Title: Determining the Hours of Sunlight for a Day in Washington, D.C.

Brief Overview:

Students will utilize actual data from the Naval Observatory to determine a linear regression model that can predict the number of sunlight hours in a day for Washington, D.C. Two models will be developed. One model will be developed for data between January and June and another for data between June and December. For Algebra II classes and above quadratic and sinusoidal regression models can be determined and compared with the linear regression models.

NCTM Content Standard/National Science Education Standard:

- Number and Operations – understand numbers, ways of representing numbers, relationships among numbers and number systems
- Algebra – understand patterns, relations and functions; represent and analyze mathematical situations and structures using algebraic symbols; use mathematical models to represent and understand quantitative relationships; analyze change in various contexts
- Data Analysis and Probability – select and use appropriate statistical methods to analyze data; develop and evaluate inferences that are based on data
- Representation – create and use representations to organize, record and communicate mathematical ideas; use representations to model and interpret physical, social and mathematical phenomena

Grade/Level:

9-12; the lesson can include quadratic and sinusoidal regressions that are appropriate for Algebra II and above classes

Duration/Length:

Approximately three 60-minute periods.

Student Outcomes:

Students will (be able to):

- Learn to import data from the Internet.
- Compile subsets of data for use in the assignment.
- Learn how to enter lists in a TI-83
- Understand how to perform linear regression on a TI-83
- Utilize a mathematical model to predict values for desired criteria.
- Compare linear, quadratic and sinusoidal regression models (for Algebra II and above classes).
- Produce a scatter plot of the data and graph the regression model(s) on a TI-83

Materials and Resources:

- TI-83 or computer with Excel 97 or later
- Internet access or worksheets with the appropriate data
- Graph Paper
- Teacher Answer Keys

Development/Procedures:

Lesson 1

Preassessment – Query students on their experiences with the number of hours of sunlight in a day as the year progresses. Students should realize that daylight is shorter in winter and longer in summer. Discuss why this change in daylight hours occurs. Students most likely discussed the tilt of the earth in 7th or 8th grade science. After some discussion, distribute Handout 1 that shows the Earth as viewed from the sun on the first day of each month. Ask the students to analyze the images for clues about the amount of daylight hours and to make a reasonable guess as to which months have the longest and shortest days.

Launch – Distribute the Handout 3 to the students. Review information on the Handout, i.e. city, year, etc. Note that times are given in military notation. Handout 2 provides a tutorial in deciphering military time and can be distributed as an aid to students.

Teacher Facilitation – Distribute Worksheet 1. Explain that students will use data for the 1st day of the month and the 15th day of the month. Both sets of data will be plotted on the same graph.

Student Application – Students will produce a scatter plot of the data and answer questions on the Worksheet.

Embedded Assessment – Walk around and review each student's progress. Make sure axes are properly labeled, scale is appropriate, etc.

Reteaching/Extension –

- Review basics of plotting graphs to meet student's needs. Focus on ordered pair notation and the relation to the Cartesian coordinate system.
- Have the students do a search of the web for information on the solar system. The following websites may be helpful.

    1. http://space.jpl.nasa.gov/
    2. http://www.nineplanets.org/
    3. http://sse.jpl.nasa.gov/index.cfm
    4. http://liftoff.msfc.nasa.gov/academy/space/solarsystem/solarststemjava.html

5. http://www.fourmilab.ch/solar/solar.html
6. http://www.usno.navy.mil/

Lesson 2

Preassessment and Launch - Ask the students to discuss what is meant by the line of best fit of a set of data points and ask them to draw what they think is the line of best fit for the sets of data points on Worksheet 2.

Teacher Facilitation – Discuss the procedure for using the TI-83 to find the equation of the line of best fit and for graphing that line along with the scatter plot of the data points. It might be a good idea to demonstrate the example on a TI-83 projector as it reviewed by the instructor. Distribute Handout 4.

Student Application- The students are to use the included sunrise/sunset data to obtain the equation of the line of best fit relating the number of hours of daylight to the day of the year for several sets of data: first using the first day of each month for January through June, then using the first and 15th of each of the same months and finally using the first 10th, 15th and 20th, of each months. The students are then to use these models to predict the number of hours of daylight for the 23rd of each month and compare the predicted value with the value given in the data. These procedures are then repeated for the months of June through January.

Embedded Assessment - Built into Worksheet 2

Reteaching/Extension

- The teacher should form the students into small groups to discuss the lesson.
- Have the students discuss the following question: Will the number of hours of sunlight in a day be the same for cities located in the northern and southern hemispheres?
- Discuss the need for two models. Have students determine the hours of daylight for November 10 using the January-June model. Students should explain the discrepancy between the results of the two models for November 10.

Lesson 3

Preassessment and Launch – Students will obtain sunrise and sunset data for other cities. Distribute Handout 5. Discuss how the latitude is similar or different compared to Washington, DC.

Teacher Facilitation – Distribute Worksheet 3. Discuss how the number of hours of daylight may be similar or different for each of the chosen cities. Have students predict an estimated change in slope for the linear regression model, i.e. will the

slope be flatter, steeper or about the same. Obtain student opinion on how appropriate a linear regression model fits the data. Discuss the reasons why the linear regression model is or is not a good fit. Encourage students to think about the tilt of the earth discussed in Lesson 1 and how this affects the hours of sunlight the further away from the equator one moves.

Student Application – Students will determine a linear regression model for each city. Have students plot the data and the linear regression lines and compare to the model and graph produced in Lesson 2.

Embedded Assessment – Check each student's work. Examine the graph for appropriate scale, proper labeling of axes, etc.

Reteaching/Extension – For students in Algebra II and above have them determine quadratic and sinusoidal models. Pick a few data points and have students predict the daylight hours using the linear regression model, quadratic model and sinusoidal model. Compare the results to actual data and discuss why one model worked better than the others. Discuss r-values for each linear model. Discuss how the r-value relates to the "fit" of the line.

Summative Assessment:

The summative assessment quiz has the students use the TI-83 to find the line of best fit for data points without context to asses the degree to which they have mastered the technique. It also has the students find a line of best fit that models the illumination of the moon and asks them to use the model to make a prediction and to analyze the difference between their prediction and the observed value.

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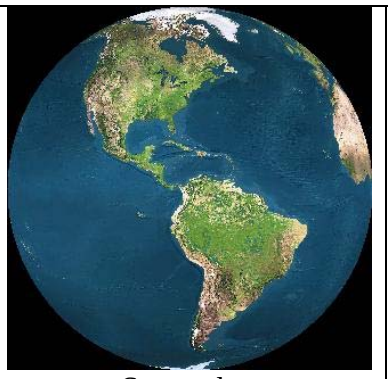
Handout 1: The Earth viewed from the sun on the first day of each month.

January	February	March
April	May	June

		
July	August	September
October	November	December

Handout 2 – How to Tell Military Time

Military time is based on a 24-hour clock and eliminates the use of A.M. and P.M.

- Note that the military clock begins with 0000 hours that occurs one-second midnight.

- Morning hours are designated using the following pattern: 1 a.m. is 0100 hours, called "oh one hundred hours"; 2 a.m. is 0200, "oh two hundred hours"; and so forth. This pattern continues for the morning hours just as with the standard method of telling time until noon, which is 1200 hours ("twelve hundred hours").

- For afternoon hours the following pattern is used: 1 p.m. is 1300 hours, called “thirteen hundred hours”, 2 p.m. is 1400 hours, “fourteen hundred hours” and so forth until 12 a.m. which is 2400 hours, “twenty-four hundred hours”.

- Times between hours are designated as follows: 5:45 a.m. is 0545 hours, 3:10 p.m. is 1510 hours.

- A simple way to determine afternoon hours in p.m. standard time is by subtracting 1200 from the military time. For example, 1800 hours is 6 p.m. in standard time: 1800-1200=600; 2040 hours is 8:40 p.m.: 2040-1200=840.

Handout 3 – Hours of Sunlight for Washington, D.C.

Sunrise and Sunset Times in Washington, DC

Location: W 077° 02' N 38° 53'

	January		February		March		April	
Day	*Rise*	*Set*	*Rise*	*Set*	*Rise*	*Set*	*Rise*	*Set*
1	7:27	16:56	7:15	17:29	6:40	18:01	5:52	18:32
2	7:27	16:57	7:14	17:30	6:39	18:02	5:51	18:33
3	7:27	16:58	7:13	17:31	6:37	18:03	5:49	18:34
4	7:27	16:59	7:12	17:32	6:36	18:04	5:48	18:35
5	7:27	17:00	7:11	17:34	6:34	18:05	5:46	18:36
6	7:27	17:01	7:10	17:35	6:33	18:07	5:45	18:37
7	7:27	17:02	7:09	17:36	6:31	18:08	5:43	18:38
8	7:27	17:03	7:08	17:37	6:30	18:09	5:41	18:39
9	7:27	17:04	7:07	17:38	6:28	18:10	5:40	18:40
10	7:27	17:05	7:06	17:39	6:27	18:11	5:38	18:41
11	7:26	17:06	7:05	17:40	6:25	18:12	5:37	18:42
12	7:26	17:07	7:04	17:42	6:23	18:13	5:35	18:43
13	7:26	17:08	7:02	17:43	6:22	18:14	5:34	18:44
14	7:26	17:09	7:01	17:44	6:20	18:15	5:33	18:45
15	7:25	17:10	7:00	17:45	6:19	18:16	5:31	18:46
16	7:25	17:11	6:59	17:46	6:17	18:17	5:30	18:47
17	7:25	17:12	6:58	17:47	6:16	18:18	5:28	18:48
18	7:24	17:13	6:56	17:48	6:14	18:19	5:27	18:49
19	7:24	17:14	6:55	17:49	6:13	18:20	5:25	18:49
20	7:23	17:15	6:54	17:51	6:11	18:21	5:24	18:50
21	7:23	17:16	6:52	17:52	6:09	18:21	5:23	18:51
22	7:22	17:17	6:51	17:53	6:08	18:22	5:21	18:52
23	7:22	17:19	6:50	17:54	6:06	18:23	5:20	18:53
24	7:21	17:20	6:48	17:55	6:05	18:24	5:19	18:54
25	7:20	17:21	6:47	17:56	6:03	18:25	5:17	18:55
26	7:20	17:22	6:46	17:57	6:02	18:26	5:16	18:56
27	7:19	17:23	6:44	17:58	6:00	18:27	5:15	18:57
28	7:18	17:24	6:43	17:59	5:58	18:28	5:14	18:58
29	7:17	17:25	6:41	18:00	5:57	18:29	5:12	18:59
30	7:17	17:27			5:55	18:30	5:11	19:00
31	7:16	17:28			5:54	18:31		

Sunrise and Sunset Times in Washington, DC

Location: W 077° 02' N 38° 53'

	May		June		July		August	
Day	*Rise*	*Set*	*Rise*	*Set*	*Rise*	*Set*	*Rise*	*Set*
1	5:10	19:01	4:45	19:28	4:47	19:37	5:09	19:19
2	5:09	19:02	4:44	19:29	4:47	19:37	5:10	19:18
3	5:07	19:03	4:44	19:29	4:48	19:37	5:11	19:17
4	5:06	19:04	4:44	19:30	4:48	19:37	5:12	19:16
5	5:05	19:05	4:43	19:30	4:49	19:37	5:13	19:14
6	5:04	19:06	4:43	19:31	4:49	19:36	5:14	19:13
7	5:03	19:07	4:43	19:32	4:50	19:36	5:15	19:12
8	5:02	19:08	4:43	19:32	4:51	19:36	5:16	19:11
9	5:01	19:09	4:42	19:33	4:51	19:35	5:17	19:10
10	5:00	19:10	4:42	19:33	4:52	19:35	5:18	19:09
11	4:59	19:10	4:42	19:34	4:53	19:34	5:18	19:07
12	4:58	19:11	4:42	19:34	4:53	19:34	5:19	19:06
13	4:57	19:12	4:42	19:34	4:54	19:34	5:20	19:05
14	4:56	19:13	4:42	19:35	4:55	19:33	5:21	19:04
15	4:55	19:14	4:42	19:35	4:55	19:32	5:22	19:02
16	4:54	19:15	4:42	19:36	4:56	19:32	5:23	19:01
17	4:54	19:16	4:42	19:36	4:57	19:31	5:24	19:00
18	4:53	19:17	4:43	19:36	4:58	19:31	5:25	18:58
19	4:52	19:18	4:43	19:36	4:58	19:30	5:26	18:57
20	4:51	19:19	4:43	19:37	4:59	19:29	5:27	18:55
21	4:51	19:19	4:43	19:37	5:00	19:29	5:28	18:54
22	4:50	19:20	4:43	19:37	5:01	19:28	5:28	18:53
23	4:49	19:21	4:44	19:37	5:02	19:27	5:29	18:51
24	4:49	19:22	4:44	19:37	5:03	19:26	5:30	18:50
25	4:48	19:23	4:44	19:37	5:03	19:25	5:31	18:48
26	4:47	19:23	4:45	19:37	5:04	19:25	5:32	18:47
27	4:47	19:24	4:45	19:38	5:05	19:24	5:33	18:45
28	4:46	19:25	4:45	19:37	5:06	19:23	5:34	18:44
29	4:46	19:26	4:46	19:37	5:07	19:22	5:35	18:42

30	4:45	19:26	4:46	19:37	5:08	19:21	5:36	18:41
31	4:45	19:27			5:09	19:20	5:37	18:39

Sunrise and Sunset Times in Washington, DC

Location: W 077° 02' N 38° 53'

	September		**October**		**November**		**December**	
Day	*Rise*	*Set*	*Rise*	*Set*	*Rise*	*Set*	*Rise*	*Set*
1	5:37	18:38	6:04	17:50	6:36	17:07	7:08	16:46
2	5:38	18:36	6:05	17:49	6:37	17:06	7:09	16:46
3	5:39	18:35	6:06	17:47	6:38	17:05	7:10	16:46
4	5:40	18:33	6:07	17:45	6:39	17:04	7:11	16:46
5	5:41	18:32	6:08	17:44	6:40	17:03	7:12	16:46
6	5:42	18:30	6:09	17:42	6:41	17:02	7:13	16:46
7	5:43	18:28	6:10	17:41	6:42	17:01	7:14	16:46
8	5:44	18:27	6:11	17:39	6:44	17:00	7:14	16:46
9	5:45	18:25	6:12	17:38	6:45	16:59	7:15	16:46
10	5:45	18:24	6:13	17:36	6:46	16:58	7:16	16:46
11	5:46	18:22	6:14	17:35	6:47	16:57	7:17	16:46
12	5:47	18:21	6:15	17:33	6:48	16:56	7:18	16:46
13	5:48	18:19	6:16	17:32	6:49	16:56	7:18	16:47
14	5:49	18:17	6:17	17:30	6:50	16:55	7:19	16:47
15	5:50	18:16	6:18	17:29	6:51	16:54	7:20	16:47
16	5:51	18:14	6:19	17:28	6:52	16:53	7:20	16:48
17	5:52	18:12	6:20	17:26	6:53	16:53	7:21	16:48
18	5:53	18:11	6:21	17:25	6:55	16:52	7:22	16:48
19	5:53	18:09	6:22	17:23	6:56	16:51	7:22	16:49
20	5:54	18:08	6:23	17:22	6:57	16:51	7:23	16:49
21	5:55	18:06	6:24	17:21	6:58	16:50	7:23	16:50
22	5:56	18:04	6:25	17:19	6:59	16:50	7:24	16:50
23	5:57	18:03	6:26	17:18	7:00	16:49	7:24	16:51
24	5:58	18:01	6:27	17:17	7:01	16:49	7:25	16:51
25	5:59	18:00	6:28	17:16	7:02	16:48	7:25	16:52
26	6:00	17:58	6:29	17:14	7:03	16:48	7:25	16:53
27	6:01	17:56	6:30	17:13	7:04	16:48	7:26	16:53

28	6:02	17:55	6:31	17:12	7:05	16:47	7:26	16:54
29	6:03	17:53	6:33	17:11	7:06	16:47	7:26	16:55
30	6:03	17:52	6:34	17:09	7:07	16:47	7:26	16:56
31			6:35	17:08		16:47	7:27	16:56

Name_____ Date_____ Period_____

Worksheet 1

You operate a business which provides sight seeing tours of Washington DC via airplane. You must fly during daylight hours. Since the number of daylight hours varies with the time of year, you would like to develop a mathematical model to predict how many hours of daylight for a given day. This will help determine how many hours you can operate your business and help maximize profits.

Part I

Directions: Using the data for Washington, DC for the first day of each month, calculate the number of hours of sunlight and write your answers in Table 1. Make a scatter plot of this data on graph paper. The x-axis should represent the number of the day since January 1. For example February 1 is day 32. Label the axes and choose a suitable scale. Answer the questions.

To calculate the hours of sunlight for each day use the following process:

Step 1: Determine the number of hours between noon and the sunset time. Remember that noon in military time is 1200 hours.
Step 2: Determine the number of hours between the sunrise time and noon.
Step 3: Add the two answers from Step 1 and Step 2. Remember that you cannot have more than 60 minutes. If the number of minutes exceeds 59 subtract 60 and add 1 to the number of hours.
Step 4: Convert the time to decimal hours by dividing the minutes by 60.

Example 1: How many hours of sunlight for March 26?

Step 1: Number of hours of sunlight between noon and sunset time = 1826 – 1200 = 626

This is read as 6 hours and 26 minutes.

Step 2: Number of hours of sunlight between sunrise time and noon = 1200 – 602 = 558

This is read as 5 hours and 98 minutes.

Step 3: Total number of hours of sunlight = Answer in Step 1 + Answer in Step 2
Total number of hours of sunlight = 626 + 598
Total number of hours of sunlight = 1224

which is read 12 hours and 24 minutes

Step 4: Convert 12 hours 24 minutes to decimal hours. Divide 24 by 60 = 0.4

The number of hours of sunlight in decimal hours is 12.4 hours.

Table 1: Hours of Daylight for the 1st Day of the Month

Month	Day of the Year	Hours of Sunlight
January	1	
February	32	
March	60	
April	91	
May	121	
June	152	
July	182	
August	213	
September	244	
October	274	
November	305	
December	335	

Questions:

1) Why does the amount of sunlight vary over time?

_____

_____

_____

2) What type of pattern does the data create?

_____

_____

_____

3) Do you think the pattern will be different if you use another day of the month? (e.g. you use the 10th day of each month instead of the 1st) Why or why not?

_____

_____

_____

Part II

Directions: Using the data for Washington, DC make a table of values for the hours of sunlight for the 15th day of each month using Table 2 below. Make a scatter plot of this data on graph paper. Label the axes and choose a suitable scale. Answer the questions.

Table 2: Hours of Sunlight for the 15th Day of the Month

Month	Day of the Year	Hours of Sunlight
January	15	
February	46	
March	74	
April	105	
May	135	
June	166	
July	196	
August	227	
September	258	
October	288	
November	319	
December	349	

Questions:

1) Compare and contrast this scatter plot to the scatter plot made in Part I.

_____

_____

_____

2) Why do you think the pattern was the same or different?

Name_____ Date_____ Period_____

Worksheet 1 – Answer Key

You operate a business which provides sight seeing tours of Washington DC via airplane. You must fly during daylight hours. Since the number of daylight hours varies with the time of year, you would like to develop a mathematical model to predict how many hours of daylight for a given day. This will help determine how many hours you can operate your business and help maximize profits.

Part I

Directions: Using the data for Washington, DC for the first day of each month, calculate the number of hours of sunlight and write your answers in Table 1. Make a scatter plot of this data on graph paper. The x-axis should represent the number of the day since January 1. For example February 1 is day 32. Label the axes and choose a suitable scale. Answer the questions.

To calculate the hours of sunlight for each day use the following process:

Step 1: Determine the number of hours between noon and the sunset time. Remember that noon in military time is 1200 hours.
Step 2: Determine the number of hours between the sunrise time and noon.
Step 3: Add the two answers from Step 1 and Step 2. Remember that you cannot have more than 60 minutes. If the number of minutes exceeds 59 subtract 60 and add 1 to the number of hours.
Step 4: Convert the time to decimal hours by dividing the minutes by 60.

Example 1: How many hours of sunlight for March 26?

Step 1: Number of hours of sunlight between noon and sunset time = 1826 – 1200 = 626

This is read as 6 hours and 26 minutes.

Step 2: Number of hours of sunlight between sunrise time and noon = 1200 – 602 = 558

This is read as 5 hours and 98 minutes.

Step 3: Total number of hours of sunlight = Answer in Step 1 + Answer in Step 2
Total number of hours of sunlight = 626 + 598
Total number of hours of sunlight = 1224

which is read 12 hours and 24 minutes

Step 4: Convert 12 hours 24 minutes to decimal hours. Divide 24 by 60 = 0.4

The number of hours of sunlight in decimal hours is 12.4 hours.

Table 1: Hours of Daylight for the 1st Day of the Month

Month	Day of the Year	Hours of Sunlight
January	1	9.48
February	32	10.23
March	60	11.35
April	91	12.67
May	121	13.85
June	152	14.72
July	182	14.83
August	213	14.17
September	244	13.02
October	274	11.77
November	305	10.52
December	335	9.63

Questions:

1) Why does the amount of sunlight vary over time?

_Due to the tilt of the Earth, there is less daylight during winter and more daylight during summer.

2) What type of pattern does the data create?

__The data creates a hill or curve.

3) Do you think the pattern will be different if you use another day of the month? (e.g. you use the 10th day of each month instead of the 1st) Why or why not?

__No, the pattern will be the same. Since the Earth changes it's tilt at a relatively constant rate, the day of the month will not alter the pattern. The pattern would be different if you picked a different day of each month.

Part II

Directions: Using the data for Washington, DC make a table of values for the hours of sunlight for the 15th day of each month using Table 2 below. Make a scatter plot of this data on graph paper. Label the axes and choose a suitable scale. Answer the questions.

Table 2: Hours of Sunlight for the 15th Day of the Month

Month	Day of the Year	Hours of Sunlight
January	15	9.75
February	46	10.75
March	74	11.95
April	105	13.25
May	135	14.32
June	166	14.88
July	196	14.62
August	227	13.67
September	258	12.43
October	288	11.18
November	319	10.05
December	349	9.45

Questions:

1) Compare and contrast this scatter plot to the scatter plot made in Part I.

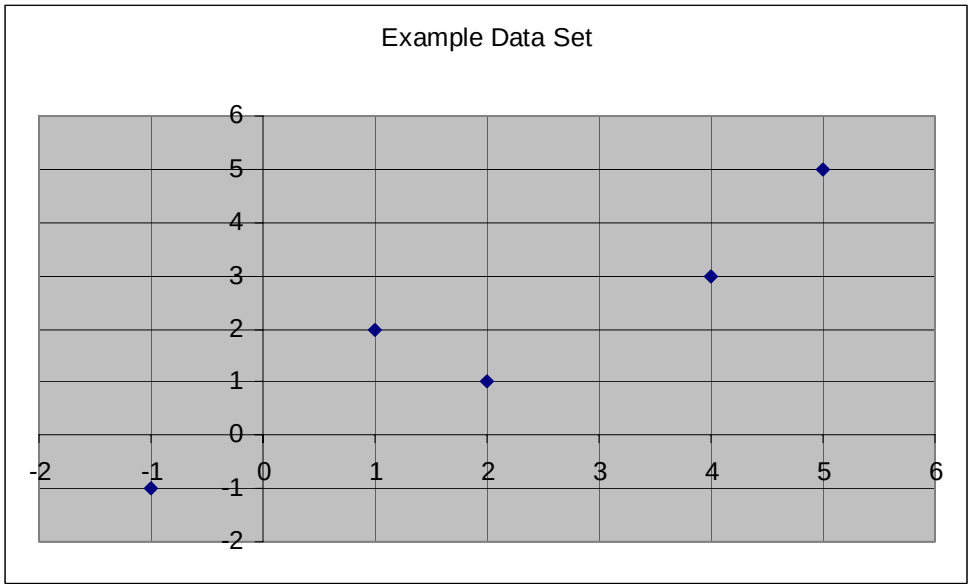
___This scatter plot is very similar to the plot made in Part I. The pattern is almost identical.

2) Why do you think the pattern was the same or different?

___The pattern is the same because the Earth changes its tilt at a relatively constant rate. Therefore, the change in daylight hours for a one-month period will be the same regardless of the day chosen.

Handout 4: Instructions for finding the equation of the line of best fit for a set of data points on the TI-83.

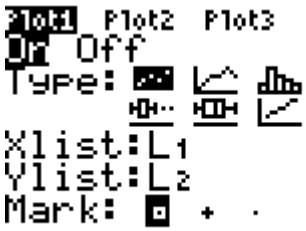
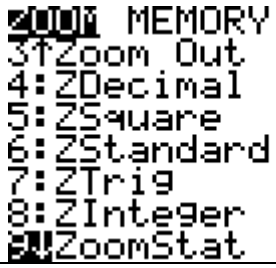
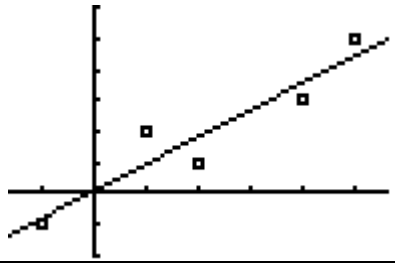
Given a set of non-collinear points in the plane, we would like to find the line that "fits these points the best." This process is called linear regression, and the TI-83 calculator will perform this operation for you. Here is an example:



Develop a data table for the five points above. Find the equation of the line of best fit and graph this line together with a scatter plot of these points on the TI-83.

Procedure	TI-83 Screen
To begin, we enter the x-coordinates and the y-coordinates as lists. Press {-1, 1, 2, 4, 5} sto-> 2nd L_1 ENTER and {-1, 2, 1, 3, 5} sto-> L_2 ENTER Be careful to keep the same order for both sets of coordinates.	{-1,1,2,4,5}→L_1 {-1 1 2 4 5} {-1,2,1,3,5}→L_2 {-1 2 1 3 5}
Next, press STAT	

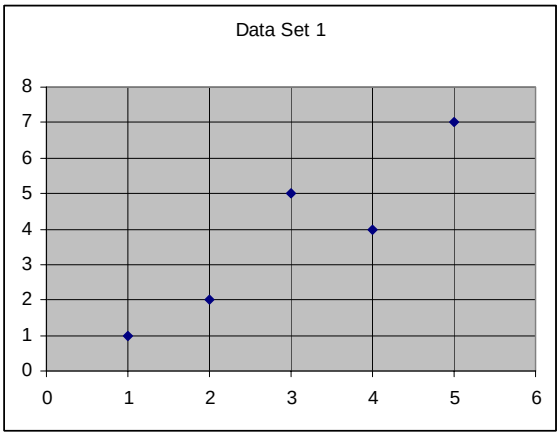
	EDIT CALC TESTS 1:Edit… 2:SortA(3:SortD(4:ClrList 5:SetUpEditor
Then right arrow to select the CALC menu.	EDIT CALC TESTS 1:1-Var Stats 2:2-Var Stats 3:Med-Med 4:LinReg(ax+b) 5:QuadReg 6:CubicReg 7↓QuartReg
Select option 4: LinReg(ax+b). You are returned to the home screen. Press 2nd L1 , 2nd L2 VARS > Y-VARS 1 1 ENTER	{-1,1,2,4,5}→L1 {-1 1 2 4 5} {-1,2,1,3,5}→L2 {-1 2 1 3 5} LinReg(ax+b) L1, L2,Y1
The calculator returns the line of best fit.	LinReg y=ax+b a=.8771929825 b=.0701754386
Press Y= and observe that the equation of the line is entered as Y_1	Plot1 Plot2 Plot3 \Y1=.87719298245 614X+.0701754385 965 \Y2= \Y3= \Y4= \Y5=
In order to plot the line along with the given points press 2nd STAT PLOT	STAT PLOTS 1:Plot1…Off L1 L2 2:Plot2…Off L1 L2 3:Plot3…Off L1 L2 4↓PlotsOff

Then select 1: for Plot 1. Press Enter to turn this plot on. No other modifications need to be made.	
Press ZOOM	
Choose 9: ZoomStat	
The calculator then displays the data points along with the line of best fit.	

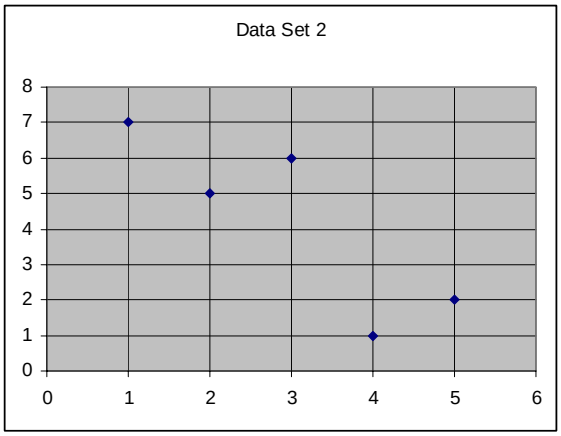
Name_____ Date_____ Period_____

Worksheet 2

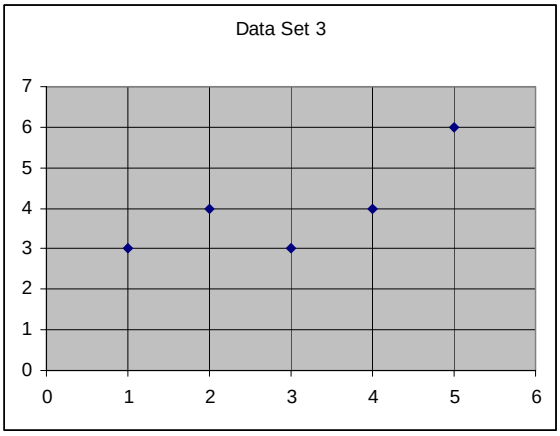
1. Draw the line that you think fits the data points below the best. It will not be possible to find a line that passes through all of the points. Create a data table for the points. Then use your TI-83 to find the equation of the line of best fit for these sets of data and use the STAT PLOT utility to plot the points and the line of best fit.



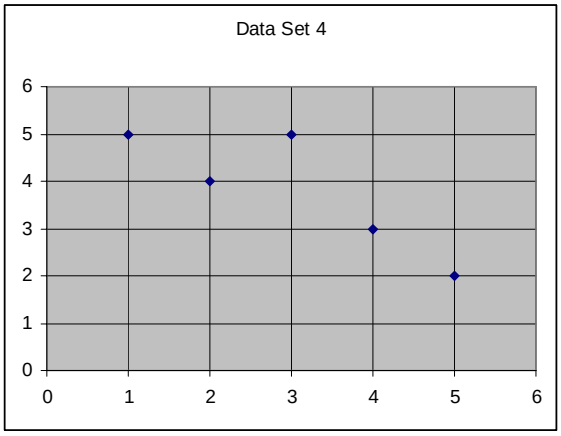
Equation_____



Equation_____



Equation_____



Equation_____

2. For the included data for Washington DC, in 2004, use your TI-83 to write the equation of the line of best fit that relates the number of hours of daylight to the day of the year for the months of January to June using the first day of each month (6 data points). Use as your x-coordinates the number of the day since January first. For example February 1 is day 32. Then use your model to predict the number of hours of daylight on March 17, day 76, and compare this predicted value with the actual data from the table that is 12.03 hours.

   a) The first day of each month: Equation:_____

   March 17:_____

   Repeat the above procedure using data for the 1st and 15th days of each month (12 data points)

   b) The first and 15th of each month: Equation:_____

   March 17:_____

   Repeat again using data for the 1st, 10th, 15th and 20th day of each month (48 data points)

   c) The first, 10th, 15th, and 20th of each month: Equation: _____

   March 17th:_____

3. For the included data for Washington DC, in 2004, use your TI-83 to write the equation of the line of best fit that relates the number of hours of daylight to the day of the year for the months of July through December using the first day of each month (6 data points). Use as your x-coordinates the number of the day since January first. For example February 1 is day 32. Also use your model to predict the number of hours of daylight on November 17, day 321, and compare the calculated value with the actual data from the table which is 10.00 hours.

   a) The first day of each month: Equation:_____

   November 17:_____

   Repeat the above procedure using data for the 1st and 15th days of each month (12 data points)

   b) The first and 15th of each month: Equation:_____

   November 17:_____

Repeat again using data for the 1st, 10th, 15th and 20th day of each month (48 data points)

c) The first, 10th, 15th and 20th of each month: Equation: _____

November 17:_____

4. Discuss your results for questions 2 and 3. Did your model make better or worse predictions as you used more data points?

_____

_____

_____

5. Does there seem to be a relationship between models in questions 2 and 3?

_____

_____

_____

6. Do you think that a linear model is the most appropriate model for this situation? Explain your reasoning.

_____

_____

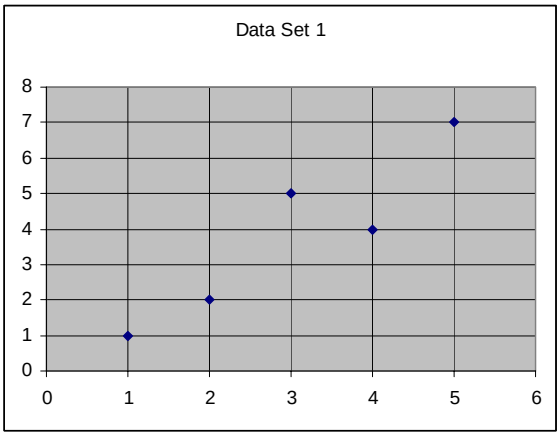
_____

_____

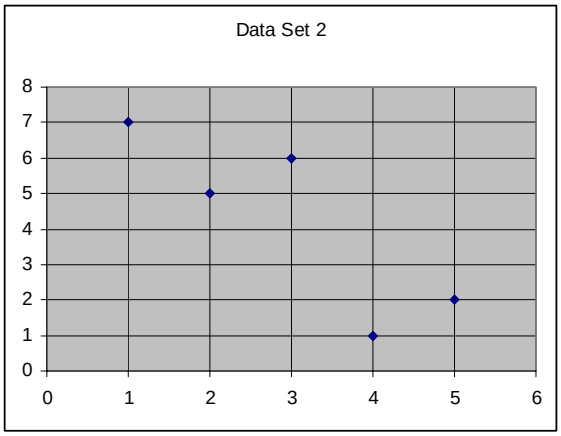
Name_____Key_____ Date_____ Period_____

Worksheet 2 – Answer Key

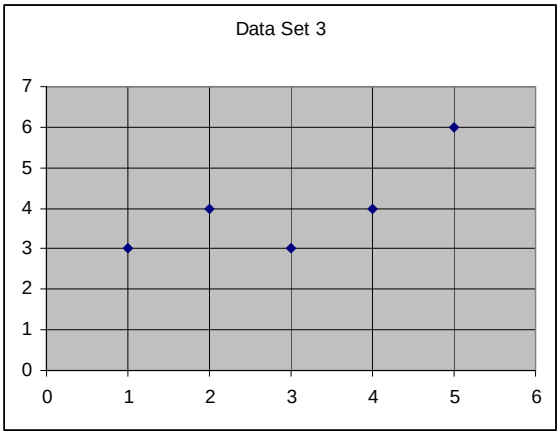
1. Draw the line that you think fits the data points below the best. It will not be possible to find a line that passes through all of the points. Then use your TI-83 to find the equation of the line of best fit for these sets of data and use the STAT PLOT utility to plot the points and the line of best fit.



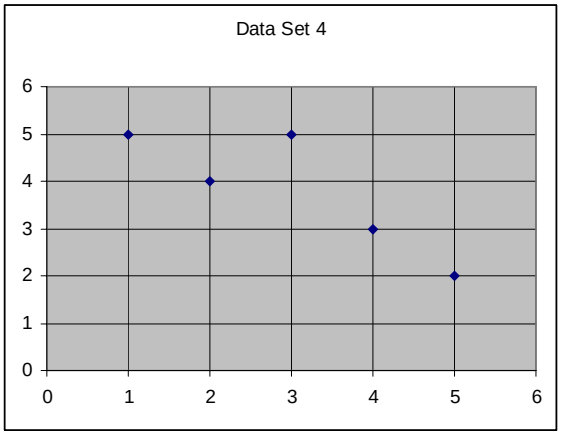
Equation__y = x+0.4_____



Equation____y = -1.4x+8.4_____



Equation____y = 0.6x+2.2_____



Equation_____y= -0.7x+5.9_____

2. For the included data for Washington DC, in 2004, use your TI-83 to write the equation of the line of best fit that relates the number of hours of daylight to the day of the year for the months of January to June using the first day of each month (6 data points). Use as your x-coordinates the number of the day since January first. For example February 1 is day 32. Then use your model to predict the number of hours of daylight on March 17, day 76, and compare this predicted value with the actual data from the table that is 12.03 hours.

   a) The first day of each month: Equation:___y=0.036x+9.275_____

   March 17:___12.056 hours_____

   Repeat the above procedure using data for the 1st and 15th days of each month (12 data points)

   b) The first and 15th of each month: Equation:_y=0.036x+9.273____

   March 17:_12.022 hours_____

   Repeat again using data for the 1st, 10th, 15th and 20th day of each month (48 data points)

   c) The first, 10th, 15th, and 20th of each month:
   Equation: _y=0.032x+9.271_

   March 17th:__12.006 hours___

3. For the included data for Washington DC, in 2004, use your TI-83 to write the equation of the line of best fit that relates the number of hours of daylight to the day of the year for the months of July through December using the first day of each month (6 data points). Use as your x-coordinates the number of the day since January first. For example February 1 is day 32. Also use your model to predict the number of hours of daylight on November 17, day 321, and compare the calculated value with the actual data from the table that is 10.00 hours.

   a) The first day of each month: Equation:___y=-0.031x+20.175__

   November 17:____10.287 hours____

   Repeat the above procedure using data for the 1st and 15th days of each month (12 data points)

   b) The first and 15th of each month: Equation:_y=-0.032x+20.437

November 17:___10.272 hours____

Repeat again using data for the 1st, 10th, 15th and 20th day of each month (48 data points)

c) The first, 10th, 15th and 20th of each month:

Equation: _y=-0.032x+20.569

November 17:____10.270 hours___

4. Discuss your results for questions 2 and 3. Did your model make better or worse predictions as you used more data points?

   The number of data points did not significantly improve the model.

5. Does there seem to be a relationship between models in questions 2 and 3?

   The slopes of the models were very similar in magnitude, but the January to June model had a positive slope while the July to December model had a negative slope.

6. Do you think that a linear model is the most appropriate model for this situation? Explain your reasoning.

   A linear model can be used, however, two equations are needed. A better model would require only one equation. A quadratic or sinusoidal model would probably be better.

Handout 5 – Other City Data

	Denver, CO		Anchorage, AK		Miami, FL	
Month	*Rise*	*Set*	*Rise*	*Set*	*Rise*	*Set*
January	721	1646	1014	1552	707	1741
February	708	1719	921	1706	705	1804
March	632	1753	758	1826	644	1823
April	543	1825	621	1947	612	1838
May	500	1855	449	2106	544	1852
June	434	1922	335	2221	529	1908
July	436	1932	328	2238	533	1916
August	459	1912	433	2136	547	1907
September	528	1830	553	2004	601	1840
October	557	1742	707	1829	613	1807
November	629	1657	829	1656	630	1739
December	702	1636	947	1551	651	1729
17-Mar	607	1809	708	1908	628	1831
17-Nov	647	1642	912	1616	641	1731

All data is for the 1st day of the month.

Latitude and longitude for each city is as follows:

Denver, CO: W104° 59’ N39° 43’
Anchorage, AK: W149° 52’ N61° 13’
Miami, FL: W80° 13’ N25° 47’

Name_____ Date_____ Period_____

Worksheet 3

You are planning to expand your business to other cities. You need to determine how many hours of sunlight each city has to calculate your hours of operation.

1. For the included data for Denver, CO in 2004, use your TI-83 to write the equation of the line of best fit that relates the number of hours of daylight to the day of the year for the months of January to June using the data in Handout 5. Use as your x-coordinates the number of the day since January first. For example February 1 is day 32. Also use your model to predict the number of hours of daylight on March 17, day 76, and compare that value with the actual data.

   The first day of each month: Equation: _____

   March 17:_____

2. For the included data for Denver, CO in 2004, use your TI-83 to write the equation of the line of best fit that relates the number of hours of daylight to the day of the year for the months of July through December using the data in Handout 5 below. Use as your x-coordinates the number of the day since January first. For example February 1 is day 32. Also use your model to predict the number of hours of daylight on November 17, day 321, and compare that value with the actual data.

   The first day of each month: Equation: _____

   November 17:_____

3. Compare the linear regression model you calculated for Denver with the linear regression model you calculated for Washington, DC. Discuss similarities and differences in slope and y-intercepts.

_____

_____

_____

4. Why are the models similar or different?

_____

_____

_____

5. Repeat Steps 1 and 4 for Anchorage, AK.

January to June: _____ March 17: _____

July to December: _____ November 17: _____

Compare the linear regression model you calculated for Anchorage with the linear regression model you calculated for Washington, DC. Discuss similarities and differences in slope and y-intercepts.

_____

_____

_____

Why are the models similar or different?

_____

_____

_____

6. Repeat Steps 1 and 4 for Miami, FL.

January to June: _____ March 17: _____

July to December: _____ November 17: _____

Compare the linear regression model you calculated for Miami with the linear regression model you calculated for Washington, DC. Discuss similarities and differences in slope and y-intercepts.

_____

_____

_____

Why are the models similar or different?

Name_____ Date_____ Period_____

Worksheet 3 – Answer Key

See the attached spreadsheet for answers to the number of daylight hours for the 1st day of the month for each city.

You are planning to expand your business to other cities. You need to determine how many hours of sunlight each city has to calculate your hours of operation.

For the included data for Denver, CO in 2004, use your TI-83 to write the equation of the line of best fit that relates the number of hours of daylight to the day of the year for the months of January to June using the data in Handout 5. Use as your x-coordinates the number of the day since January first. For example February 1 is day 32. Also use your model to predict the number of hours of daylight on March 17, day 76, and compare that value with the actual data.

The first day of each month: Equation: $y = 0.04x + 9.15$

March 17: 12.2 hrs (Actual 12.02 hrs)

1. For the included data for Denver, CO in 2004, use your TI-83 to write the equation of the line of best fit that relates the number of hours of daylight to the day of the year for the months of July through December using the data in Handout 5 below. Use as your x-coordinates the number of the day since January first. For example February 1 is day 32. Also use your model to predict the number of hours of daylight on November 17, day 321, and compare that value with the actual data.

   The first day of each month: Equation: $y = -0.04x + 23.3$

   November 17: 10.42 hrs (Actual 10.6 hrs)

2. Compare the linear regression model you calculated for Denver with the linear regression model you calculated for Washington, DC. Discuss similarities and differences in slope and y-intercepts.

The models are very similar especially for January to June. The slopes are very close in both cases. The y-intercepts vary, but with a “best-fit” linear regression model some variation is expected since data is slightly different between the two cities.

3. Why are the models similar or different?

The models are similar because Denver and Washington, DC are on a similar latitude (Denver is N39° 43’ and Washington, DC is N38° 53’). Consequently both cities will have the same tilt towards the sun and receive similar amounts of sunlight.

4. Repeat Steps 1 and 4 for Anchorage AK.

January to June: $y = 0.092x + 5.46$ March 17: 12.45 hrs (Actual 12.0 hrs)

July to December: $y = -0.089x + 35.8$ November 17: 7.1 hrs (Actual 7.1 hrs)

Compare the linear regression model you calculated for Anchorage with the linear regression model you calculated for Washington, DC. Discuss similarities and differences in slope and y-intercepts.

The slope for Anchorage is about twice that of Washington, DC. Reviewing the data indicates that Anchorage has a much greater range of sunlight hours compared with Washington, DC. Therefore a steeper slope is reasonable. The y-intercepts also vary considerably which is expected since the slopes are different.

Why are the models similar or different?

The models are different because Anchorage is much farther North. Consequently during the summer the tilt of the Earth provides Anchorage with much more sunlight. During the winter Anchorage tilts away from the sun and receives much less sunlight.

5. Repeat Steps 1 and 4 for Miami, FL.

January to June: $y = 0.017x + 10.95$ March 17: 12.24 hrs (Actual 12.1 hrs)

July to December: $y = -0.018x + 17.1$ November 17: 11.3 hrs (Actual 11.5 hrs)

Compare the linear regression model you calculated for Miami with the linear regression model you calculated for Washington, DC. Discuss similarities and differences in slope and y-intercepts.

The slope for Miami is about one-half of the slope for Washington, DC. Reviewing the data reveals that Miami has a smaller range of data than Washington, DC and therefore the slope will flatten more horizontally. The y-intercepts vary, but not as much as might be expected.

Why are the models similar or different?

The models are different because Miami is much farther South. As you near the equator you will find the number of hours of daylight to vary less compared with cities farther North or South of the equator.