial with multiple random candidates, the spinner is divided.

1 point - Student explains that the simulation has one trial with candidates, the spinner doesn’t exist.
Simulation and Probability
MODULE 3
Generating Random Numbers Using Your Graphing Calculator

Objectives:

Students will be able to:

• determine the type of learning style they have.
• set up and conduct a simulation.
• produce random integers using the graphing calculator.
• analyze data and find probabilities.

Materials:

• Graphing calculator, TI-83 or TI-83 Plus
• Lesson worksheet
  1. Graphing Calculator Procedure Guide
  2. Student Simulation Worksheet
  3. Learning Style Inventory Worksheet

Lesson:

1. Introduction. Ask student to turn to the student next to them. Have the students orally exchange phone or student ID numbers and give them one minute to be able to recite it back without reading it from a paper. Ask students to share the different methods they used to learn the number. Introduce the concept of learning styles.

   If you want more information on learning styles go to http://www.visionstream.com/mx/clc/lstyles.htm. This will also give them studying tips that are conducive to their learning style, or you can have the students research this information.

2. Learning Style Inventory. Have the students complete the learning style inventory worksheet to determine which learning style they are. You might want to point out that a student can be split between two or three learning styles, but a dominant learning style should be present and that is what he or she will use in class today.

3. Conduct the simulation by following the direction on the Teacher’s Key to the Student Simulation Worksheet. Space for 50 trials is provided. Teacher should decide how many trials to have students complete.

4. Discuss and compare the answers on the Student Simulation Worksheet
What Is Your Learning Style?

Read each sentence carefully and think about how it applies to you. On each question, circle the number that best describes your reaction.

**5-Almost always  4-Often  3-Sometimes  2-Rarely  1-Almost Never**

**Part I.**

1. I remember what people look like better than what they say.
   5  4  3  2  1
2. I would rather read instructions than have them read to me.
   5  4  3  2  1
3. I can picture my notes in my head when I take a test.
   5  4  3  2  1
4. I write things down to remember them better.
   5  4  3  2  1
5. It is hard to concentrate with a lot of background noise.
   5  4  3  2  1
6. I cannot remember a new website address long enough to use it later.
   5  4  3  2  1
7. I prefer to look at the person who is speaking.
   5  4  3  2  1
8. I prefer to get work done in a quiet place.
   5  4  3  2  1
9. When I’m lost I picture the map in my head to determine which way to go.
   5  4  3  2  1
10. When trying to remember new information I form a picture in my mind.
    5  4  3  2  1

**Part II.**

1. I talk to myself when doing a difficult task.
   5  4  3  2  1
2. I would rather hear instructions than read them.
   5  4  3  2  1
3. I prefer to study by telling somebody about what I know.
   5  4  3  2  1
4. I can still concentrate on a speaker even if I’m looking at something else.
   5  4  3  2  1
5. I can easily learn the words of a song by hearing it a few times.
   5 4 3 2 1
6. When I talk about my ideas I understand them better.
   5 4 3 2 1
7. I prefer to first hear a new song on the radio rather than watching the video.
   5 4 3 2 1
8. I’m better at remembering names than faces.
   5 4 3 2 1
9. When learning something new I would rather hear it on a tape rather than read about it.
   5 4 3 2 1
10. I would prefer to tell about myself rather than write it.
    5 4 3 2 1

Part III.

1. When putting together something new, instead of reading the directions I just start working.
   5 4 3 2 1
2. When I study I move around a lot and don’t like to use a desk.
   5 4 3 2 1
3. I use my hands to describe things because I often can’t find the right word.
   5 4 3 2 1
4. When I start a book I like to look at the ending first.
   5 4 3 2 1
5. I may be disorganized, but I know where things are.
   5 4 3 2 1
6. I use my fingers in math and move my lips when I read.
   5 4 3 2 1
7. I don’t like to review or proofread what I write.
   5 4 3 2 1
8. It does not bother me to have the TV or music on while I study.
   5 4 3 2 1
9. I usually do not use my notes to study.
   5 4 3 2 1
10. Given the choice, I would rather build a model plane, than read or write about it.
    5 4 3 2 1

Add up your scores and record below

<table>
<thead>
<tr>
<th>Part I-Visual</th>
<th>Part II-Auditory</th>
<th>Part III-Kinesthetic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Learn by Seeing</td>
<td>Learn by Hearing</td>
<td>Learn by Doing</td>
</tr>
</tbody>
</table>

Total Score  __________  __________  __________

The highest score represents your dominant type of learning style.
Graphing Calculator Procedure Guide

1) Clear home screen

2) Set the Seed on your calculator. This is important so that each student in the class will generate a different list of numbers.
   (a) Choose a number between 1 and 10,000. A good idea is to use the last 4 digits of your phone number. This will ensure that everybody’s seed number is different.
   (b) Hit ENTER
   (c) Store the number, hit the STO→ key
   (d) Go to the math menu, hit the MATH key
   (e) Arrow over to the PRB menu
   (f) Since rand is already highlighted, hit ENTER
   (g) Hit ENTER again, your number is now stored; it should be displayed on the screen.

3) Generate 50 trials of 5 random integers from 0 to 9
   (a) Go to the math menu, hit the MATH key
   (b) Arrow over to PRB
   (c) Chose #5 randInt (and press ENTER. This key tells the calculator that you want random integers
   (d) Type 0 and a comma,. This determines the beginning of the range you want.
   (e) Type 9 and,. This determines the end number in your range.
   (f) Type 5 and ). This determines how many numbers you want in each trial.
   (g) Hit ENTER. Each time you hit ENTER you will generate another set of numbers.
Student Simulation Worksheet

1. State the Problem
   Our class is being put into groups of five. What is the probability that there will be at least 3 of the same learning style in any group?

2. State the outcomes and assumptions

3. Choose method of generating data

4. Define experiment/trials

5. Document results

<table>
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<tr>
<th>Trial #</th>
<th>Trial Outcome</th>
<th>3 of 5 Same</th>
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</table>
6. Analyze data
a) What is the probability that any group has at least three students that have the same learning style?
b) What is the probability that there are exactly four students with the same learning style in a group?

c) What is the probability that there are at least two other students with the same learning style as you?

d) On the first Monday of each month there is a staff meeting for all teachers. By conducting a simulation of random numbers and doing ten trials using the same integer assignment for learning styles from the problem above, what is the probability that the fourth teacher to walk into the room is a visual learner?
Student Simulation Worksheet

1. State the Problem

Our class is being put into groups of five. What is the probability that there will be at least 3 of the same learning style in any group?

2. State the outcomes and assumptions

We assume that each student reflects only one dominant learning style, and the groups of students will be chosen at random.

3. Choose method of generating data

We will use the calculator to generate a list of random integers from 0 to 9. We will then assign a percentage of the number to each learning style based on the information collected from the class.

4. Define experiment/trials

A trial will consist of 5 numbers representing 5 students in each group. We will do 50 trials. After the students determine their learning style and you have found the class percentages, assign a learning style to each random integer. For example, if the class had 30% visual, 40% auditory and 30% kinesthetic (round to the nearest ten percent), then 0, 1, and 2 will represent visual, 3, 4, 5, and 6 will represent auditory, and 7, 8, and 9 will represent kinesthetic.

5. Document results

See calculator worksheet to product the random integers.

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6. Analyze data

a) What is the probability that any group has at least three students that have the same learning style?

\[
P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}
\]

\[
P(3 \text{STYLES}) = \frac{36}{50} = 72\%
\]
b) What is the probability that there are exactly four students with the same learning styles in a group?

\[ P(\text{STYLES}) = \frac{8}{50} = 16\% \]

c) What is the probability that there are at least two other students with the same learning style as you?

d) On the first Monday of each month there is a staff meeting for all teachers. By conducting a simulation of random numbers and doing ten trials using the same integer assignment for learning styles from the problem above, what is the probability that the fourth teacher to walk into the room is a visual learner?

\[ 1. \ 1 \ 1 \ 9 \ 3 \ \text{auditory} \]
\[ 2. \ 3 \ 8 \ 6 \ 4 \ \text{auditory} \]
\[ 3. \ 7 \ 0 \ 8 \ 7 \ \text{kinesthetic} \]
\[ 4. \ 4 \ 2 \ 7 \ 7 \ \text{kinesthetic} \]
\[ 5. \ 2 \ 3 \ 6 \ 1 \ \text{visual} \]
\[ 6. \ 1 \ 8 \ 7 \ 5 \ \text{auditory} \]
\[ 7. \ 4 \ 2 \ 1 \ 1 \ \text{visual} \]
\[ 8. \ 9 \ 1 \ 1 \ 8 \ \text{kinesthetic} \]
\[ 9. \ 0 \ 4 \ 3 \ 6 \ \text{auditory} \]
\[ 10. \ 6 \ 9 \ 5 \ 4 \ \text{auditory} \]

\[ P(\text{visual}) = \frac{2}{10} = 20\% \]

**Extension:** Have the students research the learning styles of their teachers.

- How does the learning style of the teacher reflect their lesson presentations?
- Is there a correlation between subject taught and learning style?
- How should you as a student adapt to the learning styles of your teachers? Should you have to?
- How will knowing your learning style be helpful in college?
- How can you strengthen your weaker learning styles
- What study habits should you implement to help you be most successful in school?
Assessment Questions from Module 3

1. Tina and Torie found out that 70% of their school population have brown eyes, 20% have blue eyes, and 10% have green eyes. They want to set up a simulation that represents this data. They are using 0-9 as their integers. How would you distribute the digits 0-9 to accurately represent the three categories?

2. The calculator screens below show a simulation that Troy did by rolling a six-sided die four times.

```
randInt(1,6,4)
(6 1 4 6)
(6 4 1 43)
(6 4 5 13)
(1 6 3 23)
(5 8 5 1)
```

```
(5 5 2 5)
(5 6 2 1)
(4 6 3 22)
(6 2 3 63)
(5 1 3 39)
(4 9 3 33)
```

a) What does the numbers 1, 6, and 4 represent on the first line of the first screen?

b) How many trials did Troy run?

c) Using Troy’s simulation, what is the probability that exactly one two is rolled in a trial?

d) Using Troy’s simulation what is the probability that a 4 or greater is rolled in a trial?

e) Troy is taking a trip to Las Vegas, and he has $500.00 to spend. He is playing a game that for each bet he has four chances to roll the number of his choice. If his number is rolled he will double his money, and if his number is not rolled he will lose the money he bet. Troy always bets on three. Using the above data as his previous turns, how much should he bet on his next turn? Support your answer with the data above.
1. Tina and Torie found out that 70% of their school population have brown eyes, 20% have blue eyes, and 10% have green eyes. They want to set up a simulation that represents this data. They are using 0-9 as their integers. How would you distribute the digits 0-9 to accurately represent the three categories?

- 0, 1, 2, 3, 4, 5, 6, will represent brown eyes
- 7, 8 will represent blue eyes
- 9 will represent brown eyes

2. The calculator screens below show a simulation that Troy did by rolling a six-sided die four times.

![Calculator Screenshots]

a) What does the numbers 1, 6, and 4 represent on the first line of the first screen?
1 is the smallest number on the die, 6 is the largest number on the die, and 4 is the number of rolls each trial

b) How many trials did Troy run?
13

c) Using Troy’s simulation, what is the probability that exactly one two is rolled in a trial?

\[ P(\text{exactly one two}) = \frac{6}{13} = 46\% \]

d) Using Troy’s simulation what is the probability that a 4 or greater is rolled in a trail?

\[ P(4 \text{ or greater}) = \frac{12}{13} = 92.3\% \]

e) Troy is taking a trip to Las Vegas, and he has $500.00 to spend. He is playing a game that for each bet he has four chances to roll the number of his choice. If his number is rolled he will double his money, and if his number is not rolled he will lose the money he bet. Troy always bets on three. Using the above data as his previous turns, how much should he bet on his next turn? Support your answer with the data above.

Based on previous turns he has a 53.8% chance of rolling a three. Make sure the students write complete sentences and completely support their answer.
Unit Project: Lottery – Play It or Not?

Objective: The student will design and implement an investigation of the possible results of playing the Pick 3 and Pick 4 lottery games. The student should be able to state the theoretical and experimental probability of each game as a result of the investigation. Following the investigation, the student will write a one-page analysis of the investigation and draw conclusions based on the data obtained. Conclusions must be mathematically based while emotional factors could be used to explain why so many people do in fact play the lottery.

Guidelines: The student will be expected to:

- create organized data collection forms for both the Pick 3 and the Pick 4 lottery games.
- decide if the class will be included in data collection or if he/she will perform data collection by random number simulation.
- produce “daily” lottery results using one of the simulation techniques covered in this unit.
- write a paper, first analyzing the data collected, then drawing valid, well supported conclusion.

Suggested Student Instructions:

1.) Decide whether you will perform all data choices yourself or include the class in some manner.
2.) Create data collection forms for both games. A separate form would probably have to be designed if you wish to collect lottery numbers from each class member on a daily basis.
3.) Choose a method for “pulling” a daily lottery number. (Calculator, spinner)
4.) Simulate 50 days for each game with at least 100 players a day. Change your seed before drawing the lottery number and between each day.
5.) Calculate and record a theoretical probability and an experimental probability for each game.
6.) Write your one-page (five paragraph minimum) paper analyzing the results of your data and drawing conclusions from your results. Make sure that you support your conclusion using the results of your data. Emotional factors should only be used as a secondary motivation for either playing or not playing the lottery.
# UNIT PROJECT: Lottery Play It or Not?

## Grading Guide

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<td>Fair</td>
<td>Poor</td>
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<td>1 of 3</td>
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<tr>
<td>Presentation</td>
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<td>75% Accurate</td>
<td>50% Accurate</td>
<td>25% or less Accurate</td>
</tr>
<tr>
<td><strong>Writing Component</strong></td>
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<tr>
<td>Paragraph Structure</td>
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<tr>
<td>Sentences Per Paragraph</td>
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<td>Grammar and Spelling</td>
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<tr>
<td>Conclusion</td>
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<td>60% supported by Mathematical Data</td>
<td>40% supported by Mathematical Data</td>
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