Title: CSI: Circleville

Brief Overview:

Students will learn the properties of the measures of the angles of a circle and their arcs. Students will use these properties and put together clues to solve the mystery of the kidnapped Euclidean Cucumber.

NCTM Content Stand/National Science Education Standard

- Understand and use the appropriate terminology to describe the parts of a circle.
- Use both direct and indirect measurement to problem solve.
- Use deductive reasoning to problem solve.

Grade/Level:

Grades 9 – 12

Duration/Length

Three 60 minute class periods

Student Outcomes:

Students will be able to:

- Construct a circle and label its parts
- Analyze the properties of the circle angles and arcs.
- Use direct and indirect measurement.
- Explore relationships and text conjectures about circle angles and arcs.
- Use a geoboard to establish the validity of geometric using deduction.
- Use coordinate geometry along with navigational skills to problem solve

Materials and Resources:

- Compasses
- Protractors
- Geoboards
- Rubberbands
- Sheet protectors
- Overhead markers
- Rulers
- Post-it notes
- Copies of worksheets:
    - Circles and Their Parts
    - Central and Inscribed Angles

- Properties of Inscribed Angles
- The First Clue
- Central and Inscribed Angles (Lesson 1 Homework)
- It's Time for Central Angles!
- Geoboard Paper
- Angles formed by Chords, Tangents, and Secants
- Sector Smackdown
- Angles formed by Chords, Secants, and Tangents (Lesson 2 Homework)
- Memory Cards
- The Third and Final Clue
- Summative Assessment
- Summative Assessment Rubric

Development/Procedures:

Lesson 1

Preassessment - Students will review the basic parts of a circle using the worksheet, "Circles and Their Parts." Students will draw and label the parts of the circle using the words provided in the word box. Students will then describe relationships between the parts of the circle,. The teacher will solicit relationships not already listed and review the questions.

Launch- Students will work in groups with geoboards and protractors to complete student "Properties of Inscribed Angles". Prompt discussion as to what would happen if the vertex was moved from the center and onto the circle.

Would the measure of the angle change? Would the measure of the arc change? Does it matter where on the circle the vertex is placed? Collect all opinions on the board for students to analyze during the activity.

Allow students to explore properties of inscribed angles while circulating to provide assistance as needed. The goal is to provide as little guidance with regards to the relationship they are discovering. Limit help to constructing on the geoboard and using the protractor.

Project overhead copy of "Properties of Inscribed Angles"
Use random number generator on calculator to select students to contribute values to the overhead table. Prompt phrasing for the observed relationship. For example, "The measure of the inscribed angle is half of the measure of its intercepted arc" or "The measure of the intercepted arc is twice the measure of the inscribed angle." Challenge students with the following ideas:

- Does it matter which piece of information you are provided in the diagram?
- Readdress if it matters where the vertex of the inscribed angle lies on the circle?
- What do you predict will happen if a tangent intersects the diameter? Where is the vertex? What is the measure of the angle formed?

Teacher Facilitation – Provide students with two sticky notes and copies of, "Central and Inscribed Angles". The teacher will use the resource sheet to introduce students to various angles within a circle, and model the relationships between the angles and intercepted arcs. Students should attempt to answer Part A of the resource sheet before the teacher introduces the angle relationships. As each concept is introduced, students will be asked to check their answers to see how much they know.

Have a student identify the type of angle in circle 1 and justify their reasoning. Follow the reasoning by introducing the concepts of intercepted arc and sector. Instruct students to shade and label the sector, which is region *x*. Take the opportunity to point out minor and major arcs, as well as how each are named. Use the sticky notes to cover parts of the circle, to emphasize which angles form major and minor arcs. For example, have students place the sticky notes over the circle so that only ∠EOF is showing. Challenge students to find $m\angle$EOF, which is 100°. Thus, 100° = $m\overset{\frown}{EF}$. $\overset{\frown}{FTE}$ is a major arc because it measures more than 180°.

Find the measure of $\overset{\frown}{FTE}$ = 360° – 100° = 260°. Note that you need three points to name a major arc. Check answers to problems 1 and 2 at the top of the page.

Circles 3 through 6 are all examples of inscribed circles. Emphasize that the vertex of an inscribed angle is located on the circle. There are three cases of inscribed angles.

Have a student describe the angle in circle 3. The vertex is on circle while a ray goes through the center. Have students copy this characteristic to the right of circle. Use post-its to cover segments *y* and *z*. Use this visual to address the theorem that the measure of an inscribed circle is one half the measure of its intercepted arc. The measure of $\overset{\frown}{GI}$ is given, 60°. Find the measure of $\overset{\frown}{IHG}$ = 360° – 60° = 300°. Continue with circle 3 to find m∠GHI = 30° = ½ the measure of the intercepted arc.

Have another student describe the angle in circle 4. The vertex is on the circle and center of circle is in the angle. Have students use post-its as before. Continue to model inscribed angles in circle 4. Prompt students for various measures.:

- Measure of $\overset{\frown}{JL}$ = 110°, given, measure of $\overset{\frown}{LKJ}$ = 360° – 110° = 150°, m∠JKL = 55° = ½ the measure of the intercepted arc.

Have students describe the angle in circle 5. The vertex is on the circle and the center of circle is outside the angle. This is an inscribed angle. Shade a sector. Give students a few minutes to find the measures in circles 5. Students may check with their partner.

- *m*∠LMN = 21°, given, measure of $\overset{\frown}{LN}$ = 42° = twice the measure of its intercepted arc, measure of $\overset{\frown}{NML}$ = 360° - 42° = 318°.

Examine circle 6. Find the measures and justify. m∠EFG = 90°= ½ of semicircle, measure of $\overset{\frown}{GE}$ = 180° = semicircle.

Distribute "Crime Scene Maps" and "The First Clue". Also distribute sheet protectors and overhead markers to the students. Direct students to slide the Crime Scene Map into the sheet protector. This will allow them to write on the maps with the markers without fear of making mistakes. Direct students to complete the "The First Clue". Collect maps at the end of class to review. Teachers may wish to make an overhead of the "Map Key" to place over student products. This will make checking quite quick! Assign "Central and Inscribed Angles" for homework.

Embedded Assessment - Randomly call on students. Circle the room to make sure students are on task and using the geoboards and protractors correctly. Have other students repeat answers that were just given.

Reteaching/Extension - Students who understand the process help those who are having difficulty with the compasses, geoboards, and protractors. How would you find the area of the regions of central and inscribed angles?

Lesson 2 - Preassessment – Students review the "Central and Inscribed Angles" homework in groups to reaffirm concepts from Lesson 1. Use a "2 before me" strategy requesting that students ask their group members prior to seeking teacher assistance.

Prior to the beginning of class, arrange desks to form cooperative groups of three. Number each desk to according to how many students are in the class (i.e. If you have 30 students, label desk 1 to 30). This will place responsibility on each group member during presentation time.

Use the numbered desks to randomly lace students into groups of three. Provide each group with a minimum of one 12-sector geoboard (one for each student is preferred). Provide each group with a minimum of one protractor (one for each student is preferred). Provide each student with a minimum of one straightedge (one for each student is preferred.)

Distribute "It's Time for Central Angles!" handout. Allow students to complete this brief preassessment in the cooperative groups. Allow five minutes, and then project overhead copy of "It's Time for Central Angles!" Use the random number generator on your calculator to select students to report their answers to each of the six problems. Discuss as needed. (5 minutes)

Launch - Review manipulating angles on the geoboard to ensure that students are comfortable constructing and determining measures of the central angles and their intercepted arcs. Use the intervals created by the pegs. For example, if a circular geoboard is divided into 12 congruent arcs, then each arc would measure 30 degrees. Thus, a central angle constructed with rays intersecting the circle at adjacent pegs would measure 30 degrees, as well.

Teacher Facilitation - Distribute "Angles formed by Chords, Tangents, and Secants" and a sheet of "Geoboard Paper." Allow students to explore and complete the investigation of angles formed by the intersection of two chords or secants. Circulate and limit assistance to measuring with the protractor and construction of the figures on the geoboard.

Project an overhead copy of "Properties of Angles formed by Chords or Secants". Use random number generator on calculator to select students to contribute values to the overhead table. Prompt phrasing for the observed relationship. For example, "The measure of an angle formed by the intersection of two chords or

secants is half the sum of its intercepted arcs." Pose the following questions:

- Does it matter where the vertex lies for this relationship?
- Will the vertex always lie in the interior of the circle when two secants intersect?

Allow students to return to their handout and complete the investigation of angles formed by the intersection of two secants or tangents. Again, circulate and limit assistance to measuring and construction.

Project overhead copy of "Properties of Angles formed by Secants or Tangents". Use random number generator on calculator to select students to contribute values to the overhead table. Prompt phrasing for the observed relationship. For example, "The measure of an angle formed by the intersection of two secants or tangents is half the difference of its intercepted arcs." Ask,

- Does it matter where the vertex lies for this relationship?
- Will the vertex always lie in the exterior of the circle when two tangents intersect?

At this point, students should be comfortable calculating the values of the three categories of angles discussed above. Now, students will gain confidence in "working backwards." That is, setting up an equation to find a missing angle.

Project an overhead copy of "Classwork for Angles formed by Chords, Tangents, and Secants." Direct Students to the "Examples" section. Guide students through each example using the "think-aloud" strategy, modeling your thought process as you determine which relationship the angles exhibit and how to set up the problem using an equation.

Allow students time to complete the three "Group Practice" problems to reinforce the solution methods.

Student Application - Distribute "Sector Smackdown" handout and the students' "Crime Scene Maps". Take time to discuss the connection between angle measurements and cardinal directions on a compass. Help students label the angles in the top portion of their handout. Students will need a basic understanding of this topic prior to completing the remainder of the handout.

Students will take remaining time to solve the 10 questions and narrow down the location of the kidnapper on the "Crime Scene Map"

Collect the maps at the end of class. Distribute and direct students to work on the lesson two homework, “Angles formed by Chords, Secants, and Tangents”.

Reteaching - Students who are struggling may wish to use Post-It notes to cover up extraneous information in the figures. This allows them to focus on only the intercepted arcs of the angle in question. Students who do not respond to this technique generally find it useful to create a table relating the location of the vertex with respect to the circle and the rule describing the relationship between the intercepted arcs. Then students can follow a simple three-step process.

Where is the Vertex?	**Angle-Arc Relationship**
On the Circle	Half the intercepted arc
Inside of the Circle	Half the **sum** of the intercepted arcs
Outside of the Circle	Half the **difference** of the intercepted arcs

1. Find the vertex and cover up with Post-Its
2. Substitute into the relationship from the table
3. Solve for the missing arc/angle

Extension - The use of angles and sectors is an important concept in the instruction of polar coordinates, as well. Educators may wish to incorporate polar to discuss locations of buildings on the “Crime Scene Map”.

Lesson 3

Preassessment - Allow students five minutes to share homework results and discuss and resolve differences in answers. Then, take five additional minutes and randomly select students, as in previous lessons, to contribute their results for discussion. Resolve any homework concerns prior to moving in to the day's lesson

Launch - Distribute a set of "Memory Cards" to each group of three. Reinforce the importance of knowing vocabulary and angle relationships in lesson 3. Students must be very comfortable with the relationships in order to be successful in the lesson. Circulate as needed to resolve disputes and observe student knowledge of new content.

Student Application - Distribute "The Third and Final Clue" and their Crime Scene Maps. Students work in groups of three to solve the angle puzzles and complete their Crime Scene Map. Be prepared to distribute Post-It notes as needed to help students focus on only the angle or arc in question. At this point, the teacher should circulate and help students focus, rather than explain the relationships between the angles.

Embedded Assessment - Students will be stretched in the second puzzle. They must realize that the triangles formed by two radii will be isosceles. Thus, the base angles will be congruent.

After all groups have completed the puzzles and completed their Crime Scene Maps, solicit assistance in completing the puzzles on an overhead copy. Encourage students to discuss alternative methods to determine angles and arcs. Perhaps, they used a different order to determine their solution. Having students realize there are multiple solutions strategies is an important part of accepting these new concepts.

Reteaching - Often, allowing students to ignore the order of the questions and simply solve the puzzle as they see the solutions can be a great strategy. The important part of learning is that there are multiple strategies. These puzzles can be simplified by removing additional chords or secants. By solving a simplified puzzle, students can gain confidence. Return the chords and secants to the diagram one at a time, and solve the puzzle again, focusing on the repetitive nature of the solution process.

Extension -Remove all givens from the puzzles. Ask students to examine the "naked" puzzles and determining what information is needed to solve the puzzle. This will certainly reinforce the notion of angle

relationships as equations, by having students determine knowns and unknowns.

Summative Assessment:

Students have been finding angle and arc measurements throughout the unit and discussing the relationships in writing.

Rather than ask students to continue solving repetitive problems, have students create their own angle puzzles using the "Summative Assessment" handout. This will still assess vocabulary knowledge, as well as all of the angle relationships, without appearing to be a traditional assessment. You may wish to provide students with a larger circle for the assessment.

It will take some time to grade, if you want to check them yourself. This is due to an infinite number of combinations of arcs and angles. You may wish to allow students to check each others' puzzles for accuracy.

Authors:

Colin M Reinhard
Paint Branch High School
Montgomery County Public Schools, MD

Karen Zipp
Coppin Academy
Baltimore City Public Schools, MD

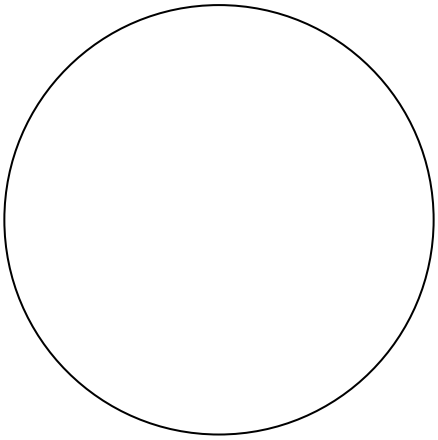
Circles and Their Parts

Name: _____

Date: _____

Part A: Draw and label the parts of the circle located in the word box.

1. List relationships between various parts of the circle.



Word Box
Center
Diameter
Radius
Chord
Circumference

Part B: Answer the following questions about how the parts of the circle relate.

1. Is the diameter of a circle a chord? Use mathematics to justify your answer.

2. Is the radius of a circle a chord? Use mathematics to justify your answer.

3. How are the radius and diameter different than the line segment labeled chord on your circle?

4. What is the relationship between the length of the diameter of a circle and the length of the radius of that circle?

5. Explain the circumference of a circle with respect to the center of the circle.

6. Identify any other relationships within the circle and its labeled parts.

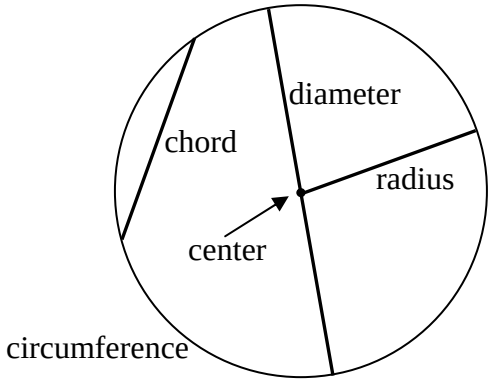
Circles and Their Parts

Name: ANSWER KEY_

Date: _____

Part A: Draw and label the parts of the circle located in the word box.

1. List relationships between various parts of the circle.



Word Box
Center
Diameter
Radius
Chord
Circumference

Part B: Answer the following questions about how the parts of the circle relate.

1. Is the diameter of a circle a chord? Use mathematics to justify your answer.
   Only the diameter is a chord because it is a segment with both endpoints on the circle.

2. Is the radius of a circle a chord? Use mathematics to justify your answer.
   The radius is not a chord because only one endpoint is on the circle.

3. How are the radius and diameter different than the line segment labeled chord on your circle?
   The diameter and radius both touch the center of the circle.

4. What is the relationship between the length of the diameter of a circle and the length of the radius of that circle?
   Radius is ½ the length of the diameter or diameter is twice the length of the circle.

5. Explain the circumference of a circle with respect to the center of the circle.
   The circumference of the circle consists of all points an equal distance from the circle.

6. Identify any other relationships within the circle and its labeled parts.

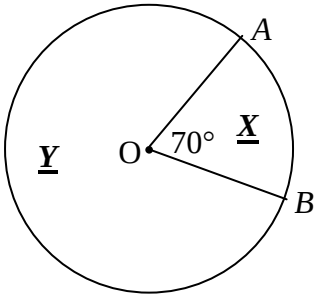
Central And Inscribed Angles Name: _____

CSI: Circleville Pd: _____ Date: _____

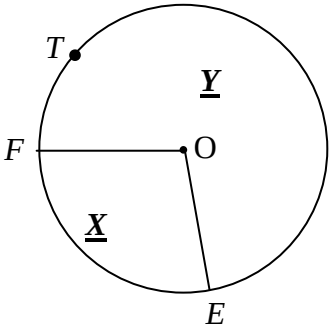
1. The vertex of a central angle is located _____.
2. The measure of a central angle and its intercepted arc are _____.
3. The vertex of an inscribed angle is located _____.
4. The measure of an inscribed angle is equal to ½ the measure of its_____.
5. The measure of a semicolon is equal to _____.
6. The measure of a minor arc is _____.
7. The measure of a major arc is _____.

Central Angles

1.



2.

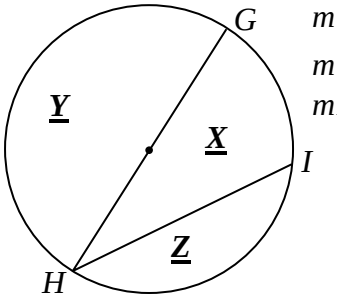


$m\angle EOF = 100°$

$m\overset{\frown}{FTE}$ = _____

Inscribed Angles

3.

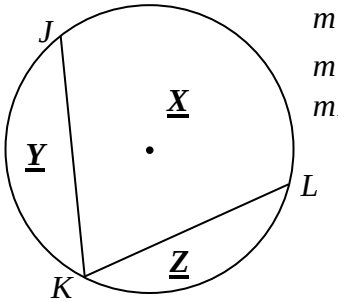


$m\overset{\frown}{GI} = 60°$

$m\overset{\frown}{IHG}$ = _____

$m\angle GHI$ = _____

4.

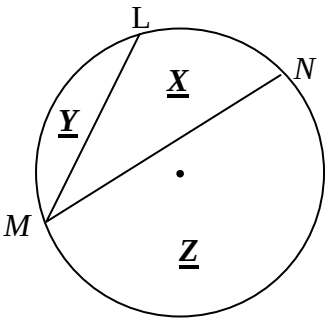


$m\overset{\frown}{JL} = 110°$

$m\overset{\frown}{LKJ}$ = _____

$m\angle JKL$ = _____

5.

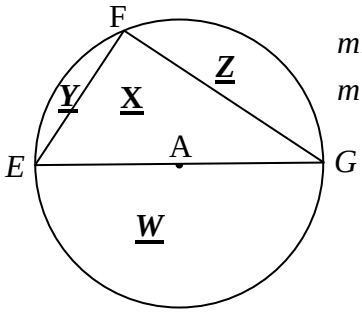


$m\angle LMN = 21°$

$m\overset{\frown}{LN}$ = _____

$m\overset{\frown}{NML}$ = _____

6.



$m\angle EFG$ = _____

$m\overset{\frown}{GE}$ = _____

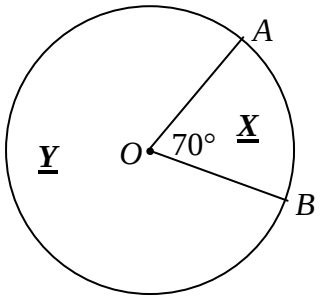
Central And Inscribed Angles Name: **_ANSWER KEY_____**

CSI: Circleville Pd: _____ Date: _____

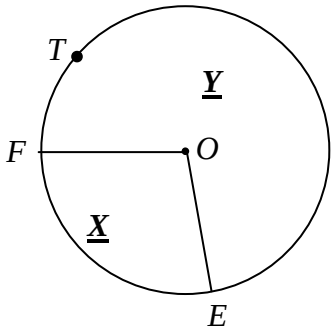
1. The vertex of a central angle is located **on the center of the circle.**.
2. The measure of a central angle and its intercepted arc are **equal**.
3. The vertex of an inscribed angle is located **on the circle.**
4. The measure of an inscribed angle is equal to ½ the measure of its **intercepted arc**.
5. The measure of a semicolon is equal to **180 degrees.**
6. The measure of a minor arc is **less than 180 degrees.**
7. The measure of a major arc is **more than 180 degrees.**

Central Angles

1.



2.

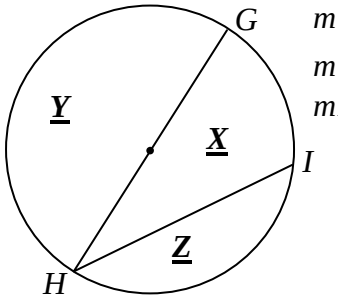


$m\angle EOF = 100°$

$m\overset{\frown}{FTE}$ = **260°**

Inscribed Angles

3.

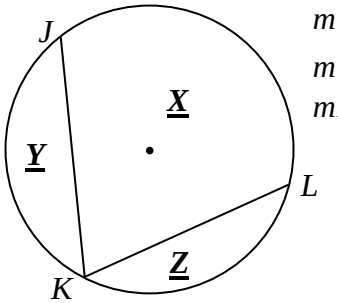


$m\overset{\frown}{GI} = 60°$

$m\overset{\frown}{IHG}$ = **100°**

$m\angle GHI$ = **60°**

4.

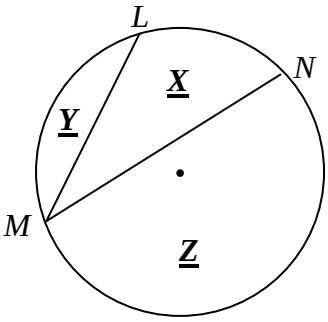


$m\overset{\frown}{JL} = 110°$

$m\overset{\frown}{LKJ}$ = **250°**

$m\angle JKL$ = **55°**

5.

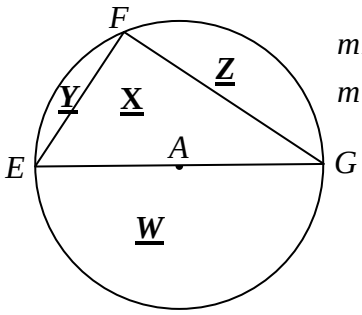


$m\angle LMN = 21°$

$m\overset{\frown}{LN}$ = **42°**

$m\overset{\frown}{NML}$ = **318°**

6.



$m\angle EFG$ = **90°**

$m\overset{\frown}{GE}$ = **180°**

Properties of Inscribed Angles Name: _____

CSI: Circleville Pd: _____ Date: _____

1. Begin by constructing a central angle on your geoboard. Your group should construct central angles measuring 60°, 120°, and 240°.

2. Now, move the vertex so that it lies on the circle, creating an inscribed angle.

3. Use your protractor to measure the inscribed angle.

4. Move the vertex to a different point on the circle. Does this change the measure of the inscribed angle? Measure with your protractor to check.

5. Complete the table below using the measurements collected by your group.

Measure of Central ∠	Measure of Inscribed ∠
60°	
120°	
240°	
x°	

6. If an inscribed angle measures 90°, what would the corresponding central angle look like? Use your geoboard to explore this question.

Properties of Inscribed Angles Name: __**ANSWER KEY**_____

CSI: Circleville Pd: _____ Date: _____

1. Begin by constructing a central angle on your geoboard. Your group should construct central angles measuring 60°, 120°, and 240°.

2. Now, move the vertex so that it lies on the circle, creating an inscribed angle.

3. Use your protractor to measure the inscribed angle.

4. Move the vertex to a different point on the circle. Does this change the measure of the inscribed angle? Measure with your protractor to check.

   No, as long as the vertex remains on the circle, the measure of the inscribed angle will remain the same

5. Complete the table below using the measurements collected by your group.

Measure of Central ∠	Measure of Inscribed ∠
60°	**30°**
120°	**60°**
240°	**120°**
x°	**(x / 2) °**

6. If an inscribed angle measures 90°, what would the corresponding central angle look like? Use your geoboard to explore this question.

   The central angle would be a straight angle. The rays of the angle would form the diameter.

The First Clue Name: _____

CSI: Circleville Pd: _____ Date: _____

The Clue

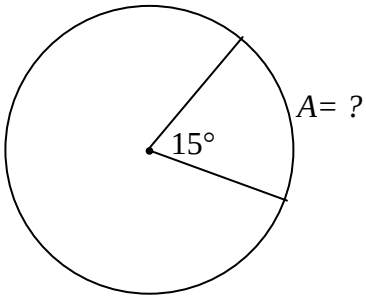
Porter Polygon, intrepid reporter for the city paper, The Circleville Circulator, steps into the CSI: Circleville office. He has turned a pale shade of green. When you ask what's wrong, with a gasping voice, he explains that this note was sent to him at the newspaper building.

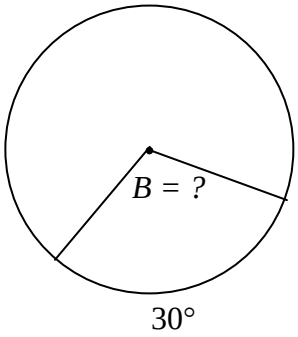
City of Circleville, I've got your number,
Not only that, I've stolen your sacred cucumber.
In searching the map, you'll need a keen mind,
But first, of a circle, the center you must find.
Then, with some skill you'll come to the point
At the end of a radius, so please don't disappoint
Your citizens worried, your mayor in a tizzy,
CSI: Circleville, your task won't be easy!
-The Green Locus

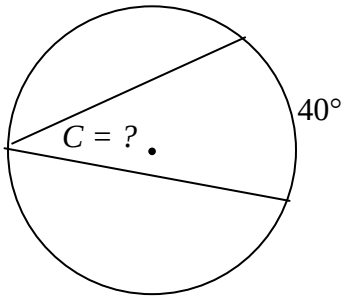
The Investigation

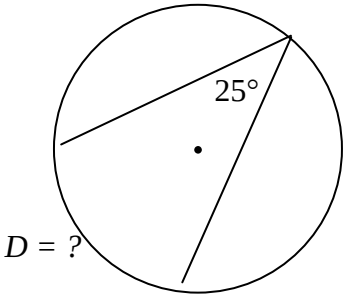
At the bottom of the note you find four problems.

Directions: Solve each problem to help pinpoint two locations on your Crime Scene Map. Drawings are **not** to scale!









Center of search area **Endpoint of radius of search area**

A , ***B*** ***C*** , ***D***

Plot these **points** on your **Crime Scene Map.**

The First Clue Name: **_ANSWER KEY_____**

CSI: Circleville Pd: _____ Date: _____

The Clue

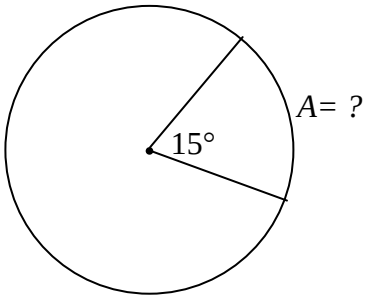
Porter Polygon, intrepid reporter for the city paper, The Circleville Circulator, steps into the CSI: Circleville office. He has turned a pale shade of green. When you ask what's wrong, with a gasping voice he explains that this note was sent to him at the newspaper building.

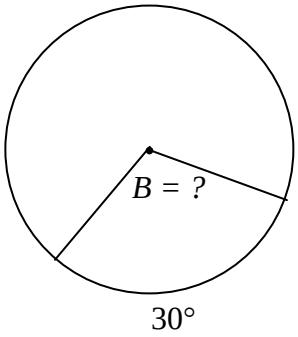
City of Circleville, I've got your number,
Not only that, I've stolen your sacred cucumber.
In searching the map, you'll need a keen mind,
But first, of a circle, the center you must find.
Then, with some skill you'll come to the point
At the end of a radius, so please don't disappoint
Your citizens worried, your mayor in a tizzy,
CSI: Circleville, your task won't be easy!
-The Green Locus

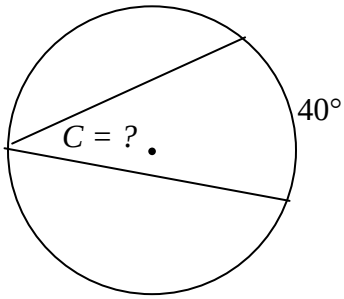
The Investigation

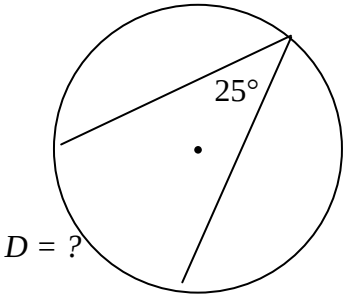
At the bottom of the note you find four problems.

Directions: Solve each problem to help pinpoint two locations on your Crime Scene Map. Drawings are **not** to scale!









Center of search area **Endpoint of radius of search area**

15 , 30 **20 , 50**

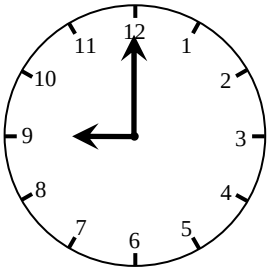
Plot these **points** on your **Crime Scene Map.**

It's Time for Central Angles! Name: _____

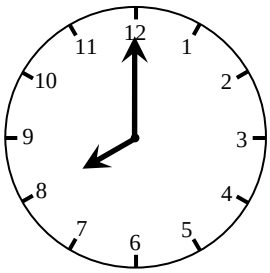
CSI: Circleville Pd: _____ Date: _____

Directions: Complete each of the questions below by using properties of central angles, inscribed angles, and their intercepted arcs.

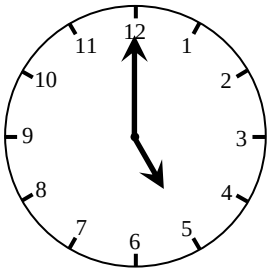
1. Find the measure of the smallest central angle generated by the hands of a clock at 9:00.



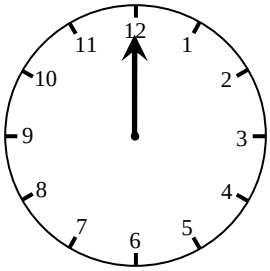
2. Find the measure of the largest central angle generated by the hands of a clock at 8:00.



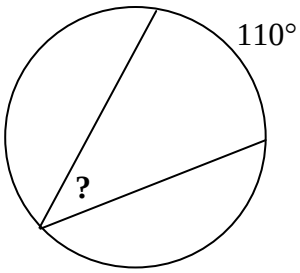
3. Find the measure of the minor arc generated by the hands of a clock at 5:00.



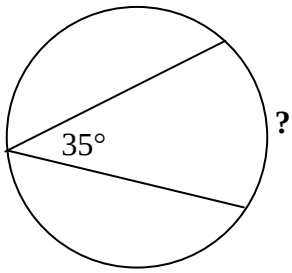
4. If the long hand of a clock is at 12, where would the short hand lie so that the central angle measures 180°?



5. Find the degree measure of the labeled angle.



6. Find the degree measure of the labeled minor arc.



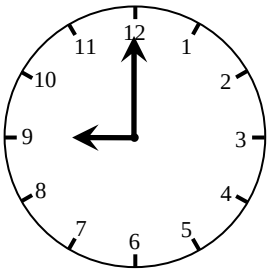
It's Time for Central Angles! Name: __**ANSWER KEY**_____

CSI: Circleville Pd: _____ Date: _____

Directions: Complete each of the questions below by using properties of central angles, inscribed angles, and their intercepted arcs.

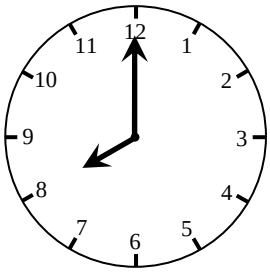
1. Find the measure of the smallest central angle generated by the hands of a clock at 9:00.

90°



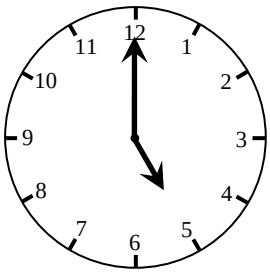
2. Find the measure of the largest central angle generated by the hands of a clock at 8:00.

240°



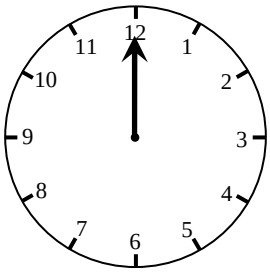
3. Find the measure of the minor arc generated by the hands of a clock at 5:00.

150°



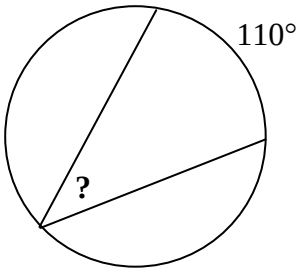
4. If the long hand of a clock is at 12, where would the short hand lie so that the central angle measures 180°?

6



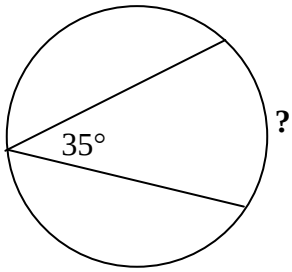
5. Find the degree measure of the labeled angle.

55°



6. Find the degree measure of the labeled minor arc.

70°

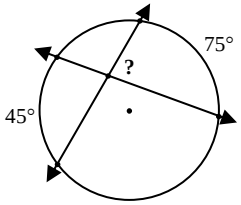


Angles formed by Chords, Tangents, and Secants Name: _____

CSI: Circleville Pd: _____ Date: _____

Properties of Angles formed by Chords or Secants

1. Begin by constructing two intersecting chords (or secants) on your geoboard. Your group should construct three unique figures. An example is shown below.



2. Now, use the pegs to determine the measure of two opposite intercepted arcs, as shown in the diagram above.

3. Use your protractor to measure the angle formed by the two intersecting chords (or secants).

4. Complete the table below using the measurements collected by your group.

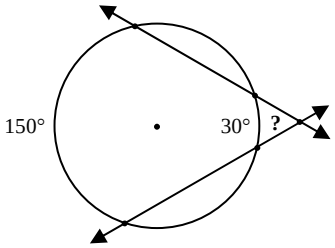
Measure of Intercepted Arc	Measure of Opposite Intercepted Arc	Measure of ∠ formed by Chords
$x°$	$y°$	

5. How would you determine the measures of the remaining three angles? Justify using terms we have learned in geometry.

Properties of Angles formed by Secants or Tangents

For the next figure, we will not be able to use the geoboard. Use your ***geoboard paper*** to construct your figure.

1. Begin by constructing two intersecting secants or tangents on the grid below. Your group should construct three unique figures. An example is shown below.



2. Now, use the "pegs" on the geoboard paper to determine the measure of two opposite intercepted arcs, as shown in the diagram above.

3. Use your protractor to measure the angle formed by the two intersecting secants (or tangents).

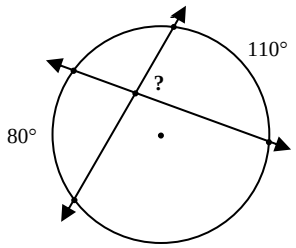
4. Complete the table below using the measurements collected by your group.

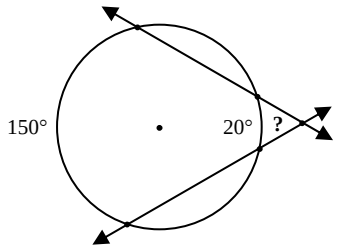
Measure of Intercepted Arc	Measure of Opposite Intercepted Arc	Measure of ∠ formed by Secants
$x°$	$y°$	

5. Describe how you would find the measure of the angle created by two intersecting tangents.

Classwork for "Angles formed by Chords, Tangents, and Secants"

EXAMPLES

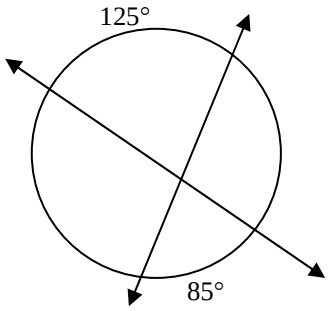




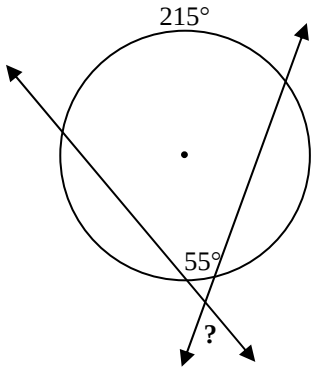
GROUP PRACTICE

Directions: Complete the following three questions in your group. Be prepared to explain how you found your answer!

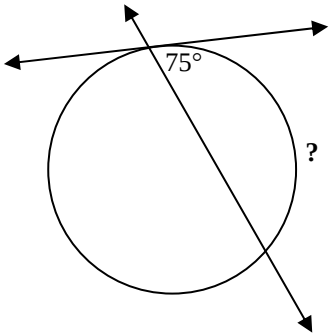
1.



2.



3.

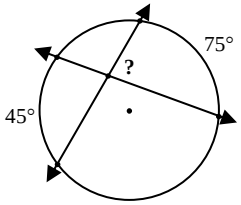


Angles formed by Chords, Tangents, and Secants Name: ___**ANSWER KEY**_____

CSI: Circleville Pd: _____ Date: _____

<u>Properties of Angles formed by Chords or Secants</u>

1. Begin by constructing two intersecting chords (or secants) on your geoboard. Your group should construct three unique figures. An example is shown below.



2. Now, use the pegs to determine the measure of two opposite intercepted arcs, as shown in the diagram above.

3. Use your protractor to measure the angle formed by the two intersecting chords (or secants).

4. Complete the table below using the measurements collected by your group.

Measure of Intercepted Arc	Measure of Opposite Intercepted Arc	Measure of ∠ formed by Chords
i.e. 45°	**i.e. 75°**	**i.e. 60°**
$x°$	$y°$	$(x + y)° / 2$

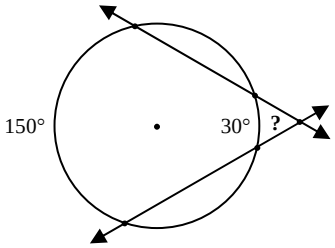
5. How would you determine the measures of the remaining three angles? Justify using terms we have learned in geometry.

   The known angle is part of a vertical pair, which means the angle across the vertex is congruent. The known angle is also part of a linear pair, meaning the adjacent angle is supplementary.

Properties of Angles formed by Secants or Tangents

For the next figure, we will not be able to use the geoboard. Use your ***geoboard paper*** to construct your figure.

6. Begin by constructing two intersecting secants or tangents on the grid below. Your group should construct three unique figures. An example is shown below.



7. Now, use the "pegs" on the geoboard paper to determine the measure of two opposite intercepted arcs, as shown in the diagram above.

8. Use your protractor to measure the angle formed by the two intersecting secants (or tangents).

9. Complete the table below using the measurements collected by your group.

Measure of Intercepted Arc	Measure of Opposite Intercepted Arc	Measure of ∠ formed by Secants
i.e. 30°	**i.e. 150°**	**i.e. 60°**
$x°$	$y°$	$\mathbf{\lvert x - y \rvert° / 2}$

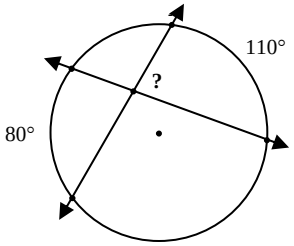
10. Describe how you would find the measure of the angle created by two intersecting tangents.

The tangents would create only two points of tangency on the circle. Thus, there would only be two arcs defined on the circle. The difference of the measures of these arcs, divided by two will find the measure of the angle formed by the intersecting tangents.

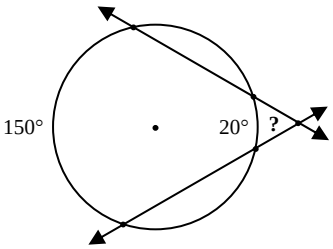
Classwork for "Angles formed by Chords, Tangents, and Secants" (KEY)

EXAMPLES

95°



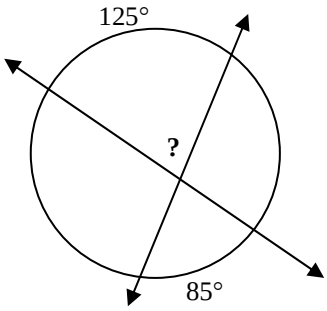
65°



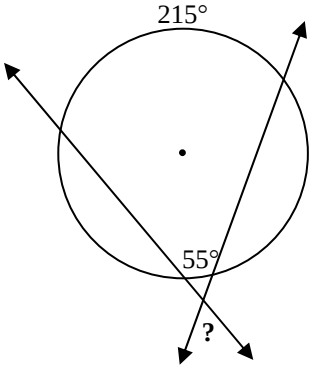
GROUP PRACTICE

Directions: Complete the following three questions in your group. Be prepared to explain how you found your answer!

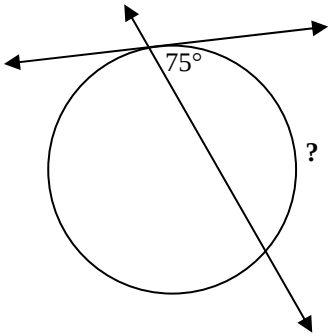
1. **105°**



2. **80°**



3. **150°**



Sector Smackdown Name: _____

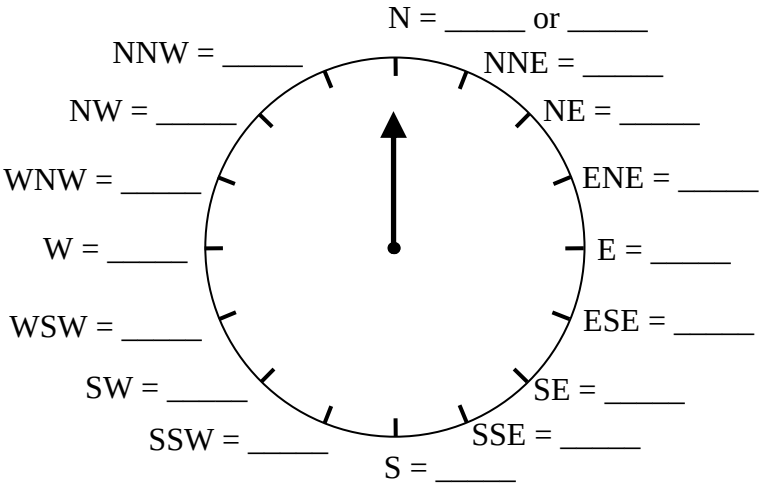
CSI: Circleville Pd: _____ Date: _____

Detective Skills

An important part of any large investigation is using the police force efficiently. You do not want to waste time searching areas that will not provide clues. When dealing with search areas on maps, officers refer to North, South, East, West, and all of the directions in between by using angle measurements. This allows the investigators a quick way to describe exactly where a suspect may be located or may be traveling.

- Look at the compass below. Begin by labeling North with 0° and 360°, because the circle contains 360°.

- The angles that correspond to the directions on the compass are named by moving clockwise around the compass. Thus, East would be 90°. Finish labeling South and West with their angles.



- Look for NE on the compass. Northeast lies on the angle bisector of the central angle formed by North and East. What is the corresponding angle? _____

- Label NE, SE, SW, and NW with their corresponding angles.

- Look for NNE on the compass. North-Northeast lies on the angle bisector of the central angle formed by North and Northeast. What is the corresponding angle? _____

- Label NNE, ENE, ESE, SSE, SSW, WSW, WNW, and NNW with their corresponding angles.

Now you have a complete compass marked with the corresponding angles. You will be using this to narrow down your search area and use the police force more efficiently.

The Second Clue

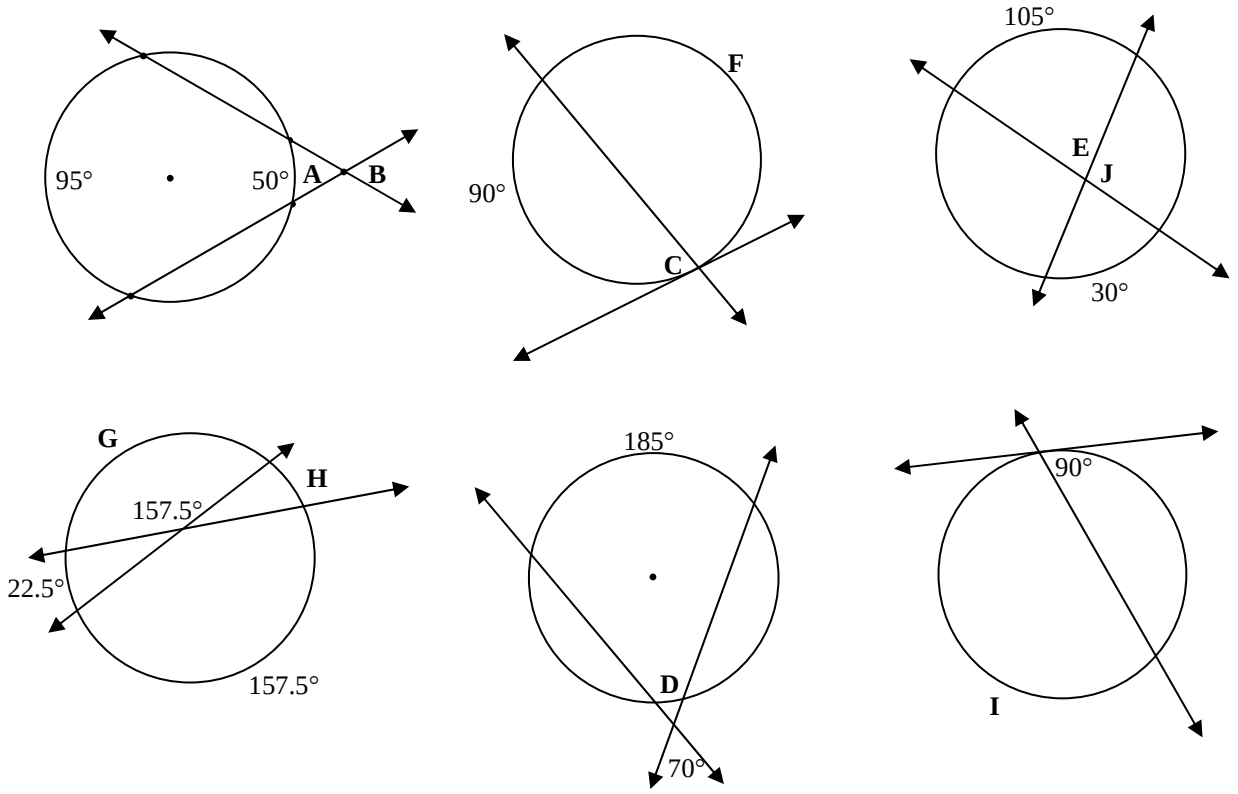
A reporter from the Circleville Circulator dropped off a new clue today. It reads as follows:

While it seems like you are closing in
And your discoveries are sweeping,
With chords and secants and tangents to draw
With sectors, you are sneaking.
Solve my puzzlers to find where I am ***not****,*
Or your lucky cucumber will surely rot!
-The Green Locus

<u>The Investigation</u>

After carefully studying the note, you find the following problems written in invisible ink.

Directions: Solve each problem to help eliminate sectors on your Crime Scene Map. Drawings are **not** to scale!



1. Eliminate the **smallest** sector created by the compass angles **A** and **D**.
2. Eliminate the **smallest** sector created by the compass angles **B** and **F**.
3. Eliminate the **smallest** sector created by the compass angles **E** and **J**.
4. Eliminate the **smallest** sector created by the compass angles **C** and **G**.
5. Eliminate the **smallest** sector created by the compass angles **H** and **I**.

Sector Smackdown Name: __**ANSWER KEY**_____

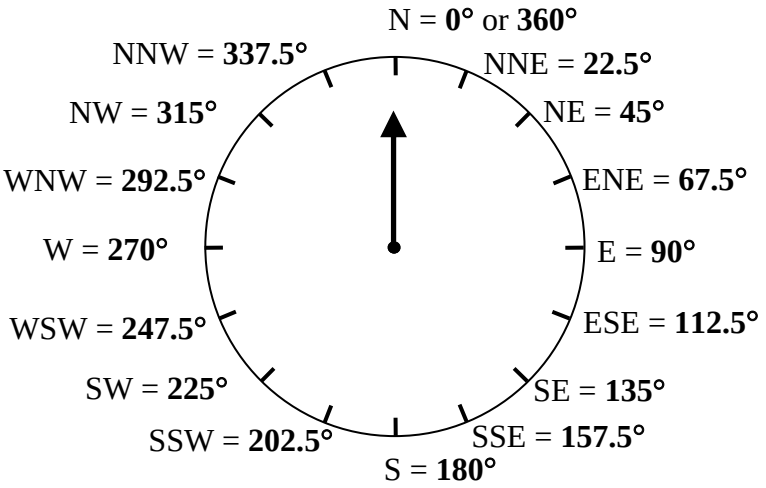
CSI: Circleville Pd: _____ Date: _____

Detective Skills

An important part of any large investigation is using the police force efficiently. You do not want to waste time searching areas that will not provide clues. When dealing with search areas on maps, officers refer to North, South, East, West, and all of the directions in between by using angle measurements. This allows the investigators a quick way to describe exactly where a suspect may be located or may be traveling.

- Look at the compass below. Begin by labeling North with 0° and 360°, because the circle contains 360°.

- The angles that correspond to the directions on the compass are named by moving clockwise around the compass. Thus, East would be 90°. Finish labeling South and West with their angles.



- Look for NE on the compass. Northeast lies on the angle bisector of the central angle formed by North and East. What is the corresponding angle? **45°**

- Label NE, SE, SW, and NW with their corresponding angles.

- Look for NNE on the compass. North-Northeast lies on the angle bisector of the central angle formed by North and Northeast. What is the corresponding angle? **22.5°**

- Label NNE, ENE, ESE, SSE, SSW, WSW, WNW, and NNW with their corresponding angles.

Now you have a complete compass marked with the corresponding angles. You will be using this to narrow down your search area and use the police force more efficiently.

The Second Clue

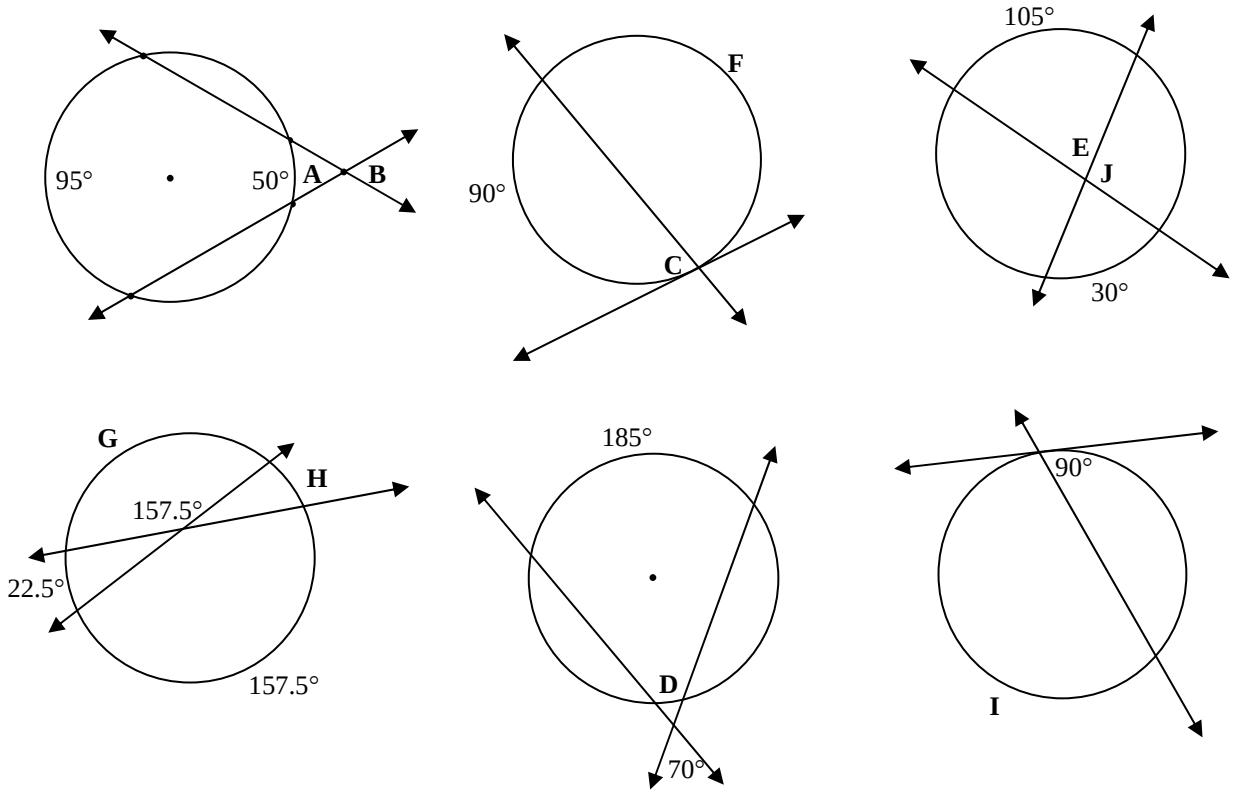
A reporter from the Circleville Circulator dropped off a new clue today. It reads as follows:

While it seems like you are closing in
And your discoveries are sweeping,
With chords and secants and tangents to draw
With sectors, you are sneaking.
Solve my puzzlers to find where I am ***not****,*
Or your lucky cucumber will surely rot!
-The Green Locus

<u>The Investigation</u>

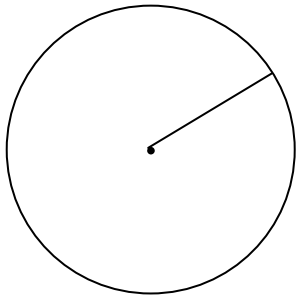
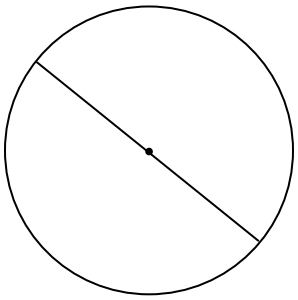
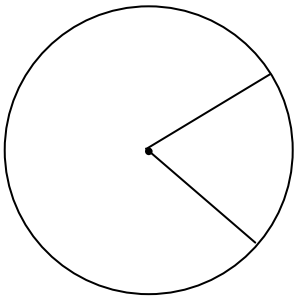
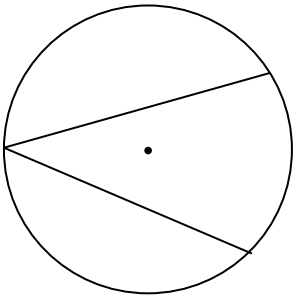
After carefully studying the note, you find the following problems written in invisible ink.

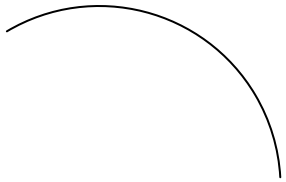
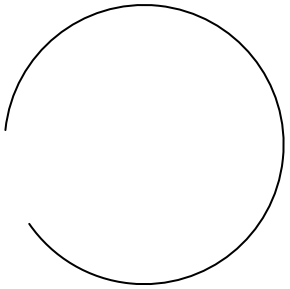
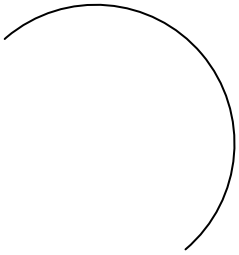
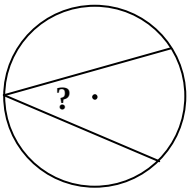
Directions: Solve each problem to help eliminate sectors on your Crime Scene Map. Drawings are **not** to scale!

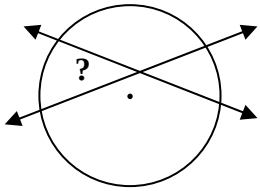
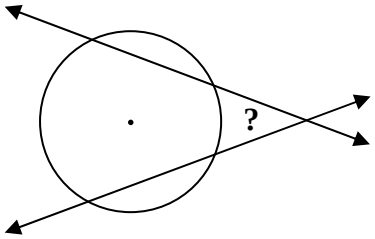
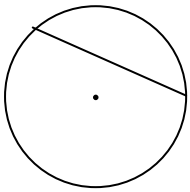
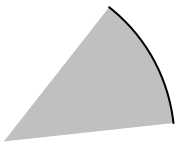


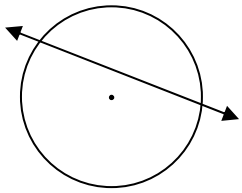
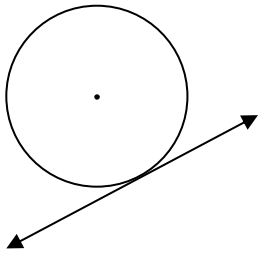
6. Eliminate the **smallest** sector created by the compass angles **A** and **D**. **(22.5° and 45°)**
7. Eliminate the **smallest** sector created by the compass angles **B** and **F**. **(22.5° and 270°)**
8. Eliminate the **smallest** sector created by the compass angles **E** and **J**. **(67.5° and 112.5°)**
9. Eliminate the **smallest** sector created by the compass angles **C** and **G**. **(45° and 157.5°)**
10. Eliminate the **smallest** sector created by the compass angles **H** and **I**. **(22.5° and 180°)**

Lesson 3, Circle Vocabulary / Angle Relationship Memory

Radius	
Diameter	
Central Angle	
Inscribed Angle	

Minor Arc	
Major Arc	
Semicircle	
What is the measure of the missing angle? 	Half the measure of the intercepted arc

What is the measure of the missing angle? 	Half the sum of the measures of the intercepted arcs
What is the measure of the missing angle? 	Half the difference of the measures of the intercepted arcs
Chord	
Sector	

Secant	
Tangent	
What angle is formed when a line is tangent to the circle on the endpoint of a radius?	A right angle
What is the measure of the intercepted arc of an inscribed angle that intersects the endpoints of a diameter?	180 degrees

The Third and Final Clue | Name: _____

CSI: Circleville | Pd: _____ Date: _____

The Clue

Another day of learning about circles can only mean one thing…another delivery from your friendly Circleville Circular journalist.

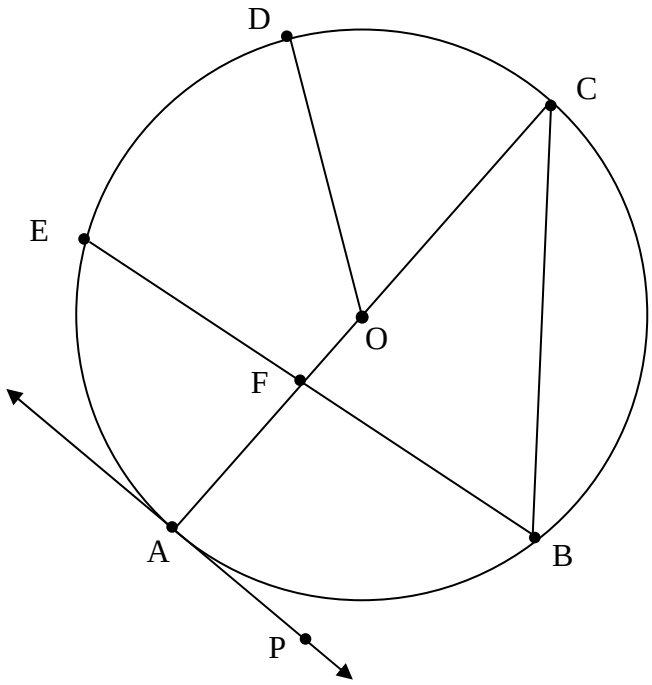
Two days of searching has yielded results,
But if you take any longer you'll only find pulp
Of the cucumber that you consider so highly.
I'll make it a pickle I can munch on quite slyly!
Maybe you'll find where I've hidden it today?
You are so close to the building, yet so far away.
-The Green Locus

The Investigation

Directions: Find the missing degree measures by using properties of central and inscribed angles, as well as angles formed by chords, secants, and tangents. Then, use your answers to fill in the blanks underneath each figure. Good Luck!

Puzzle 1 – Find the Neighborhood

Given: $\overline{AC}$ is a diameter. $\overleftrightarrow{AP}$ is tangent to the circle at A. $m\overset{\frown}{BC} = 110°$, $m\angle EFA = 95°$, and $m\angle COD = 75°$.



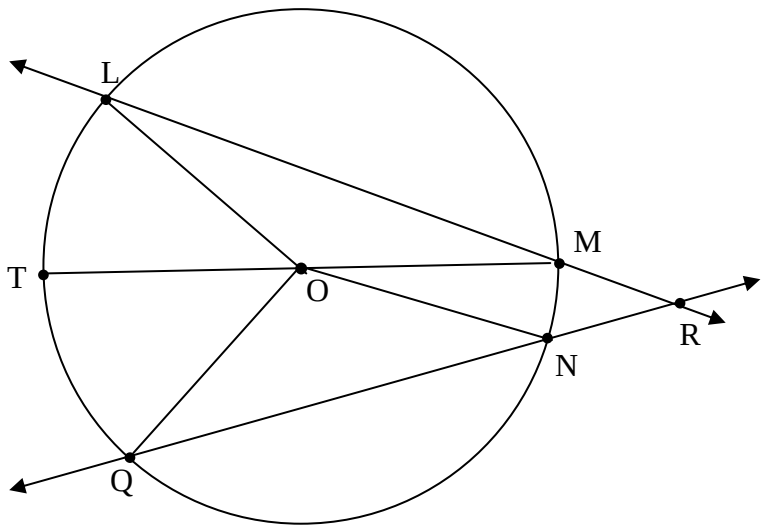
W	$m\angle FAP$ = _____
O	$m\overset{\frown}{AB}$ = _____
T	$m\angle BCF$ = _____
D	$m\angle BFC$ = _____
W	$m\overset{\frown}{AE}$ = _____
F	$m\angle CBF$ = _____
L	$m\angle EFC$ = _____
N	$m\overset{\frown}{DE}$ = _____

_____ _____ _____ _____ _____ _____ _____

85° 70° 90° 95° 70° 80° 25°

Puzzle 2 – Find the Building

Given: $\overline{OT}$ is the angle bisector of $\angle LOQ$. $m\overset{\frown}{LQ} = 100°$ and $m\angle MON = 20°$.



A	$m\angle LOQ =$ _____
B	$m\overset{\frown}{NQ} =$ _____
V	$m\angle NQO =$ _____
L	$m\angle TOQ =$ _____
N	$m\overset{\frown}{LT} =$ _____
F	$m\angle LMO =$ _____
G	$m\angle LOM =$ _____
H	$m\overset{\frown}{MN} =$ _____
I	$m\angle MRN =$ _____

_____ _____ _____ _____ _____ _____ _____ _____ _____ _____ _____

20° 100° 35° 40° 50° 130° 100° 110° 100° 50° 50°

When you have found the location of the Euclidean Cucumber, quickly report your results to your teacher.

Great Job, CSI: Circleville!

The Third and Final Clue Name: ___**ANSWER KEY**_____

CSI: Circleville Pd: _____ Date: _____

The Clue

Another day of learning about circles can only mean one thing…another delivery from your friendly Circleville Circular journalist.

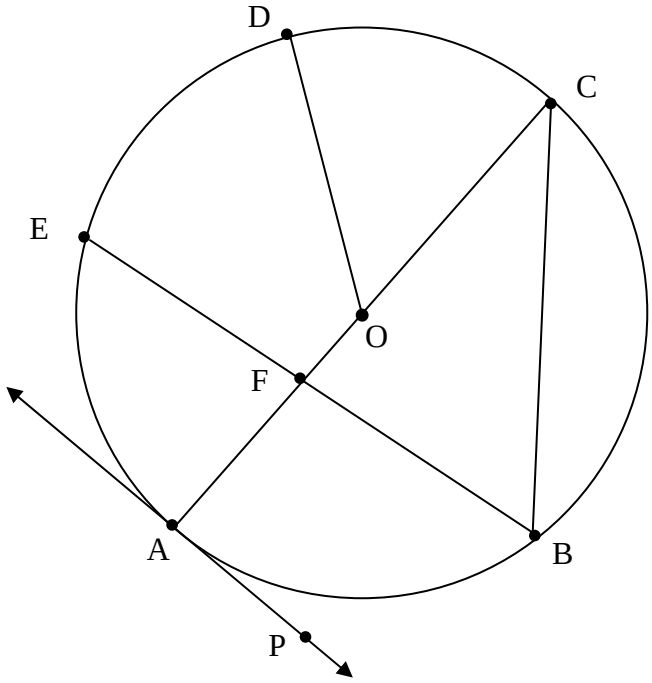
Two days of searching has yielded results,
But if you take any longer you'll only find pulp
Of the cucumber that you consider so highly.
I'll make it a pickle I can munch on quite slyly!
Maybe you'll find where I've hidden it today?
You are so close to the building, yet so far away.
-The Green Locus

The Investigation

Directions: Find the missing degree measures by using properties of central and inscribed angles, as well as angles formed by chords, secants, and tangents. Then, use your answers to fill in the blanks underneath each figure. Good Luck!

Puzzle 1 – Find the Neighborhood

Given: $\overline{AC}$ is a diameter. $\overleftrightarrow{AP}$ is tangent to the circle at A. $m\overset{\frown}{BC} = 110°$, $m\angle EFA = 95°$, and $m\angle COD = 75°$.



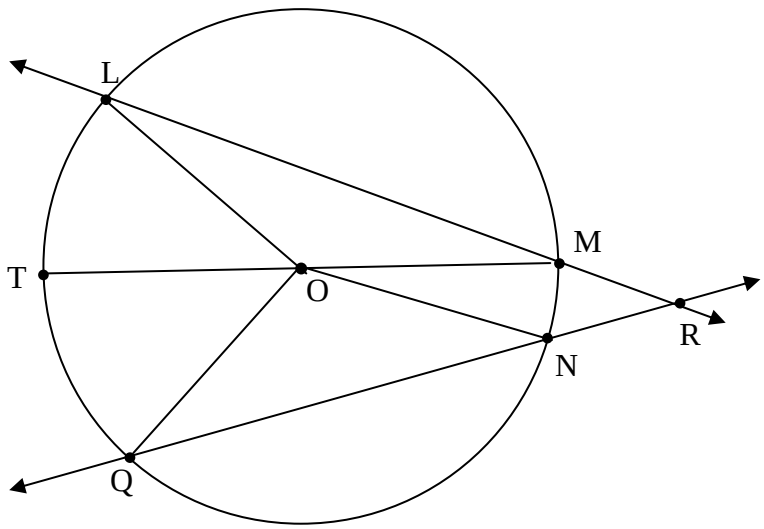
W	$m\angle FAP$ = ____**90°**_____
O	$m\overset{\frown}{AB}$ = _____**70°**_____
T	$m\angle BCF$ = ____**35°**_____
D	$m\angle BFC$ = _____**95°**_____
W	$m\overset{\frown}{AE}$ = ____**80°**_____
F	$m\angle CBF$ = _____**50°**_____
L	$m\angle EFC$ = _____**85°**_____
N	$m\overset{\frown}{DE}$ = ____**25°**_____

__**L**_ __**O**__ __**W**_ __**D**__ __**O**__ _**W**____**N**___

85° 70° 90° 95° 70° 80° 25°

Puzzle 2 – Find the Building

Given: $\overline{OT}$ is the angle bisector of $\angle LOQ$. $m\overset{\frown}{LQ} = 100°$ and $m\angle MON = 20°$.



A	$m\angle LOQ$ = _____**100**°____
B	$m\overset{\frown}{NQ}$ = ____**110**°_____
V	$m\angle NQO$ = ____**35**°_____
L	$m\angle TOQ$ = _____**50**°_____
N	$m\overset{\frown}{LT}$ = ____**50**°_____
F	$m\angle LMO$ = ____**25**°_____
G	$m\angle LOM$ = ____**130**°_____
H	$m\overset{\frown}{MN}$ = ___**20**°_____
I	$m\angle MRN$ = ____**40**°_____

Teacher Notes: Remember that a triangle formed by two radii is isosceles! Thus, base angles are congruent.

__H__	__A__	__V__	__I__	__N__	__G__	__A__	__B__	__A__	__L__	__L__
20°	100°	35°	40°	50°	130°	100°	110°	100°	50°	50°

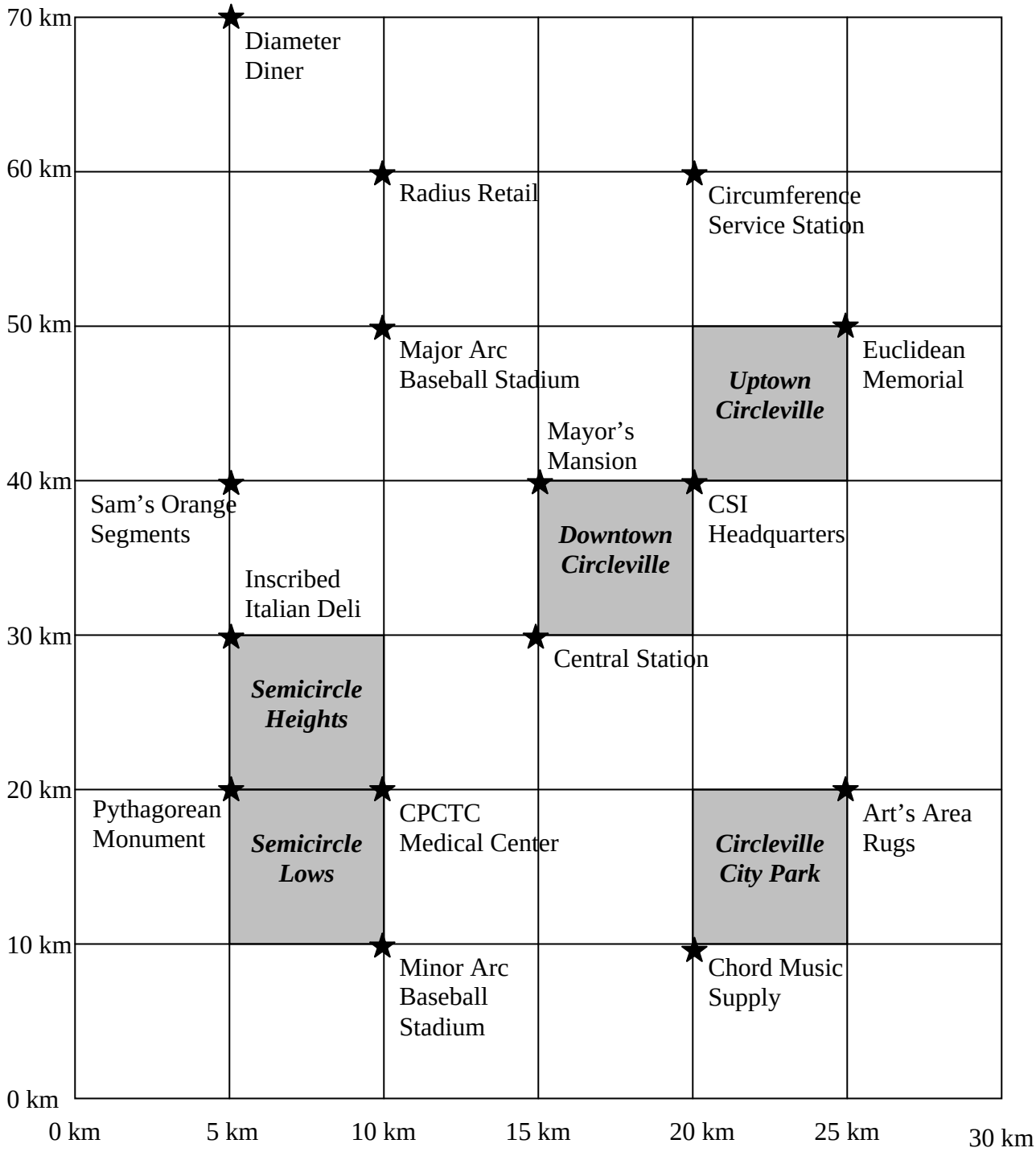
When you have found the location of the Euclidean Cucumber, quickly report your results to your teacher.

Great Job, CSI: Circleville!

Crime Scene Map Name: _____

CSI: Circleville Pd: _____ Date: _____

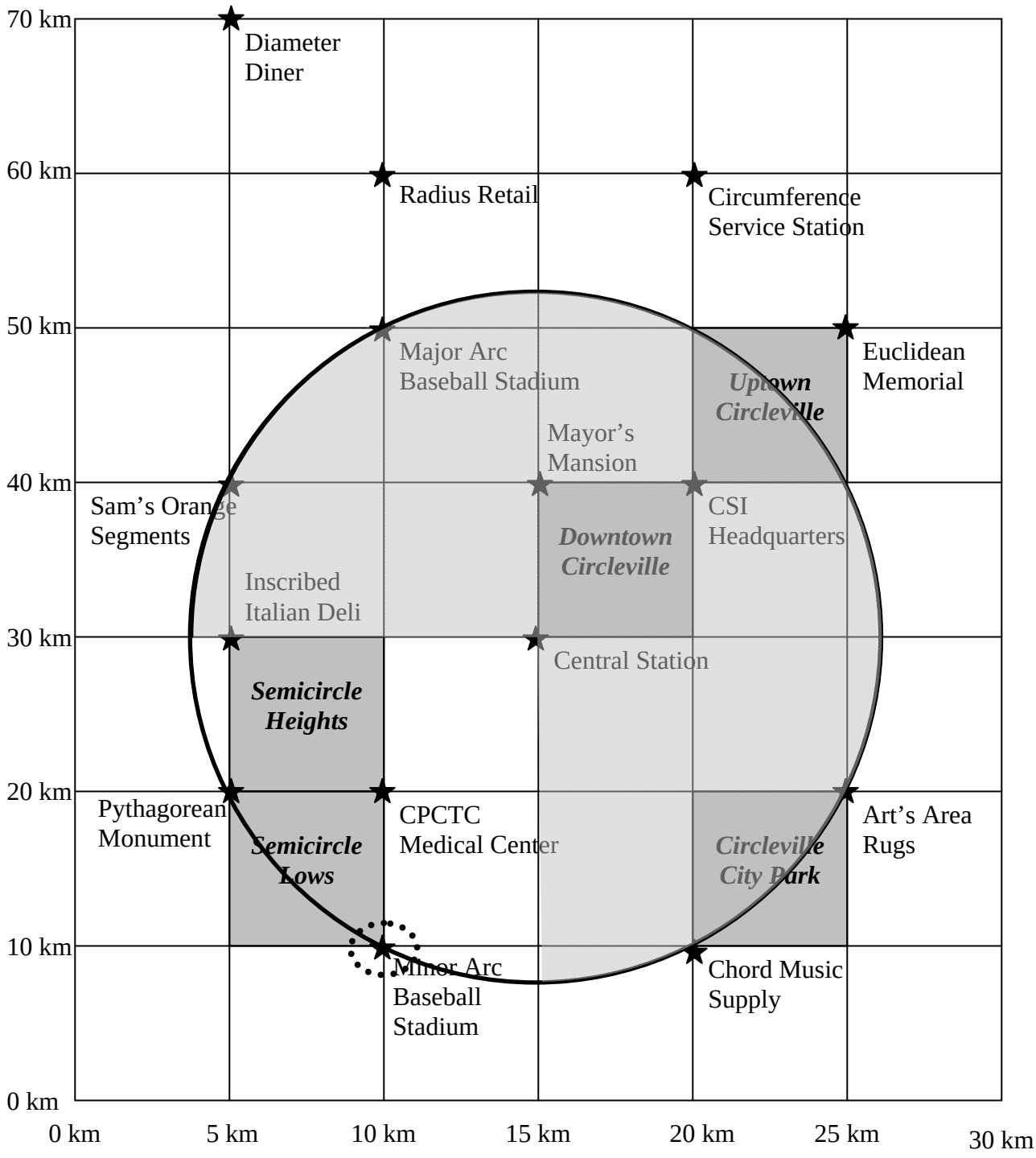
The Green Locus, the evil scourge of Circleville, has stolen Circleville's precious symbol of freedom, The Euclidean Cucumber. Track down The Green Locus and the Euclidean Cucumber with the help of your CSI team and the clues you find each day. Good Luck!



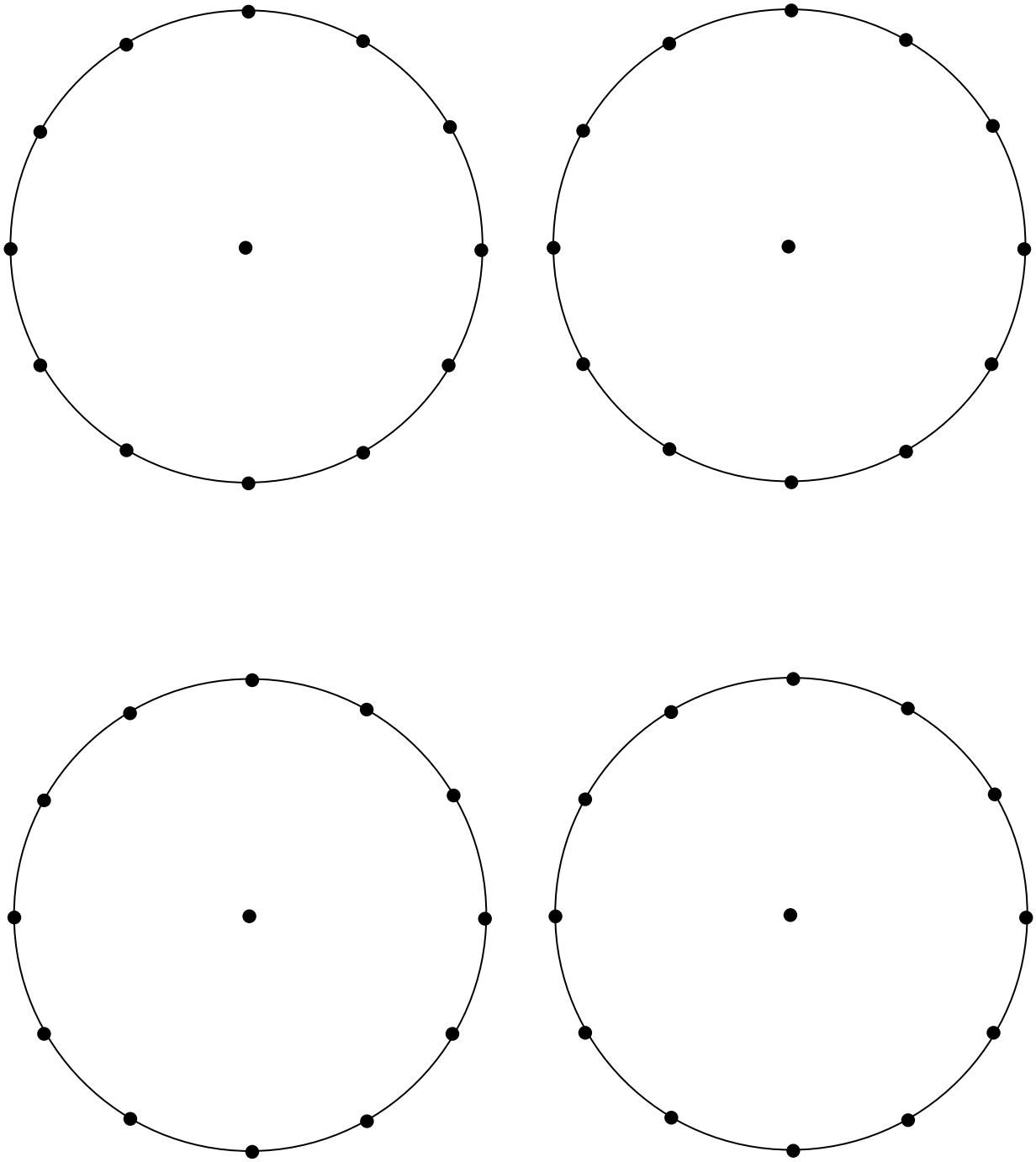
Crime Scene Map Name: _____**KEY**_____

CSI: Circleville Pd: _____ Date: _____

The Green Locus, the evil scourge of Circleville, has stolen Circleville's precious symbol of freedom, The Euclidean Cucumber. Track down The Green Locus and the Euclidean Cucumber with the help of your CSI team and the clues you find each day. Good Luck!



Geoboard Paper



Central and Inscribed Angles Name: _____

Lesson 1 Homework Pd: _____ Date: _____

1. Compare and contrast a chord, a radius, and a diameter. Include a diagram to help in your explanation.

_____

_____

_____

_____

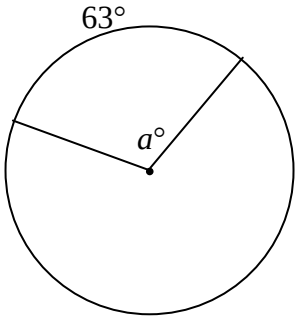
_____

Describe in words and construct a diagram of each angle.

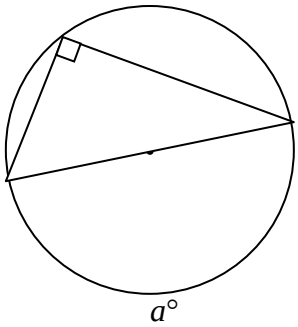
2. A central angle
3. An inscribed angle.

Find the measures of each missing angle.

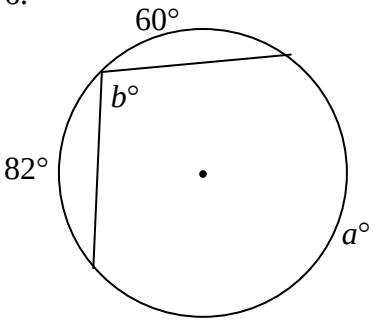
4.



5.



6.



Angles formed by Chords, Secants, and Tangents Name: _____

Lesson 2 Homework Pd: _____ Date: _____

1. Describe all possible ways that a 90° angle can be formed using a circle, chords, secants, and/or tangents.

_____

_____

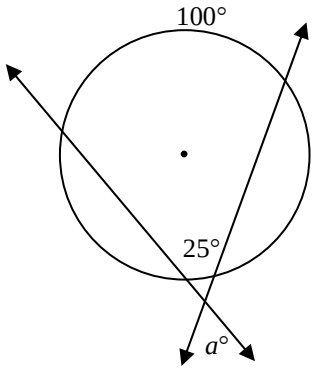
_____

_____

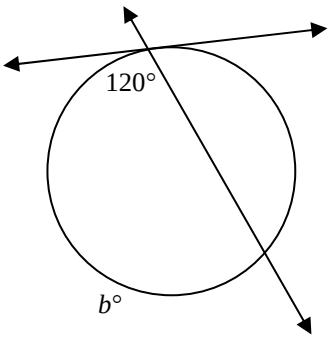
_____

Find the measures of each missing angle.

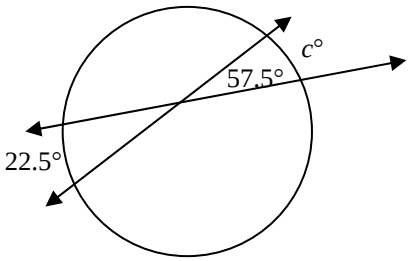
2.



3.



4.

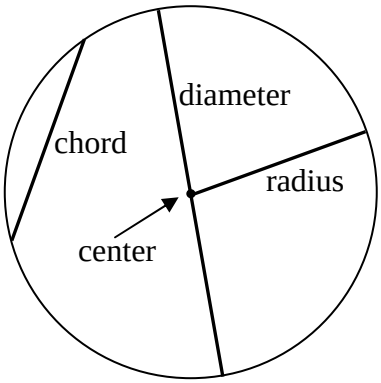


Central and Inscribed Angles Name: **_ANSWER KEY_____**

Lesson 1 Homework Pd: _____ Date: _____

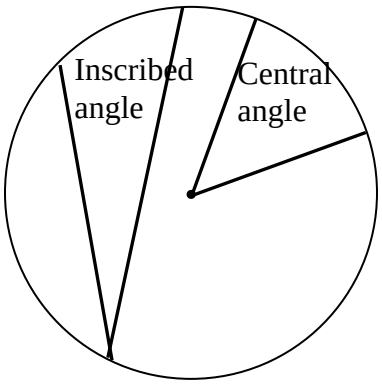
1. Compare and contrast a chord, a radius, and a diameter. Include a diagram to help in your explanation.

A chord, radius, and diameter are all line segments. In fact a diameter is a special type of chord, because a chord must have its two endpoints on the circle. A radius is half of the measure of a diameter. To construct a radius, only one endpoint lies on the circle, while the other lies on the center. In the case of a diameter, the midpoint of the diameter must lie on the center.



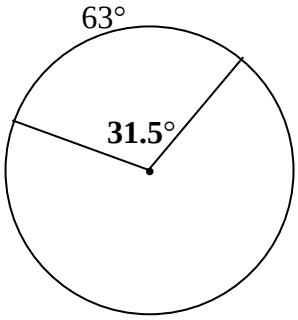
Describe in words and construct a diagram of each angle.

2. A central angle **Vertex on the center**
3. An inscribed angle. **Vertex on the circle**

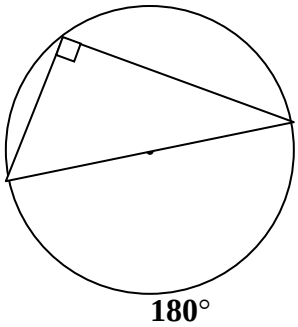


Find the measures of each missing angle.

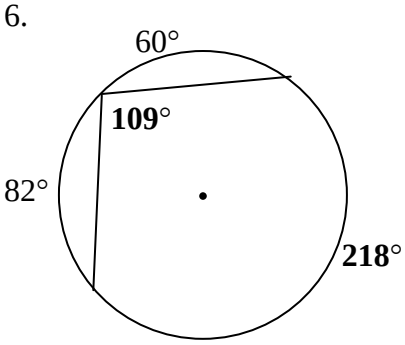
4.



5.



6.



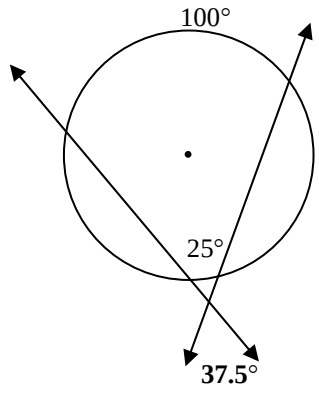
Angles formed by Chords, Secants, and Tangents Name: **_ANSWER KEY_____**

Lesson 2 Homework Pd: _____ Date: _____

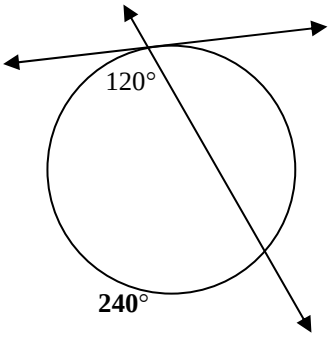
1. Describe all possible ways that a 90° angle can be formed using a circle, chords, secants, and/or tangents.
   - **Form a central angle of 90° using two chords**
   - **Use two secants/chords to intersect inside the circle so that half the sum of the intercepted arcs equals 90°.**
   - **Use two secants/chords to intersect outside the circle so that half the difference of the measures of the intercepted arcs equals 90°.**
   - **Use two tangents to intersect outside the circle so that half the difference of the measures of the intercepted arcs equals 90°.**
   - **Use one tangent and a secant or chord to form an inscribed angle with an intercepted arc measuring 180°.**

Find the measures of each missing angle.

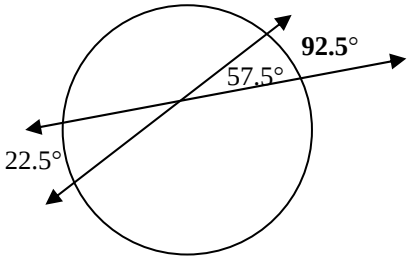
2.



3.



4.



Summative Assessment **Name: _____**

CSI: Circleville **Pd: _____ Date: _____**

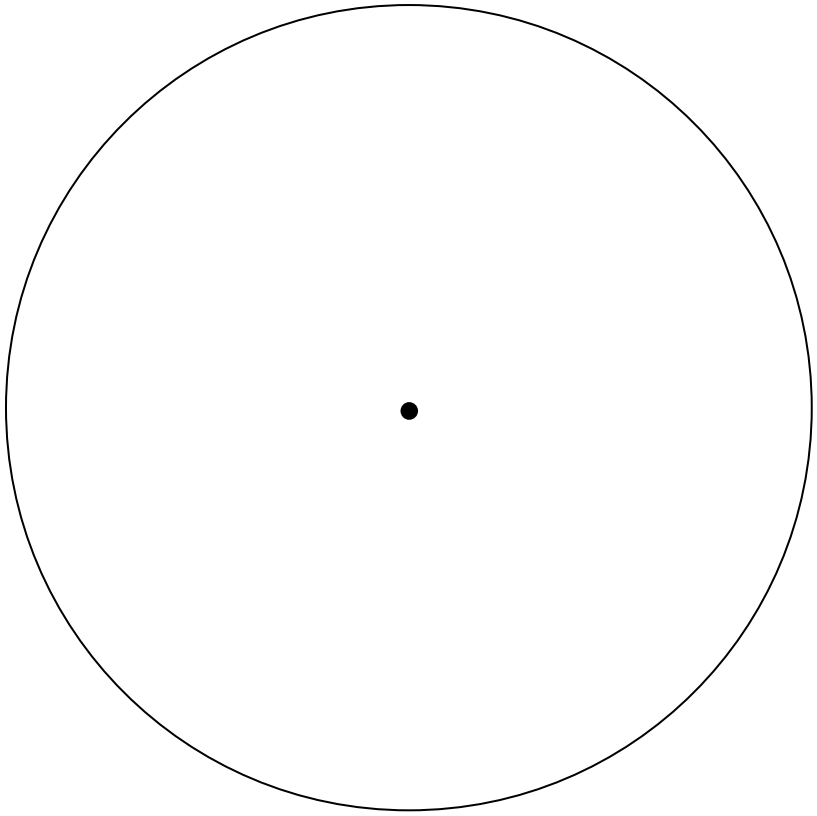
Create an angle puzzle of your own on the circle below. Use a pencil with an eraser so you can make changes, if needed.

Your puzzle must include at least one of each of the following:
A diameter, a radius, a chord, a secant, and a tangent.
Label each of these on your diagram.

Your puzzle must include at least one of each of the following:
A central angle and an inscribed angle.
Label each of these on your diagram

Your puzzle must include:
An angle with a vertex INSIDE the circle and
An angle with a vertex OUTSIDE the circle

Final Step:
Now that you have created your puzzle, choose a few arcs and assign degree measures so that you can completely solve your puzzle. Find the degree measures for each arc and each angle formed in the puzzle. This will take some time, so choose your givens carefully! If you are having trouble choosing the givens, consult your teacher to get you started.



Summative Assessment Rubric **Name: _____**

CSI: Circleville

Includes a labeled diameter	_____ / 1
Includes a labeled radius	_____ / 1
Includes a labeled chord	_____ / 1
Includes a labeled secant	_____ / 1
Includes a labeled tangent	_____ / 1
Includes a labeled central angle	_____ / 2
Includes a labeled inscribed angle	_____ / 2
Includes an angle with a vertex INSIDE the circle	_____ / 2
Includes an angle with a vertex OUTSIDE the circle	_____ / 2
Given values are labeled on puzzle	_____ / 2
Puzzle can be solved with givens	_____ / 5
Accurate solution is included	_____ / 10
Neatness and presentation	_____ / 5
Total	**_____ / 35**