Title: Super Size It!

Brief Overview:

The Geometer's Sketchpad is a powerful tool for solving problems that calls for maximizing and minimizing a function.

In this unit we will use the Geometer's Sketchpad along with the TI-84 graphing calculator to solve a real world problem.

NCTM Content Standard/National Science Education Standard:

- Use symbolic algebra to represent and explain mathematical relationship
- Use mathematical models to represent and understand quantitative relationships
- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.

Grade/Level:

- Grade 11 – 12 / Algebra II and Pre Calculus.

Duration/Length:

Three 45-minute lessons

Student Outcomes:

Students will:

- Determine the dimensions of a cone with maximum volume that can be made from a circle with a fixed radius.

Materials and Resources:

- Geometer's Sketchpad
- Graphing Calculator
- Student copies of Resource Sheets
- Markers
- Construction paper
- Scissors
- Compasses
- Tape

Development/Procedures:

Lesson 1

Preassessment / Launch – Warm Up. This activity is to ensure that each student knows and can apply the Pythagorean Theorem. It also gives the students a chance to explore the relationship between the base and height of a right triangle when the length of the hypotenuse is constant.
(See WorkSheet 1)

Teacher Facilitation / Student Application – Students are given a problem in which they are asked to maximize the volume of a cone. Lesson 1 is the first step in *trying* to solve the problem. The class is to be divided into groups of 3 to 4 students. Each group is given materials; construction paper, rulers with a cm scale, tape, compasses, scissors and WorkSheet 2, in which to construct a cone. Each cone should have a *different* radius.

Embedded Assessment – Teach by walking around, assessing students' skills with measuring the radius and height of a cone. Use this time to casually check the students' math computation in determining the volume of the cone. Assess how well they are working in their groups.

Reteaching/Extension.

- For those who have not completely understood the lesson, review the formula for finding the volume of a cone and demonstrate how to find the radius of the cone (measure the diameter and divide by 2.) and height of the cone.

- As an extension, during the last 5 or 10 minutes of class have a whole class discussion. Discuss questions on the worksheet and ask some leading questions such as…
  As the radius increases does the volume of the cone increase?
  What is the largest possible radius of the cone? What is the volume at this point?
  What is the smallest possible radius of the cone? What is the volume at this point?
  Based on your answers to the last two questions, can the relationship of the volume and radius of a cone be modeled by a parabola?

Lesson 2

Preassessment / Launch – Warm Up. This activity is to ensure that each student knows and can apply the formula for finding the volume of a cone and understands the relationship between the radius and height of a cone. (See Worksheet 3)

Teacher Facilitation / Student Application – The teacher should prepare the Geometer's Sketchpad file (*KrustyKones*) in advance (steps 1 through 12). This lesson can be teacher centered, done individually or in pairs. If done individually or in pairs the teacher must save the file to the student's computers. See directions, Cone Problem Simulation using the Geometer's Sketchpad. Students will need a copy of the Krusty Kones file, a computer containing the Geometer' Sketchpad software, graphing calculator and WorkSheet 4.

Embedded Assessment – Assess by walking around listening to students' conversations. Make sure that students carefully follow the directions on the worksheet. When existing the program it is VERY IMPORTANT That Student **DO NOT SAVE CHANGES** (They will need the origin file to do WorkSheet 6)

Reteaching/Extension

- As an extension, during the last 5 or 10 minutes of class have a whole class discussion. Discuss questions such as…
  What is the maximum volume of a cone construct from a circle of radius 10 cm?
  How was the Geometer's Sketchpad helpful in helping you determine the maximum volume of the cone?
  Can you find the maximum volume of a cone without using the Geometer's Sketchpad? If so, how? If not, why not?
  Did you enjoy using the Geometer's Sketchpad?

Lesson 3

Preassessment / Launch – Warm Up. This activity is to ensure that each student is able:

1. State and apply the formulas for finding the volume of spheres and hemispheres,
2. Write an equation for the volume of a cone in terms of the radius (given slant height) and
3. Find the maximum volume of a cone using the equation of the volume and the graphing calculator (review of lesson 2). (See Worksheet 5)

Teacher Facilitation / Student Application – Student will need the *KrustyKone*s file to complete this lesson. This lesson can be teacher centered, done individually or in pairs. If done individually or in pairs the teacher must save the file to the student's computers. Students will need a copy of the *KrustyKones* file, a computer containing the Geometer's Sketchpad software, graphing calculator and WorkSheet 6.

Embedded Assessment – Assess by walking around listening to students' conversations. Make sure that students carefully follow the directions on the worksheet. When existing the program it is VERY IMPORTANT That Student **DO NOT SAVE CHANGES.** (You may want to use this file again.)

Reteaching/Extension

As an extension, during the last 5 or 10 minutes of class have a whole class discussion. Allow groups to discuss their finding and how they arrived at their solution.

Discuss questions such as…
Did you think that the maximum volume would be obtained with the same dimension as Lisa's cone? Why or Why not?
Did you think the maximum volume would be obtained when the radius was 10 cm?

Summative Assessment: Each student will be given the radii of six circles. He/she must find the radii of the cones with maximum volume that can be formed form the circle of the given radii. Each student will compile his/her data and determine an equation that relates the maximum volume of a cone to the radius of the circle it was formed from. (See WorkSheet 7)

Author:

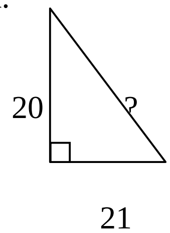
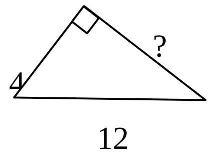
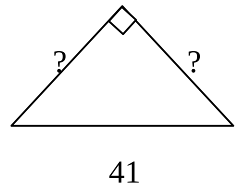
Joanne Nelson
F.W. Ballou High School
Washington D.C.

Name_______________________________________

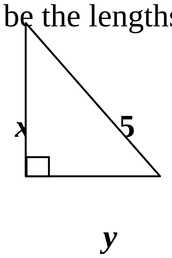
Date ______________________________ Period ____

SUPER SIZE IT
WORKSHEET 1

1. Use the Pythagorean Theorem to determine the measure of the lengths of the missing sides of the following right triangles.

a.	b.	c.
		

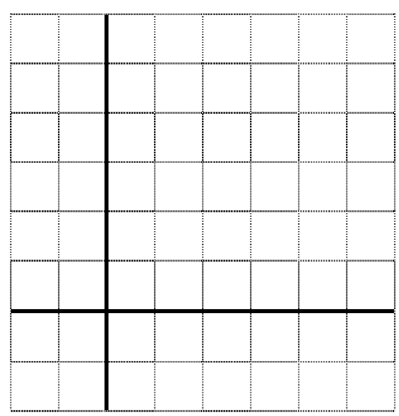
2. Determine at least five pairs of values for ***x*** and ***y*** that can be the lengths of the legs of the right triangle below.



x						
y						

3. Using the data in the table in question 2.

- Plot the data in your table in question 2.
- Draw a smooth curve through the points.
- Describe the shape of the curve formed.



____

____

____

____

Name ____

Date ______________________________ Period ____

SUPER SIZE IT
WORKSHEET 2

"SUPER SIZE IT!"

Maximizing the volume of a cone, part1

Krusty the Clown decided to give up making Krusty Burger and decided to open up an ice cream parlor called, "Krusty Kones". At Krusty Kones, for $2.25 you can make your own ice cream cone.

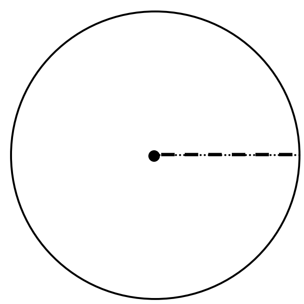
Krusty's waffle iron makes circular waffles with a radius of 10 cm. You can twist each into cones by first making one cut along a radius.

Lisa Simpson loves ice cream and is especially fond of waffle cones. She likes cones which hold as much ice cream as possible, but without ice cream sticking over the edge of the cone (too messy). Lisa is very frugal and wants to get as much ice cream as she can for her money. Help Lisa by determining the dimensions (radius and height) of the cone (formed from a circular waffle of radius 10 cm) that holds the most ice cream (flush with the top)?

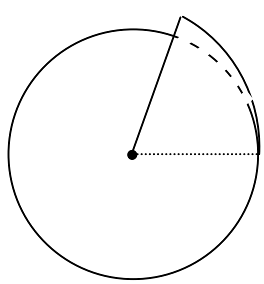
Each student will need:

8.5" × 11" piece of construction paper	Ruler (with centimeter scale)	Pair of Scissors
tape	Compass	Resource Sheet 2

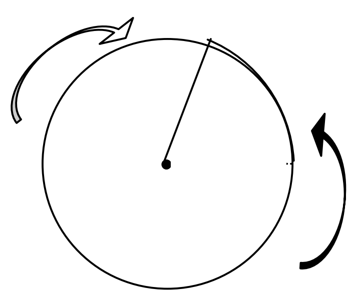
Each team member will cut a circle of radius 10 cm from the construction paper and make a cut along a radius.

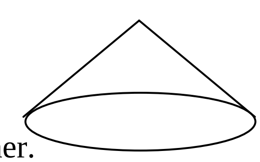


Overlap the cut edges of the radius.



Slide the cut edges of the radius away from each other and along the outer edge of the circle, forming a cone. Each team member should form a cone of different dimensions.



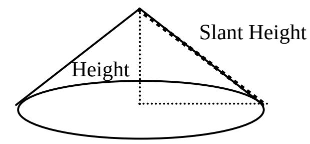


Tape the edges to keep the cone together.

Write your name inside your cone.

Each member should determine the measurement of the following: slant height, height and radius. Record the data of each team member in the table below.

Slant Height (cm)	Height (cm)	Radius (cm)



1. What measurement is the same for all the cones? ____________________________

2. Explain why this is true.

   ____

   ____

3. Which cone do you think has the largest volume? ____________________

   Why?____

   ____

   ____

4. If ***r*** *represents the radius of the cone*, ***h*** *represents the height of the cone and* ***s*** *represents the slant height of the cone*, write an equation that represents the relationship between *the radius, the height and the slant height* of a cone.

   ____

5. If ***r*** *represents the radius of the cone*, ***h*** *represents the height of the cone and* ***V*** *represents the volume of the cone*, write an equation that represents the relationship between *the radius, the height and the volume* of a cone.

   ____

6. Calculate the volume of each cone. Show how you arrived at your answer.

Volume of Cone 1	Volume of Cone 2
Volume of Cone 3	Volume of Cone 4

7. Write the volume, the length of the radius, and the length of the height on each cone.

8. Arrange the cones of everyone in your class according to their volume.

9. What is the radius and height of the cone with the largest volume? Radius: __________ Height: ____________

10. What is the volume of this cone? ________________

11. Do you think you can find a cone with a larger volume (than the volume of the cone in exercise 10) that can be constructed from a circle of radius 10 cm? Explain.

____

____

____

____

Name ____

Date ______________________________ Period ____

SUPER SIZE IT
WORKSHEET 3

1. If ***r*** *represents the radius of the cone,* ***h*** *represents the height of the cone and* ***s*** *represents the slant height of the cone,* write an equation that represents the relationship between *the radius, the height and the slant height* of a cone.

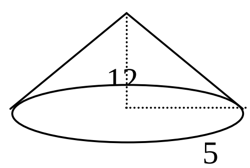
____

2. If ***r*** *represents the radius of the cone,* ***h*** *represents the height of the cone and* ***V*** *represents the volume of the cone,* write an equation that represents the relationship between *the radius, the height and the volume* of a cone.

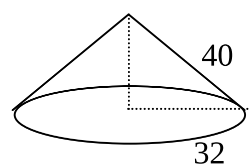
____

3. Calculate the volume of the following cones.

a.



b.



Name ____

Date ______________________________ Period ____

SUPER SIZE IT
WORKSHEET 4

"SUPER SIZE IT!"

Maximizing the volume of a cone, part 2

Using The Geometer's Sketchpad to Solve The Cone Problem:

Start the Geometer's Sketchpad program. Open the krustykones document. The diagram on the screen represents the top and side views of your cone. AB represents the radius of the base of the cone. EF represents the slant height of the cone and EG represents the height of the cone.
Click on the point not labeled on the circle. Slide the point toward point A. The circle changes in size. The dimensions of the cone also change. You may have to move point B on the circle to keep the cone upright.

1. Click the arrow on the tool menu. Click on **MEASURE**→***Calculate.*** Click on the left parenthesis. Click on **VALUE** →π. Click the right parenthesis. Click on the multiplication symbol. Click on the measure of AB. Click on the caret symbol, $\boxed{\text{L}}$ Click on 2. Click on the measure of EG. Click on **OK**.

   The equation $(\frac{\pi}{3})\cdot AB^2\cdot EG = 92.37cm^2$ appears on the screen. (your measure may be different)

   What does this equation represents? ____________________________Click on the measure of AB, the measure of EG and $(\frac{\pi}{3})\cdot AB^2\cdot EG = 92.37cm^2$. Click **GRAPH**→***Tabulate***. A table should appear on the screen. The first row of the table should have the values on the desktop. To get a second row click the table twice.

2. Select the point (not labeled) on circle A. Slide the point. The circle should change in size. The corresponding values for on the measure of AB, the measure of EG and volume should also change. Click the table twice these values now appear in the table and a third row appears. Continue this process until you get 20 data points.

Radius	Height	Volume

Radius	Height	Volume

Radius	Height	Volume

Radius	Height	Volume

3. Click **GRAPH→*Plot Table Data…*.** *Click on* **y** → $(\frac{\pi}{3})\cdot AB^2\cdot EG$ **.** Click **OK**. Since the $(\frac{\pi}{3})\cdot AB^2\cdot EG$ values are so large you must adjust the y-scale. Click **GRAPH**. Select ***Grid Form → rectangle***. Click on the point on the y-axis. Side it toward the origin. The scale on the y-axis should change. Continue to slide toward the origin until you see your data points.

   Describe the shape of the curve formed.

   ____

   ____

   ____

   What are the dimensions of the cone that appears to give you the largest volume?

   Radius: __________ Height:______________ Volume:______________________

   Exit the program. DO NOT SAVE CHANGES.

4. Type the information from your table in exercise 3 into your TI Graphing calculator. Press □. Type in the data for the radius in column L1. Type in the data for the volume in column L2. Press ψο. Press ⊆ twice. Select ∀. Press ⊆. Next to xList press ψℵ. Next to yList press ψℑ. For Mark, select, . Press ⊆. Press θ. Select **9:ZoomStat**. Press ⊆. The data points should appear on the screen.

   If ***x*** represents the radius of the cone, write an equation in terms of ***x*** for the height of the cone.

   (Refer back to Resource Sheet 2 if necessary.) Height = ________________________

   Use the information above to write an equation for the volume of a cone in terms of ***x***. Volume = ________________________________

5. Press ο. Enter the expression for the volume you found in exercise 5 next to y1=. Press σ.

   How well does your equation fit the data points?

   ____

6. Press ο. Press the up cursor key to highlight Plot1. Press ⊆. Press σ. Press ψρ. Use the down cursor key to highlight **4:maximum**. Press⊆. Use the right and left cursor key to move to a point to the left of the highest point on the graph. Press ⊆. Use the right and left cursor key to move to a point to the right of the highest point on the graph. Press ⊆ twice. The radius and volume of the cone with maximum volume appears at the bottom of the screen. Record these values below. Round your answer to the nearest tenth. Use your equation in exercise 5 to calculate the height.

   Radius: ____________ Height:______________ Volume:______________________

7. Compare your answers in problem 5 with your answers in problem 7.

   How close are they?

   ____

   ____

8. Which one do you believe is the most accurate? Explain.

   ____

   ____

   ____

   ____

Name ____

Date ______________________________ Period ____

SUPER SIZE IT
WORKSHEET 5

1. Krusty decided to buy a larger waffle iron that makes circular waffles with a radius of 15 cm, for the true waffle cone lovers.

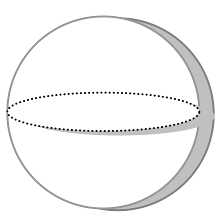
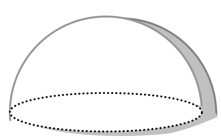
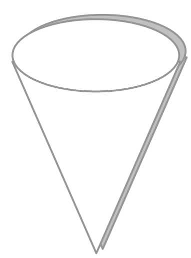
   a. If ***x*** *represents the radius of the cone,* write an equation in terms of ***x*** for the height of the cone. Height (x) = ____________________________

   b. Use the information in part (a) and the formula for finding the volume of a cone to write an equation for the volume of a cone in terms of ***x***. Volume (x) = ___________________________

   c. Determine the dimensions (radius and height) of the cone (formed from a circular waffle of radius 15 cm) that holds the most ice cream (flush with the top)? What is the volume of this cone? Use your graphing calculator and the formula in part (c). Round your answers to the nearest hundredth.

   radius: ____________ height: ____________ volume: ________________________

2. Determine the volume for the following solids.

a. Sphere	b. hemisphere	c. cone
 Radius = 9cm	 Radius = 9 cm	 Radius = 3 cm, Height = 4 cm
Volume:	Volume:	Volume:

Name ____

Date ______________________________ Period ____

SUPER SIZE IT
WORKSHEET 6

Bart loves to go to Krusty Kones as much as Lisa. He makes his ice cream by first packing as much ice cream into the cone as possible. He then tops it off with a scoop of ice cream (hemispherical in shape) that has the same radius as the cone.

Bart wants to get as much ice as he can since all ice cream cones cost the same.

1. Should he make a cone with the same dimensions as you suggested for Lisa? Should he make a cone with a radius of 10 cm? What do you think? Explain your reasoning.

    ____

    ____

    ____

    ____

    ____

Let's explore this new situation. Start the Geometer's Sketchpad program. Open the *krustykones* document. The diagram on the screen represents the top and side views of your cone. AB represents the radius of the base of the cone. EF represents the slant height of the cone and EG represents the height of the cone.
Click on the point not labeled on the circle. Slide the point toward point A. The circle changes in size. The dimensions of the cone also change. You may have to move point B on the circle to keep the cone upright.

2. Click on point G, then click on point H. Select **CONSTRUCT** → ***circle by center + point***. Click on point H, then click point F. Select **CONSTRUCT** → ***arc on circle***. Click on arc. Select **DISPLAY** → ***hide circle***. Click on arc. Select **CONSTRUCT** → ***arc interior*** → ***arc sector***.

3. What is the formula for the volume of a . . .

    sphere? ______________________ hemisphere? ______________________

4. How would you adjust the equation for the volume of Lisa's cone, $V = (\frac{\pi}{3}) \cdot AB^2 \cdot EG$ on Resource Sheet 4, to obtain an equation for the volume of Bart's cone?

    ____

    ____

    Equation for the volume for Bart's cone: ______________________________

5. Click on **MEASURE**→***Calculate.*** Type in your expression for the volume of Bart's cone. To obtain the π symbol, click on **VALUE** →π. To type in an exponent click the caret, $\boxed{\text{L}}$ and type the number of the exponent afterward.

Select the point (not labeled) on circle A. Slide the point. The size of the circle and the size of the ice cream cone change. . The corresponding values for on the measure of AB, the measure of EG and volume also changes.

6. Let ***x*** represent the radius of the cone. Rewrite the equation for the volume in exercise 3 in terms of ***x***. volume(x) = ______________________________

7. Found the dimensions for Bart's cone that will hold the most ice cream. What is the volume? Round your answers to the nearest hundredth.

   radius: __________ height: __________ volume: ______________________

Name ____

Date ______________________________ Period ____

SUPER SIZE IT
WORKSHEET 7
SUMMATIVE

Krusty's Ice Cream Parlor business has become a tremendous success. There are now several *KRUSTY KONES* in Springfield. Each *Krusty Kones* has a different size waffle iron (which makes circular waffles of a different radius).

This was great for the Simpsons, since everyone loved ice cream. But it created a new problem for Lisa. She knew the dimensions that she would use to design her cone if the waffle iron made circular waffles with a radius of 10 cm. But, since each Krusty Kones has a different size waffle iron, she does not know what dimensions to design her cone.

Develop equations for Lisa that will help her determine the dimensions of her cone (one which holds the most ice cream and flush with the top of the cone), no matter which *Krusty Kones* she goes to.

1. Let ***R*** represent the radius of the circular waffle and ***r*** represent the radius of the cone. Complete the table below by determining …

   a. an equation for the height of the cone in terms of ***r***.

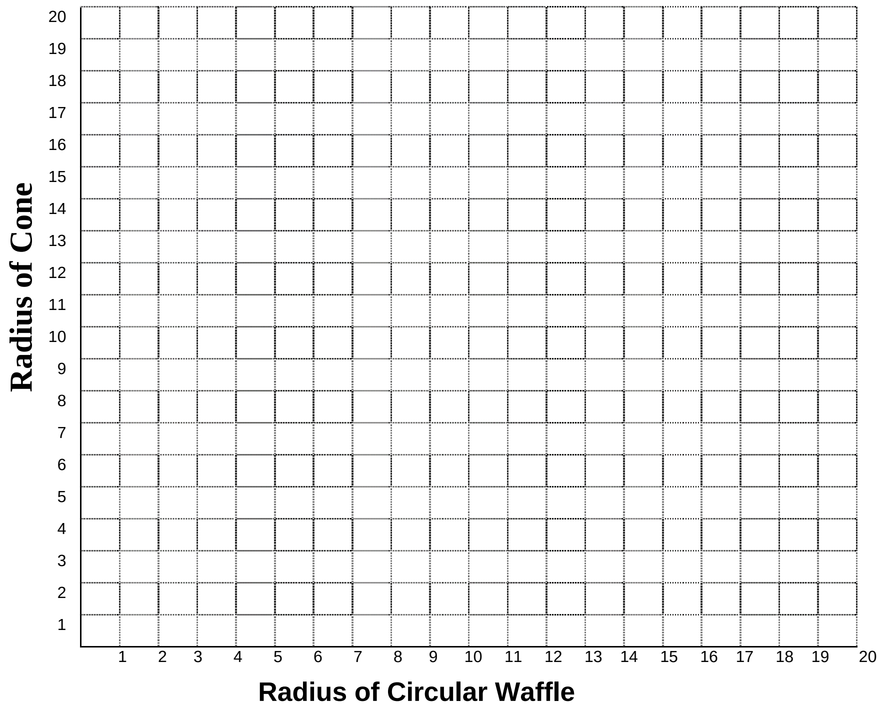
   b. an equation for the volume of the cone in terms of ***r***.

   c. the value of ***r*** of the cone formed from a circular waffle of radius ***R*** that holds the most ice cream flush with the top. (Round your answer to the nearest hundredth.)

R Radius of Circular Waffle	Height of cone in terms of **r**.	Volume of cone in terms of **r**.	**r** Radius of Cone
8 cm			
10 cm	$h = \sqrt{100 - r^2}$	$V = \frac{\pi}{3} r^2 \sqrt{100 - r^2}$	8.16
12 cm			
14 cm			
16 cm			
18 cm			

Check with other members of your group to make sure everyone has the same results.

2. Sketch a graph of your data on the grid below.



3. Determine an equation for ***r*** in terms of ***R*** that best fits the data above. Equation: ***r(R)*** = ___________________________

4. Write equations in terms of ***R*** for the height and volume of the cone that holds the most ice cream flush with the top. Simplify your equations.

Height:

h(R) = ________________________________

Volume:

V(R) = ___________________________

TO SIMULATE THE CONE PROBLEM

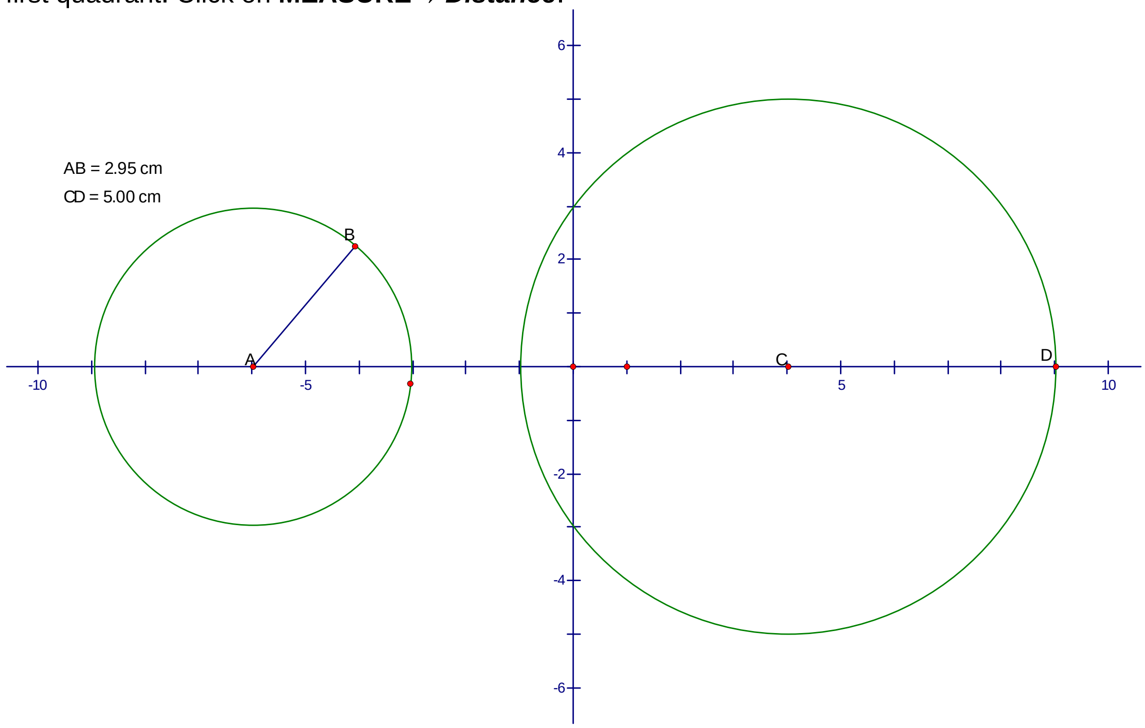
Click on **GRAPH** on the tool bar. Select ***show grid*** → ***square***.

Select the circle tool. Let the center be the point (-6, 0). Slide the pointer along the x-axis and click at the point (-3, 0). Label the center point A.

Construct a radius (with a positive slope) of the circle. Label the radius AB.

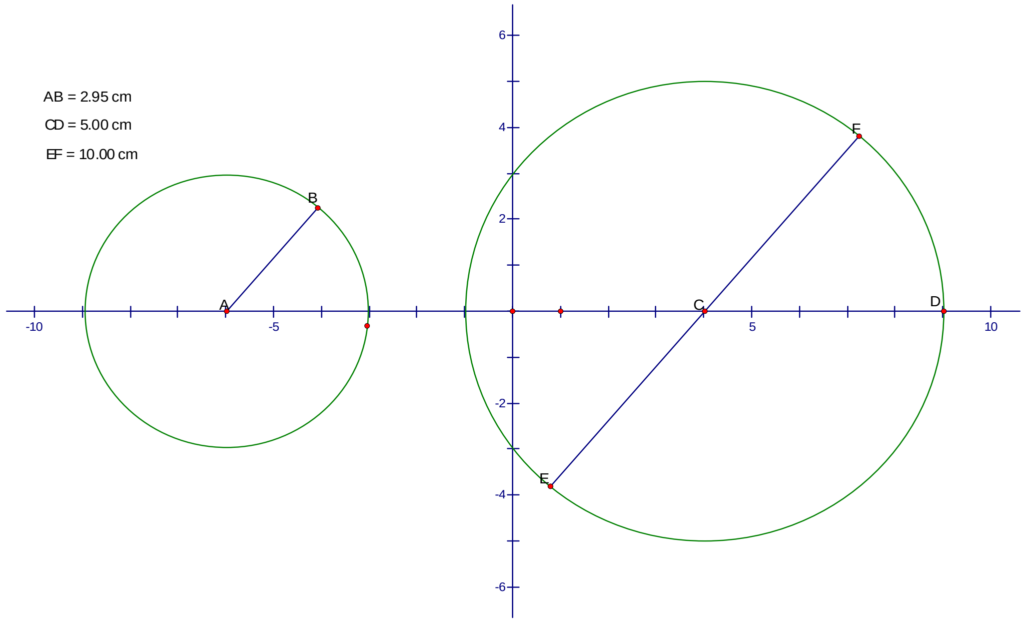
Select points **A** and **B**. Click on **MEASURE** → ***Distance.***

Select the circle tool. Let the center be located at the point (4, 0). Slide the pointer to the point (9, 0). This circle must have a diameter of 5.0 cm. Click on the center point and a point on the circle in the first quadrant. Click on **MEASURE**→ ***Distance***.



Select segment AB and point C. Click on **CONSTRUCT** → ***Parallel Line.***

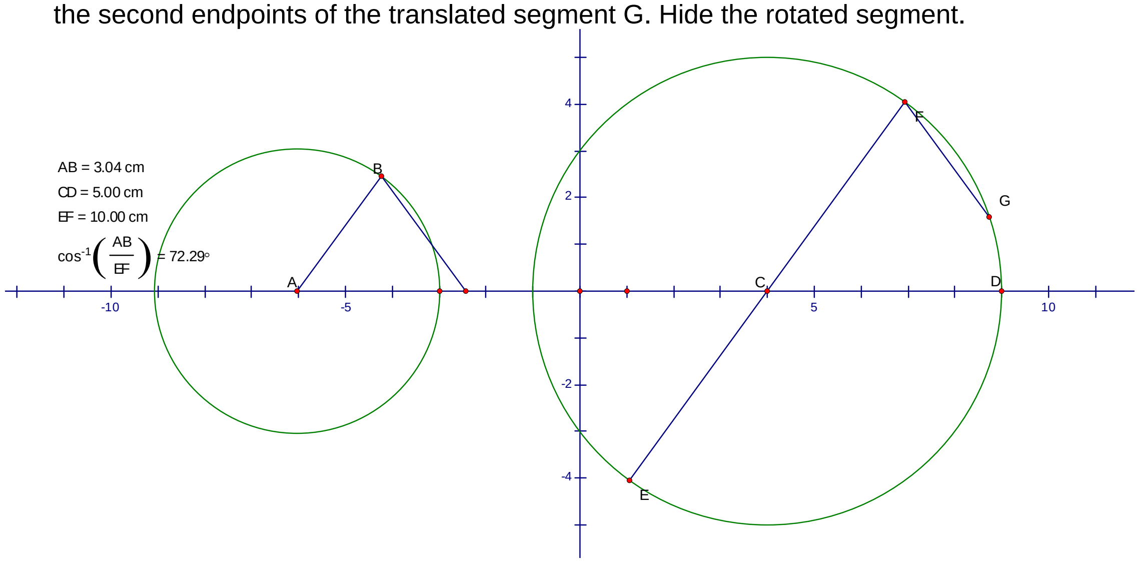
Construct the points of intersection of the line and circle C. Construct the segment containing the two points (the diameter). Deselect everything and select the parallel line. Click on **DISPLAY** and select ***Hide Parallel Line***. Click on points E and F. Click on **MEASURE**. Select ***Distance.***



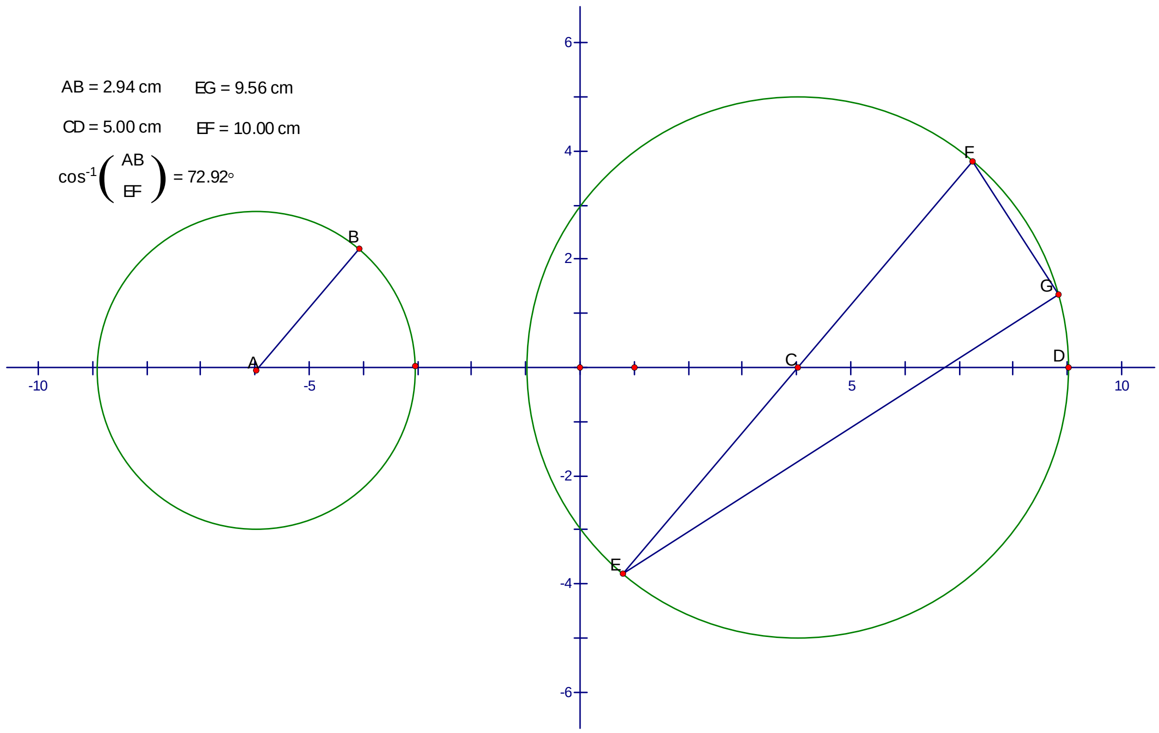
Click **MEASURE** →***calculate***. Click on **FUNCTION**. Select ***Arccos***. Click on the measure of AB. Click on the division symbol. Click on EF=10.00 cm. Click **OK**. [$\cos^{-1}\frac{AB}{EF}$ = 72.29^o appear on the screen.] Your angle measurement may be different.

Double click point B. Select segment AB along with point A. Click **TRANSFORM**… → ***Rotate***. Click $\cos^{-1}\frac{AB}{EF}$ = 72.29^o. Click **ROTATE.**

Click on point B then click on point F. Click **TRANSFORM..**. Select ***Mark Vector***. Click on the rotated segment along with the endpoints. Click **TRANSFORM**… → ***Translate***. Click **TRANSLATE.** Label the second endpoints of the translated segment G. Hide the rotated segment.

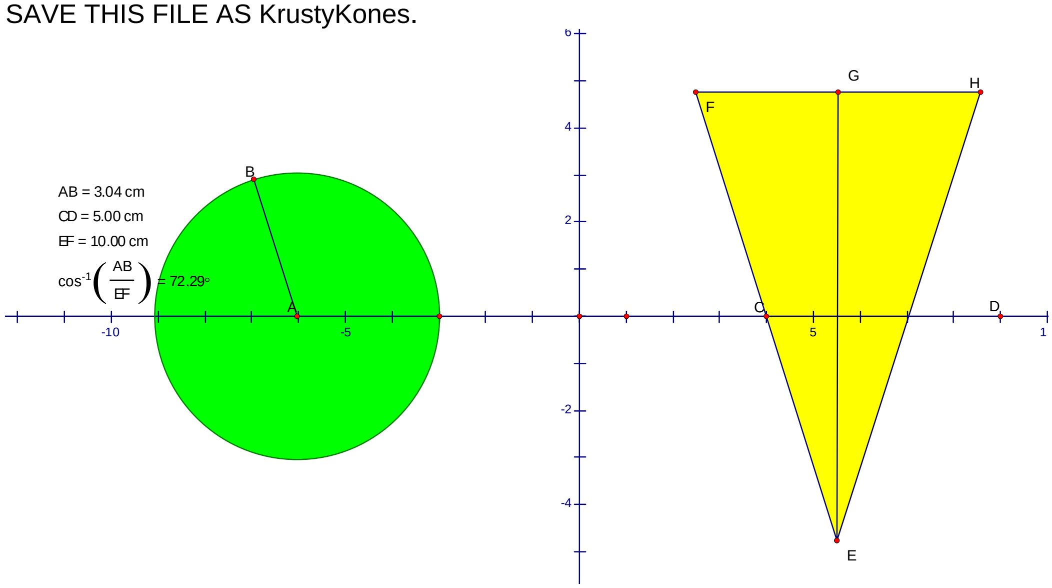


Construct segment from point E to G to complete triangle EFG to complete the triangle. Click on points E and G. Click **MEASURE** → ***distance***. Click on the rotated segment (the segment with an endpoint at B), click **DISPLAY**→***hide segment***. If you have followed all 11 steps correctly, as you increase the radius of circle A the length of EG (the radius of the cone) increases and the length of EG (the height of the cone) decreases.



12. Click on Circle C, and the measures of CD and $\cos^{-1}(\frac{AB}{EF})$. Click **DISPLAY→*hide objects***. Click on points E, F, and G. Click **CONSTRUCT → *triangle interior***. Click circle A. **CONSTRUCT → *circle interior***. Double click on segment EG. Click on segments EF and FG. Click on point F. Click on **TRANSFORM → *Reflect***. Label the vertex of the reflected angle H. Click on point F, G, and H. Click **CONSTRUCT → *triangle interior***.

    SAVE THIS FILE AS KrustyKones.

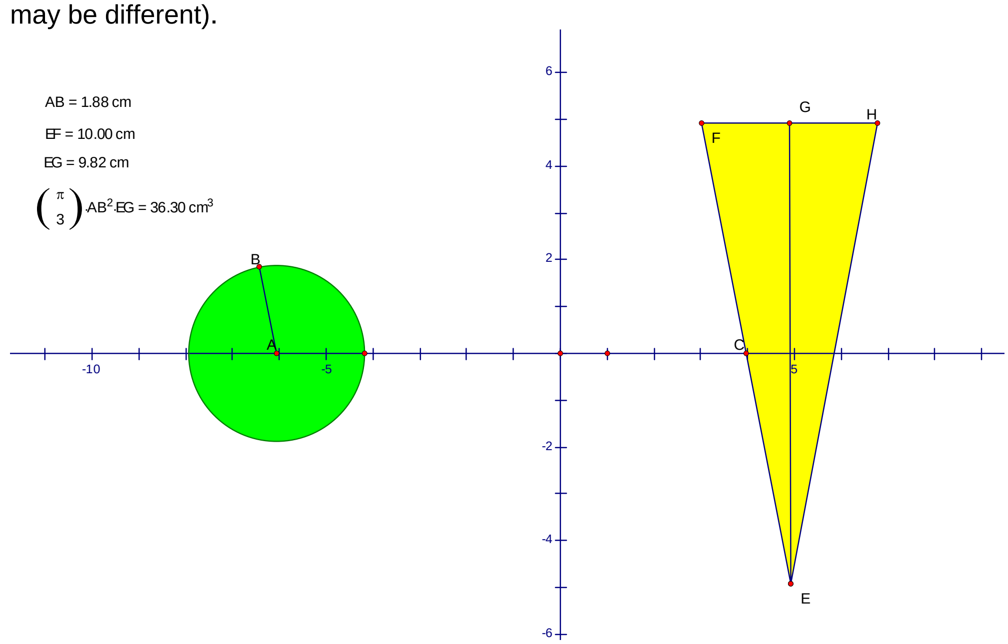


This lesson can be teacher centered or done individually. If done individually save this file to the student's computers.

This part of the directions should be done during the class period either individually or teacher–centered. These steps on reproduced on Resource Sheet 4 as problems 1 through 4 along with some leading questions

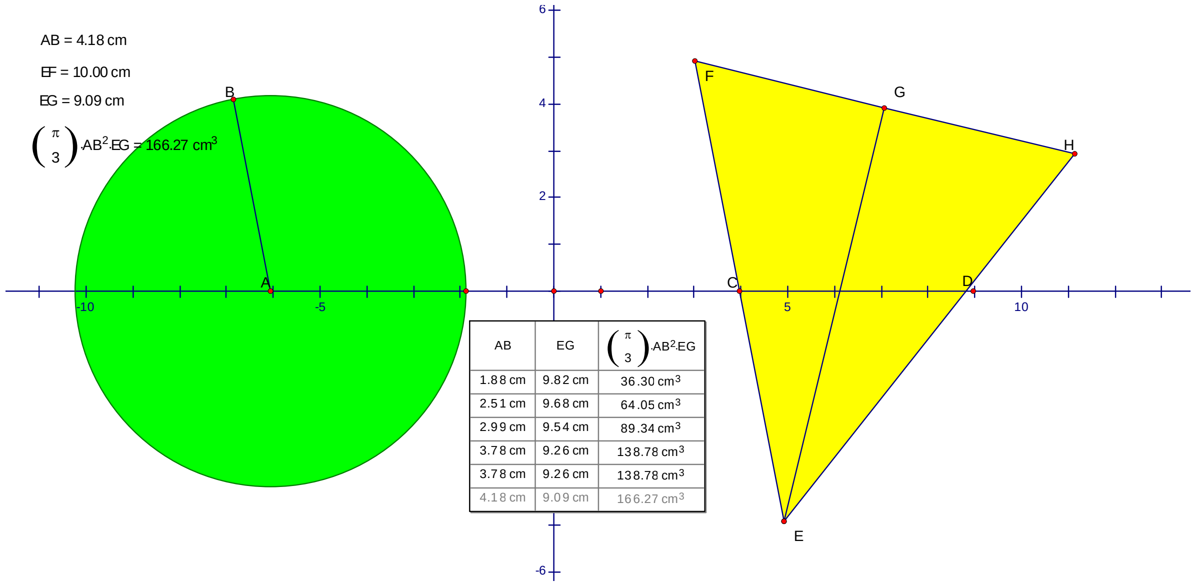
13. Click on **MEASURE** →***Calculate***. Click on **VALUE**→π**.** Click on the multiplication symbol. Click on the measure of AB. Click on the caret symbol, ∧. Click on 2. Click on the measure of EG. Click **OK**.

    The **volume** $\left(\frac{\pi}{3}\right)\cdot AB^2\cdot EG = 92.37cm^2$ should appear on the screen. (The value on you screen may be different).

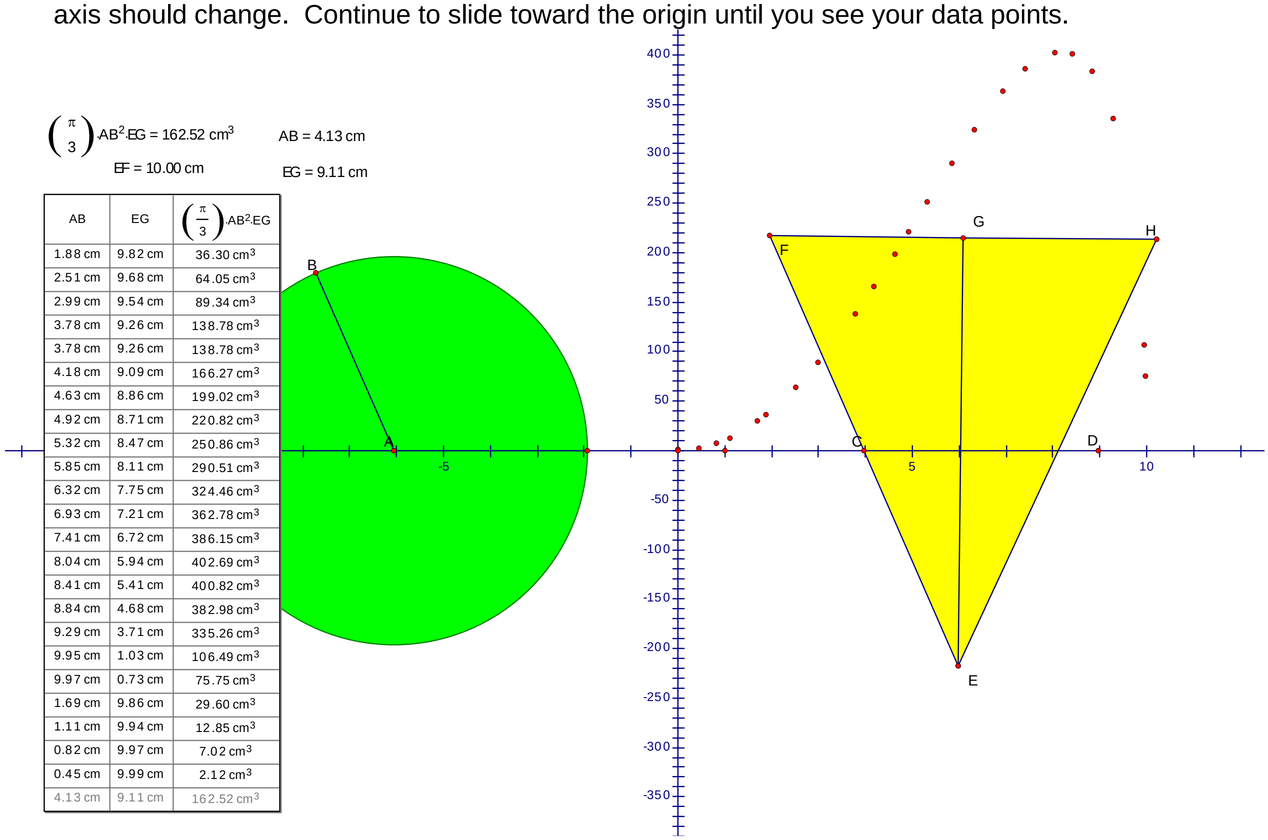


14. Click on the measure of AB, the measure of EG and the volume. Click **GRAPH** →***Tabulate****.* A table should appear on the screen. The first row of the table should have the values on the desktop. To get a second row click the table twice.

15. Select the point (not labeled) on circle A. Slide the point. The circle should change in size. The corresponding values for the measure AB, the measure of EG and the volume should also change. Click the table twice these values now appear in the table and a third row appears. Continue this process until you get 20 data points.



16. Click **GRAPH→*Plot Table Data***.... *Click on* y → $(\frac{\pi}{3}) \cdot AB^2 \cdot EG$. Click OK. Click **GRAPH →*Grid Form → rectangle.***. Click on the point on the y-axis. Slide it toward the origin. The scale on the y-axis should change. Continue to slide toward the origin until you see your data points.



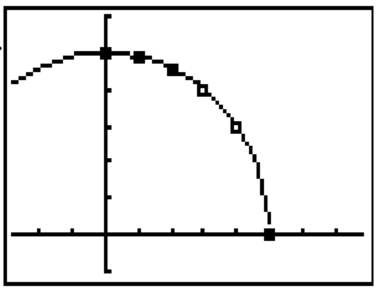
ANSWERS TO QUESTIONS

WORKSHEET 1

1 a. 29 b. $8\sqrt{2} \cong 11.31$ c. $\sqrt{\frac{41}{2}} \cong 4.53$

2. Answers may vary.

x	0	1	2	3	4	5
Y	5	4.9	4.6	4	3	0

3. 

WORKSHEET 2

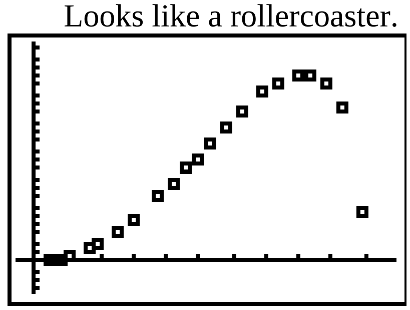
1. The slant height
2. The slant height of the cone is determined
   by the radius of the circle. Everyone used
   a circle of radius 10 cm.
3. Answer will vary
4. $r^2 + h^2 = s^2$, $r = \sqrt{s^2 - h^2}$ or
   $h = \sqrt{s^2 - r^2}$
5. $V = \frac{\pi}{3} r^2 h$
6. Answer will vary.
9. Answer will vary.
10. Answer will vary.

WORKSHEET 3

1. $r^2 + h^2 = s^2$, $r = \sqrt{s^2 - h^2}$ or
   $h = \sqrt{s^2 - r^2}$
2. $V = \frac{\pi}{3} r^2 h$
3. a. $100\pi \cong 314.16$
   b. $8192\pi \cong 25735.93$

WORKSHEET 4

1. The volume of the cone.
3. Answer will vary.
4. Answers will vary
   Looks like a rollercoaster.



radius = 8.04 height = 5.94 volume = 402.68

5. $height = \sqrt{100 - x^2}$
   $volume = \frac{\pi}{3} x^2 \sqrt{100 - x^2}$
6. Answer will vary
7. radius = 8.16, height = 5.77,
   volume = 403.07
8. Answer will vary.
9. The calculator's result. The Geometer's Sketchpad answers are based on the points selected. The calculator's results are based on ALL possible points.

WORKSHEET 5

1. a. $h = \sqrt{225 - x^2}$
   b. $V = \frac{\pi}{3} x^2 \sqrt{225 - x^2}$
   c. radius = 12.25, height = 8.66
   volume = 1360.35

2. a. $V = \frac{4}{3}\pi r^3$, V = 972π ≅ 3053.63
   b. $V = \frac{2}{3}\pi r^3$, V = 486π ≅ 1526.81
   c. $V = \frac{\pi}{3} r^2 h$, V = 12π ≅ 37.70

WORKSHEET 6

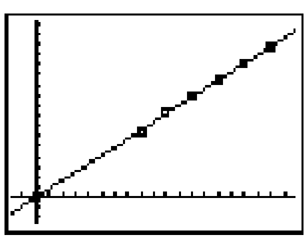
1. Answer may vary.
3. sphere: $V = \frac{4}{3}\pi r^3$ hemisphere: $V = \frac{2}{3}\pi r^3$
4. Add $\left(\frac{2}{3}\right) \cdot \pi \cdot AB^3$ $V = \left(\frac{\pi}{3}\right) \cdot AB^2 \cdot EG + \left(\frac{2}{3}\right) \cdot \pi \cdot AB^3$
6. $V = \frac{\pi}{3} x^2 \sqrt{100 - x^2} + \frac{2\pi}{3} x^3$
7. radius = 9.88 height = 1.56 volume = 2177.80

WORKSHEET 7
SUMMATIVE ASSESSMENT

1.

R	H	V	r
8	$h = \sqrt{64 - r^2}$	$V = \frac{\pi}{3} r^2 \sqrt{64 - r^2}$	6.53
10	$h = \sqrt{100 - r^2}$	$V = \frac{\pi}{3} r^2 \sqrt{100 - r^2}$	8.16
12	$h = \sqrt{144 - r^2}$	$V = \frac{\pi}{3} r^2 \sqrt{144 - r^2}$	9.80
14	$h = \sqrt{196 - r^2}$	$V = \frac{\pi}{3} r^2 \sqrt{196 - r^2}$	11.43
16	$h = \sqrt{256 - r^2}$	$V = \frac{\pi}{3} r^2 \sqrt{256 - r^2}$	13.06
18	$h = \sqrt{324 - r^2}$	$V = \frac{\pi}{3} r^2 \sqrt{324 - r^2}$	14.70

2.



3. $r(R) = R\sqrt{\frac{2}{3}}$ or $r(R) \approx 0.82R$

4. $h(R) = R\sqrt{\frac{1}{3}}$ or $h(R) \approx 0.58R$ $V(R) = \frac{2}{9}\pi R^3 \sqrt{\frac{1}{3}}$ or $V(R) \approx 0.13\pi R^3$