Title: Decode Word Is …

Brief Overview:

This learning unit is designed to make connections between properties of linear functions and cryptology. Students will code and decode using basic shift ciphering, arrow diagrams, tables, the coordinate plane, and graphs.

NCTM Content Standard/National Science Education Standard:

NCTM Content Standards

Algebra

- Understand patterns, relations, and functions
- Solve problems that arise in mathematics and in other contexts.
- Represent and analyze mathematical situations and structures using algebraic symbols.
- Analyze change in various contexts.

Geometry

- Apply transformations and use symmetry to analyze mathematical situations

Communication

- Organize and consolidate their mathematical thinking through communication.
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- Analyze and evaluate the mathematical thinking and strategies of others.

Representation

- Create and use representations to organize, record, and communicate mathematical ideas.
- Select, apply, and translate among mathematical representations to solve problems.

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Grade/Level: Grades 6-12: Pre-Algebra/Transitions, Algebra I, extensions in Pre-Calculus (modular arithmetic).

Duration/Length: Three class periods, 45-50 minutes in length.

Student Outcomes:

Students will be able to:

- Code and decode a message using shift cipher.
- Find the coded value of a plaintext letter.
- Use tables, graphs, and arrow diagrams to analyze and communicate coding processes.
- Combine a stretch (multiplication) and a shift (addition) called a **combination code** to code and decode messages.

- **Materials and Resources:**

  Scissors
  Worksheets with answer keys
  TI-83 or TI-83 plus graphing calculators

Development/Procedures:

Lesson 1

Preassessment – Start with a cryptic message from Dr. Otto Octavius to Spiderman to interest the students (give hints by numbering each space where a = 1, b = 2…etc). Ask students where coding exists in real life – possible answers may include: zip codes, social security numbers, student ids, product ids (bar codes), PINs, security verification numbers.

Launch – a discussion of the use of cryptology in history. Talk about the importance of cryptology in technology, electronic messaging, medical records, financial transactions, copyrights, national defense…etc. Discuss how Pig Latin can be used as a "secret" language. Show the alphabet table and their corresponding numbers.

Teacher Facilitation – Distribute handout of crypto-bracelets. Give students the cryptic response from Spiderman to Dr. Octavius to decode. Introduce Caesar / Shift Cipher – a method of coding messages by shifting the alphabet a given number of places. Distribute worksheet A. Shift numerically and alphabetically. The emphasis here is on the relationship between the coded values and the new letters.

Student Application / Embedded Assessment – Have students come up with their own messages for a partner to decode. The coder will choose a constant amount of shift, either addition or subtraction and code them (first translate into

numbers, then back into letters). After coding their own messages, students will pair up and decode one another's message. (Worksheet B.)

Reteaching/Extension –

Reteaching: If students have trouble translating the numbers to the correct coded letter, perhaps making two strands of alphabet bracelets (or alphabet lines) will help by stabilizing one strand and shifting the other one below it.

Extension: Incorporate the TI-83 calculator by utilizing the statistic function (enter numbers 1 through 26 in L_1, highlight L_2 and type L_1 +3 to show a plus 3 shift cipher) or using matrices to cipher (create a 1x26 matrix, enter numbers 1 through 26, label it matrix A. Create another 1x 26 matrix, enter the same number for all elements, 5, for example, call it matrix B. Either add or subtract matrix B from matrix A to get the shift cipher – multiplication can also be used if the teacher is interested in experimenting).

Lesson 2

Preassessment – Review graph interpretations (worksheet G) by discussing a real-world application graphed on the coordinate plane. Ask students questions related to the graph and the implications of their answers (such as the meaning of the x and y intercepts, rate of change, minimum and maximum values, the plausibility of positive and negative values..etc)

Launch – Review the concept and utility of functions and then introduce ways to represent them: as tables, as graphs (as on an x-y coordinate plane) and as an arrow diagram. (Worksheet G) Review how a table can represent shift cipher. Introduce functions as a *dictionary* in cryptology, and note that both the coder and the decoder must have the same *dictionary* for the process of ciphering and deciphering to work. Then introduce the arrow diagram of the coding process. After the arrow diagram, introduce how a coordinate graph for coding would look like (worksheets C and D). Remind students the relationship between the coded value and the original position of a letter can be written as an equation: $c = p \pm s$, where c is the coded value, p is the original position, and s is the shift value.

Teacher Facilitation – Discuss a few shift ciphers and decipher processes with the students. Emphasize the meaning of an ordered pair on the x-y coordinate plane (x is the input, or the original position; y is the output, or the coded value). Have the students write out the relationship between x and y as a function. Facilitate a discussion about why the points on the graph of the coding process should be on the same line.

Student Application – Have students come up with their own shift cipher rule (they should use a cipher different than the one they used at the end of day one) and

make up their own *dictionaries* (a table, an arrow diagram, and a graph on the coordinate plane).

Embedded Assessment – Break students into groups of 4 or 5 and have them assign themselves the following roles:

1. The messager - responsible for creating an original message, then becomes a decoder,
2. The coder(s) - 1 – 2 people, responsible for coding the message
3. The decoders - 1 – 2 people, responsible for decoding the message of another group.

The whole group will choose a shift cipher rule together before the messager creates a message to code. Length of the message may vary according to students' abilities. Then groups will exchange and decode one another's messages. A gallery walk (of both the cryptic messages and the decoded messages) for this activity can be exciting for the students as well.

Reteaching/Extension – If time permits, discuss formulating equations based on arrow diagram, table, or plotted points on coordinate graph. Students may require more practice on functions if this is their first exposure to the topic.

Lesson 3

Preassessment – Drill (Day 3 Pre-Assessment worksheet) will help to assess mastery of shift cipher through representations: arrow diagrams, table, plots of ordered pairs. Teacher will assess mastery of using the graphing calculator as students enter data or equations

Launch – Discuss the ease of breaking the Caesar Cipher. How could we create a code that would be more challenging to decipher? Students may brainstorm: multiplication, raising to a power, division, opposites, switch values of A to Z (making A = 26, B =25, …, Z = 1). Look at the table where we multiply by a magnitude of 2, which stretches the values (Day 3 worksheet H). Lead into combination code (combining a stretch and shift in either order.) This is an excellent time to connect coding to function notation, review of linear equations, transformations.

Teacher Facilitation – (Day 3 Worksheet H) Students explore combination code. Relationships between amount of stretch and the slope may be made. Review the decoding process (using opposite operations). Sample: Solve $y = 2x + 3$ for x. To decode a message in combination code using the graphing calculator, type your equation into y_1 and a single coded value at a time into y_2, then find the point of intersection using 2nd TRACE 5:INTERSECT, press ENTER three times. The x-coordinate is the plaintext value. Distribute Day 3 Worksheet I.

Student Application – Check Day 3 Worksheet I.

Embedded Assessment - Closure activity (Exit Pass): The classroom has been hermetically sealed from the outside. The only way to free yourselves is to decode the password.

Reteaching/Extension – Students may be challenged by omitting spaces between words and punctuation marks in messages to be decoded.

Summative Assessment:

Teachers may use the assessment provided with this unit and amend the problems according to their needs.

Authors:

Wendy Selznick
Fallston High School
Harford County, Maryland

Yiping Helen Wang
Watkins Mill High School
Montgomery County, Maryland

Doctor Octopus: TJPM RZW RJI'O NVQZ TJP IJR!

Spidyman: DO RJI'O IZZY OJ, D VGMZVYT BJO TJP!

Translation:

Doctor Octopus: YOUR WEB WON'T SAVE YOU NOW!

Spidyman: IT WON'T NEED TO, I ALREADY GOT YOU!

Secret Codes and Algebra

History of Cryptology

Cryptology has existed for thousands of years. One of the first known uses was in about 1900 BC. An Egyptian scribe used non-standard hieroglyphs while inscribing. Simple substitutions by scribes in Mesopotamia dating about 1500 BC were discovered. Long ago, writing was a safe means of communicating because few people could read. Julius Caesar used simple substitutions by shifting letters in the alphabet, offsetting (or shifting) 3 letters. For example, the word RAVENS would be coded UDYHQV. It is believed that the Spartans established the first system of cryptography for the military. Have you seen the movie Windtalkers ? An Army Commander overheard two Choctaw Indians speaking and arranged for them to become radio communicators. They spoke their own language and used common words to replace military words to become the first Code Talkers.

Is coding a foreign language to you? Here is some terminology to help!

DEFINITIONS:

Cryptology refers to the study of hidden writing. It is derived from the Greek words Kryptos and Graphen, meaning hidden and to write, respectively.

Cryptography refers to the study of codes. It is the art of communicating through insecure channels. There are two basic principles of cryptography : *substitution* (replacing one letter by another) and *transposition* (shuffling the order of the letters).

Coding/Ciphering: the method of encoding a message with the intent to conceal information.

Code breaking: Cracking a code; deciphering a word or message without the knowing how it was coded.

Decoding: Taking a word or message back to its original form (plaintext) while knowing the key.

Encoding: Changing a message to coded form.

Plaintext: Information that is not encrypted. The coder converts plaintext into coded form.

Why is encryption essential in this day and age?

Day One: Worksheet A.

Name: _____ Teacher:_____
Period: _____ Date: _____

I. Fill in the Blanks:

Plain text letter	A	B	C	D	E	F	G	H	I	J	K	L	M
Original position	1	2	3	4	5	6	7	8	9	10	11	12	13
Shift Cipher	+3	+3	+3	+3	+3	+3	+3	+3	+3	+3	+3	+3	+3
Coded Value													
New Letter													

Plain text letter	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Original position	14	15	16	17	18	19	20	21	22	23	24	25	26
Shift Cipher	+3	+3	+3	+3	+3	+3	+3	+3	+3	+3	+3	+3	+3
Coded Value													
New Letter													

II. Use the coded values from the table in part I to write a numeric code for the following sentences:

a. MICKEY LIVES IN DISNEY WORLD.

_____ _____ _____ _____ _____ _____ _____ _____ _____ _____ _____ _____ _____ _____ _____ _____ _____ _____ _____

_____ _____ _____ _____ _____.

b. HARRY POTTER GOES TO HOGWARTS.

_____ _____ _____ _____ _____ _____ _____ _____ _____ _____ _____ _____

_____ _____ _____ _____ _____

_____ _____ _____ _____ _____ _____ _____ _____.

III. Use the coded values to decode the following messages:

a. 19 21 12 17 6 8 22 22 9 12 18 17 4 12 22 12 17 15 18 25 8 26 12 23 11 22 11 21 8 14.

b. 19 8 23 8 21 19 4 21 14 8 21 12 22 22 19 12 7 8 21 16 4 17.

IV. Use the new letters from the table on the front page to code or decode the following messages:

a. Code: THE RAIN IN SPAIN FALLS MAINLY IN THE PLANE.

b. Decode: JDUILHOG ORYHV PRQGDBV.

Day One: Worksheet A. (KEY)

Name: _____ Teacher: _____ Period: _____ Date: _____

I. Fill in the Blanks:

Plain text letter	A	B	C	D	E	F	G	H	I	J	K	L	M
Original position	1	2	3	4	5	6	7	8	9	10	11	12	13
Shift Cipher	+3	+3	+3	+3	+3	+3	+3	+3	+3	+3	+3	+3	+3
Coded Value	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
New Letter	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**	**L**	**M**	**N**	**O**	**P**

Plain text letter	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Original position	14	15	16	17	18	19	20	21	22	23	24	25	26
Shift Cipher	+3	+3	+3	+3	+3	+3	+3	+3	+3	+3	+3	+3	+3
Coded Value	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**
New Letter	**Q**	**R**	**S**	**T**	**U**	**V**	**W**	**X**	**Y**	**Z**	**A**	**B**	**C**

II. Use the coded values from the table in part I to write a numeric code for the following sentences:

a. MICKEY LIVES IN DISNEY WORLD.

_16_12_6_14_8_28 _15_12_25_8_22 _12_17 _7_12_22_17_8_28 _26_18_21_15_7

b. HARRY POTTER GOES TO HOGWARTS.

_11_4_21_21_28 _19_18_23_23_8_21 _10_18_8_22 _23_18 _11_18_10_26_4_21_23_22

III. Use the coded Values to decode the following messages:

a. 19 21 12 17 6 8 22 22 9 12 18 17 4 12 22 12 17 15 18 25 8 26 12 23 11 22 11 21 8 14.

_____**PRINCESS FIONA IS IN LOVE WITH SHREK.**
_____

_____
_____

b. 19 8 23 8 21 19 4 21 14 8 21 12 22 22 19 12 7 8 21 16 4 17.

PETER PARKER IS SPIDERMAN.
_____

_____

IV. Use the new letters from the table on the front page to code or decode the following messages:

c. Code: THE RAIN IN SPAIN FALLS MAINLY IN THE PLANE.

WKH UDLQ LQ VSDLQ IDOOV PDLQOB LQ WKH SODQH._____

d. Decode: JDUILHOG ORYHV PRQGDBV.

GARFIELD LOVES MONDAYS.

_____

_____

Day One: Worksheet B.

Coder: _____; Decoder: _____

I. Use the table to help you with your shift cipher.

Plain text letter	A	B	C	D	E	F	G	H	I	J	K	L	M
Original position	1	2	3	4	5	6	7	8	9	10	11	12	13
Shift Cipher													
Coded Value													
New Letter													

Plain text letter	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Original position	14	15	16	17	18	19	20	21	22	23	24	25	26
Shift Cipher													

Coded Value													
New Letter													

II. Come up with a message and code it:

a. Your message in Plaintext:

_____

b. Your message in Numeric Code:

_____

c. Your message in Alphabet Code

_____

III. Working with a partner.

a. Copy your message in Alphabet Code below:

_____

_____

b. Identify your shift cipher expression:

_____

IV. Decoder: Please use the expression above and the table below to help you decode the message.

Plain text letter	A	B	C	D	E	F	G	H	I	J	K	L	M
Original position	1	2	3	4	5	6	7	8	9	10	11	12	13

Shift Cipher													
Coded Value													
New Letter													

Plain text letter	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Original position	14	15	16	17	18	19	20	21	22	23	24	25	26
Shift Cipher													
Coded Value													
New Letter													

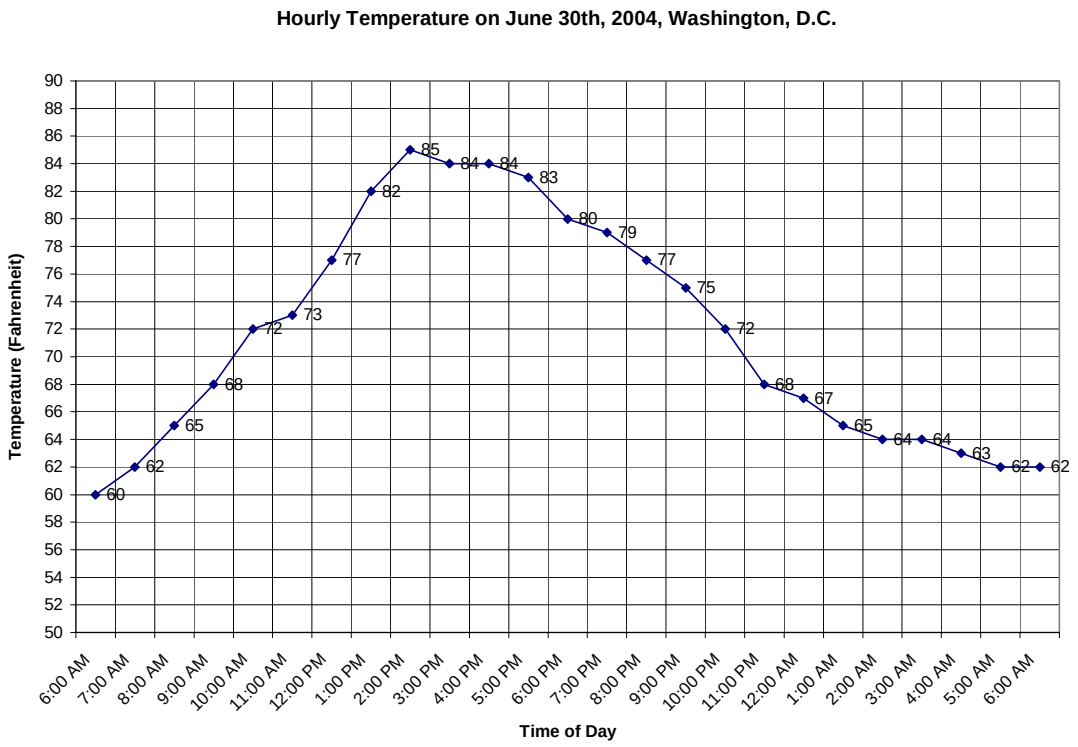
Message:

Day 2: Worksheet E.

Name: _____

Teacher:_____ Period: _____ Date: _____

I. Use the following graph to answer the questions below.



a) At what time interval was the temperature 67°F?

_____

b) What was the day's highest temperature? At what time interval did this occur?

_____

c) During what time intervals of the day was the temperature increasing? Decreasing? Staying the same?

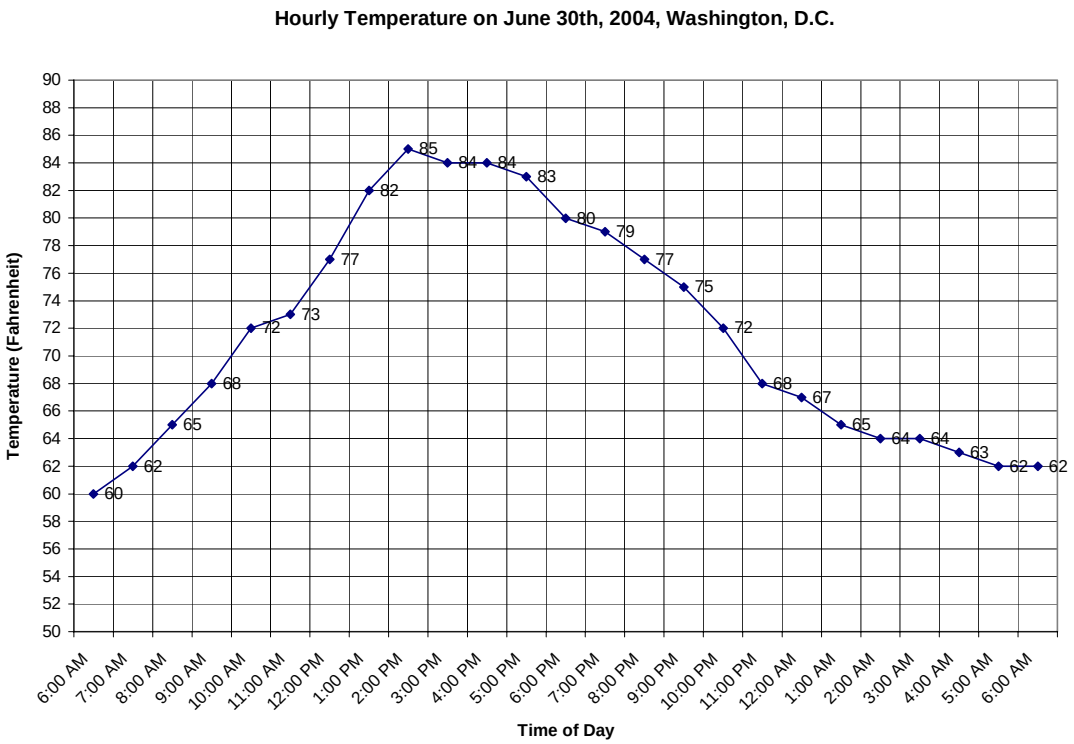
_____

d) Does it make sense to have zeros or negative numbers on either axis? Why or Why not?

Day 2: Worksheet E. (KEY)
Name: _____

Teacher:_____ Period: _____ Date: _____

II. Use the following graph to answer the questions below.



a) At what time interval was the temperature 67°F?
BETWEEN 12 AND 1 O'CLOCK IN THE MORNING.

b) What was the day's highest temperature? At what time interval did this occur?

IT IS 85°F, IT OCCURRED BETWEEN 2 AND 3 O'CLOCK IN THE AFTERNOON.

c) During what time intervals of the day was the temperature increasing? Decreasing? Staying the same? **THE TEMPERATURE WAS INCREASING FROM 6 O'CLOCK IN THE MORNING TO 3 O'CLOCK IN THE AFTERNOON. THE TEMPERATURE THEN DECREASED A LITTLE AND STAYED AT 84°F BETWEEN 3 TO 5 O'CLOCK, IT DROPPED BACK DOWN UNTIL 2 TO 3**

O'CLOCK IN THE MORNING AND 5 TO 6 IN THE MORNING WHEN THE TEMPERATURE REMAINED CONSTANT.

---

d) Does it make sense to have zeros or negative numbers on either axis? Why or Why not?

NO, BECAUSE IT DOES NOT MAKE SENSE TO HAVE ZERO OR NEGATIVE TIME ON THE HORIZONTAL AXIS. NEITHER DOES IT MAKE SENSE TO HAVE A TEMPERATURE OF ZERO OR NEGATIVE DEGREES FAHRENHEIT IN JUNE IN WASHINGTON, D.C.

Day Two: Worksheet G.

Name: _____ Teacher: _____ Period: _____ Date:_____

I. Use the Arrow Diagram below to fill in the Coded Values for the table:

Add 3

Original Position → Coded Value

Plaintext	A	B	C	D	E	F	G	H	I	J	K	L	M
Original Position													
Coded Value													

Plaintext	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Original Position													
Coded Value													

II. Use the Arrow Diagram below to fill in the Coded Values and the New Letters for the table:

Subtract 6

Original Position → Coded Value

Plaintext	A	B	C	D	E	F	G	H	I	J	K	L	M
Original Position													
Coded Value													

New Letters													

Plaintext	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Original Position													
Coded Value													
New Letters													

Day Two: Worksheet G. (KEY)

Name: _____ Teacher: _____ Period: _____ Date:_____

III. Use the Arrow Diagram below to fill in the Coded Values for the table:

Add 3

Original Position → Coded Value

Plaintext	A	B	C	D	E	F	G	H	I	J	K	L	M
Original Position	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
Coded Value	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**

Plaintext	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Original Position	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**
Coded Value	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**

IV. Use the Arrow Diagram below to fill in the Coded Values and the New Letters for the table:

Minus 6

Original Position → Coded Value

Plaintext	A	B	C	D	E	F	G	H	I	J	K	L	M
Original Position	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
Coded Value	**-5**	**-4**	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
New Letters	**U**	**V**	**W**	**X**	**Y**	**Z**	**A**	**B**	**C**	**D**	**E**	**F**	**G**

Plaintext	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Original Position	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**
Coded Value	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
New Letters	**H**	**I**	**J**	**K**	**L**	**M**	**N**	**O**	**P**	**Q**	**R**	**S**	**T**

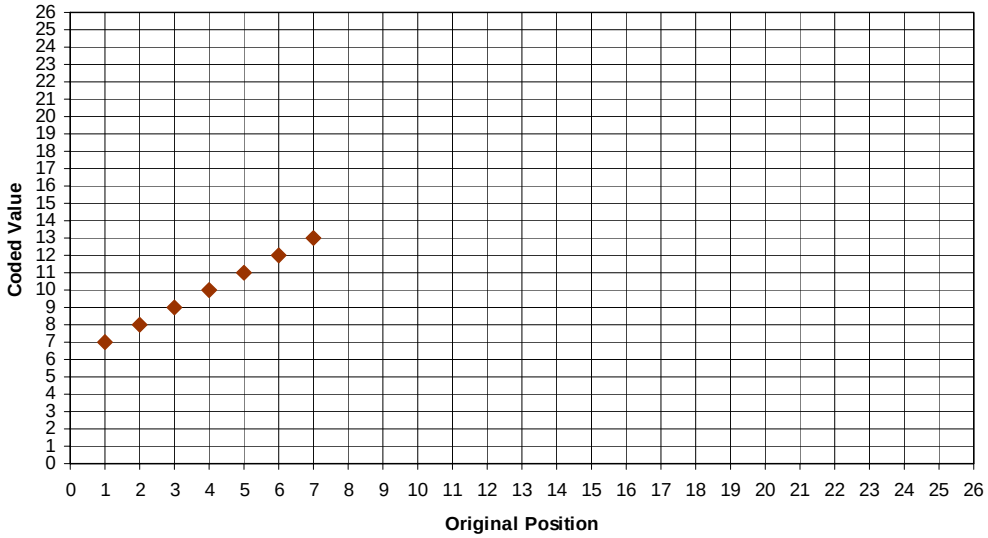
Day 2: Worksheet C.

Name: _____

Teacher:_____ **Period:** _____ **Date:** _____

I. The graph in Figure A represents part of a coding process.

Figure A



a) Describe how to determine where to plot the next two points on the graph.

_____

b) What is the meaning of the point, (5,11)?

c) Use the graph to encode the word, ***badge***, both numerically and alphabetically.

d) How else can you communicate, or represent this coding process to another person?

e) Write an equation that represents the relationship between the original position and the coded value.

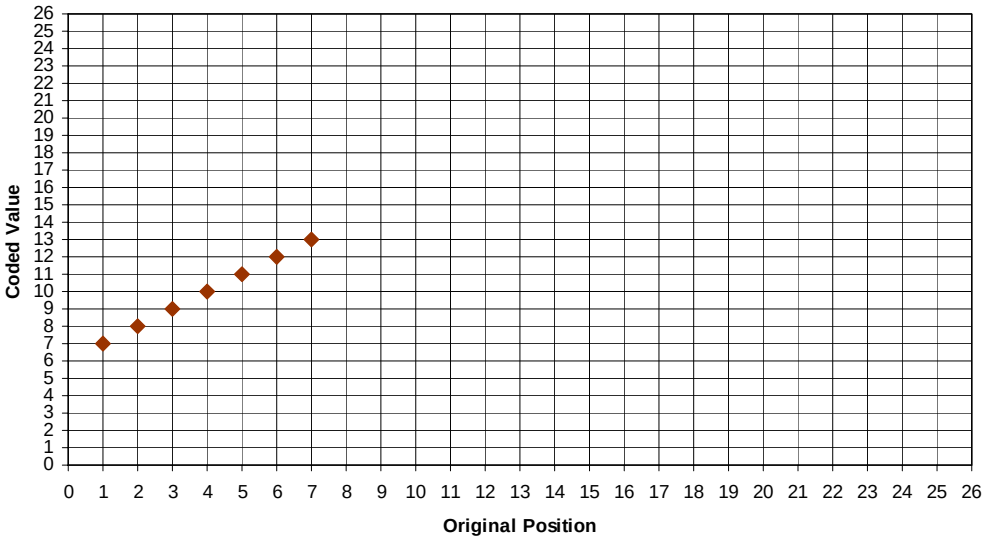
Day 2: Worksheet C. (KEY)

Name: _____

Teacher:_____ Period: _____ Date: _____

I. The graph in Figure A represents part of a coding process.

Figure A



a) Describe how to determine where to plot the next two points on the graph.
FOR EACH CONSECUTIVE X VALUE, I CAN GO UP ONE UNIT AND TO THE RIGHT ANOTHER UNIT TO FIND THE CORRESPONDING Y VALUE

b) What is the meaning of the point, (5, 11)?

IT MEANS THAT THE LETTER ORIGINALLY HAD A POSITION CODE OF 5 NOW IS CODED WITH 11 INSTEAD.

---

c) Use the graph to encode the word, ***badge***, both numerically and alphabetically.

8 7 10 13 11, HAJMK

---

d) How else can you communicate, or represent this coding process to another person?

Use an arrow diagram (original position plus 6) or a table

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e) Write an equation that represents the relationship between the original position and the coded value.
C = p + 6

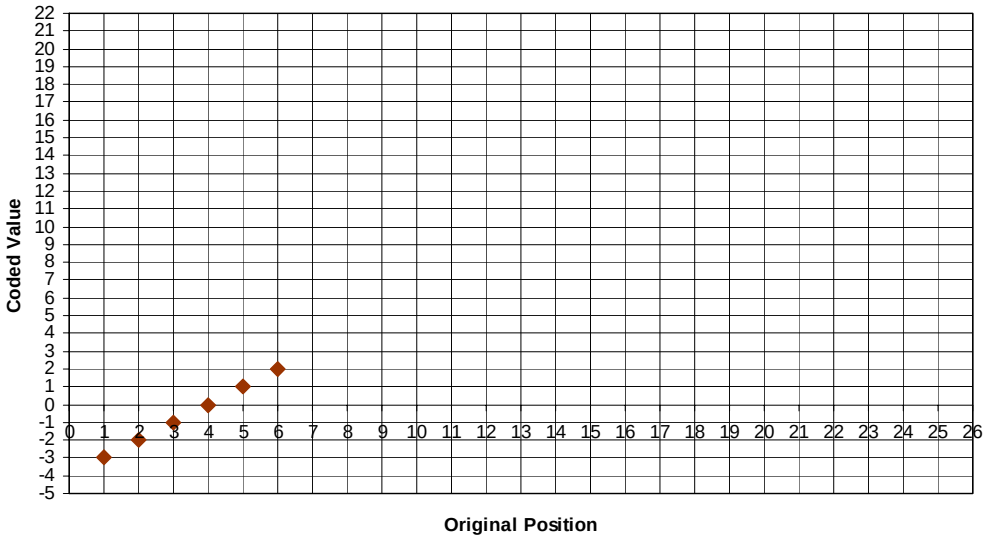
Day 2: Worksheet D.

Name: _____

Teacher:_____ Period: _____ Date: _____

I. Figure B shows the way a shift cipher codes the first six letters of the alphabet.

Figure B



a) Describe the shift cipher in your own words.

---

b) How does this shift cipher code the letter H as a letter? As a number?

---

c) Use this cipher to code the name Julius Caesar.

---

d) The word 1 8 1 12 4 -3 10 16 was coded with this shift cipher. Decode it.

---

e) Write an equation that represents the relationship between the original position and the coded value.

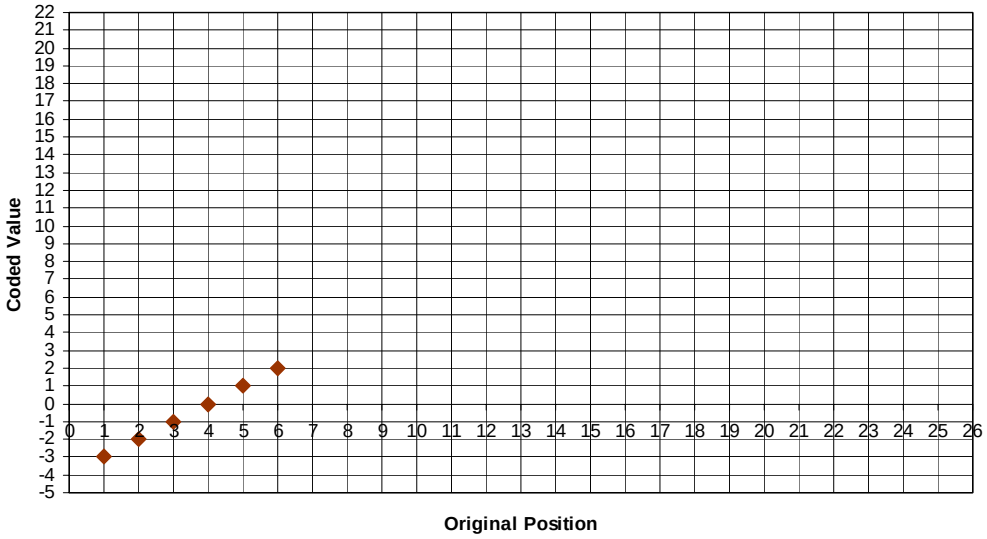
Day 2: Worksheet D. (KEY)

Name: _____

Teacher:_____ Period: _____ Date: _____

II. Figure B shows the way a shift cipher codes the first six letters of the alphabet.

Figure B



e) Describe the shift cipher in your own words.

THE CODED VALUE IS THE ORIGINAL POSITION MINUS 4 (OR SHIFT -4)

f) How does this shift cipher code the letter H as a letter? As a number?

H WILL BE CODED AS LETTER D.

g) Use this cipher to code the name Julius Caesar.

FQHEQO YWAOWN

h) The word 1 8 1 12 4 -3 10 16 was coded with this shift cipher. Decode it.

ELEPHANT

e) Write an equation that represents the relationship between the original position and the coded value.

C = p - 4

Day Two: Worksheet F.

Coding Group: _____; Decoding Group: _____

I. Use the table to help you with your shift cipher.

Plain text letter	A	B	C	D	E	F	G	H	I	J	K	L	M
Original position	1	2	3	4	5	6	7	8	9	10	11	12	13
Shift Cipher													
Coded Value													
New Letter													

Plain text letter	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Original position	14	15	16	17	18	19	20	21	22	23	24	25	26
Shift Cipher													
Coded Value													
New Letter													

II. Come up with a message and code it:

a. Your message in Plaintext:

_____

_____

b. Your message in Numeric Code:

_____

_____

c. Your message in Alphabet Code:

_____

_____

III. For the Other Team.

a. Copy your message in Alphabet Code below:

_____
_____

b. Identify your shift cipher as a table, a coordinate graph, or an arrow diagram:

IV. Decoding Group: Please use the expression above and the table below to help you decode the message.

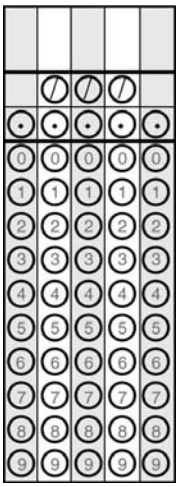
Plain text letter	A	B	C	D	E	F	G	H	I	J	K	L	M
Original position	1	2	3	4	5	6	7	8	9	10	11	12	13
Shift Cipher													
Coded Value													
New Letter													

Plain text letter	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Original position	14	15	16	17	18	19	20	21	22	23	24	25	26
Shift Cipher													
Coded Value													
New Letter													

Message:

Day 3 Pre-Assessment

1. The table below shows a relationship between *x* and *y*.

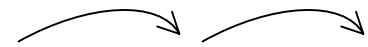


X	***Y***
0	2
1	3
2	6
3	11
4	?

What is the next value of y?

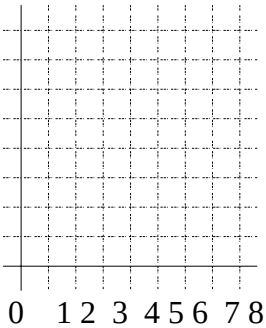
2. Represent the following arrow diagram by plotting ordered pairs and creating a table in terms of *x* and *y*.

Add 3



A 1

Graph



Table

x	***y***
1	
2	
3	
4	
5	

Can you hypothesize an equation to describe this situation?

Day 3 Worksheet H: Combination Codes

You've mastered the Caesar Cipher. It doesn't take a rocket scientist to decode, does it?

1. How could we make our code more difficult to break? List at least 3 methods.

The table provides some values using a new process.

Plaintext Letter	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**	**L**	**M**
Original Position	1	2	3	4	5	6	7	8	9	10	11	12	13
Coded Value	2	4	6	8	10	12	14	16	18	20	22	24	26

2. Describe the new process in your own words.

3. Decode the word: **2 26 2 52 18 28 14**

4. Write an equation that produces the table in terms of c and p. Then write an equation in terms of y and x.

This cipher is called a **stretch** cipher. Why do you think it's called that?

Combination codes combine a shift and a stretch. Will your answers be different if you stretch before you shift versus shifting before you stretch?

Writing combination codes as linear functions:

Recall that in the coding process where c represents the coded value, p represents the position number, and s represents the amount of shift, $c = p + s$ is a linear function where c and p are variables and s is a constant. When we write combination codes, the amount of stretch is multiplied by p in the equation. For example, a stretch of 4 followed by a shift of 3 would yield the equation $y = 4x + 3$.

5. Example: Code the message KEEP IT ON THE DOWN LOW using a stretch of 2 followed by a shift of -4.

6. How will we decode a combination code if we have the key?

7. Decode the message: **7 40 37 ‘ 55 19 -2 61 10 -2**

4 40 64 , 7 58 7 10

The key is stretch 3 followed by a shift of -5.

8. Do you think combination codes are easy to crack? (without being given a key)? Explain.

9. How do combination codes compare to shift ciphers?

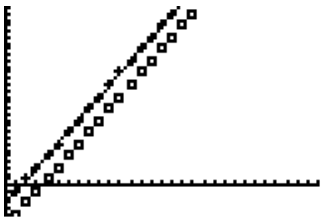
Let's explore some graphs.

11. Consider the coding processes $y = 2x - 3$ and $y = 2x - 7$.

    a. How are they similar?

    b. How are they different?

    c. Do these two processes code any letters the same?

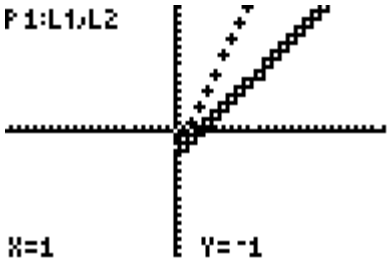


12. Consider the coding processes $y = 2x - 3$ and $y = x - 3$.

a. How are they similar?

b. How are they different?

c. Do these two processes code any letters the same?



Day 3 Worksheet H: Combination Codes (**KEY**)

You've mastered the Caesar Cipher. It doesn't take a rocket scientist to decode, does it?

1. How could we make our code more difficult to break? List at least 3 methods.

The table provides some values using a new process.

Plaintext Letter	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**	**L**	**M**
Original Position	1	2	3	4	5	6	7	8	9	10	11	12	13
Coded Value	2	4	6	8	10	12	14	16	18	20	22	24	26

3. Describe the new process in your own words.

3. Decode the word: **2 26 2 52 18 28 14** **AMAZING**

4. Write an equation that produces the table in terms of c and p. Then write an equation in terms of y and x.

$c = 2p$, $y = 2x$ (Function notation may be explored here)

This cipher is called a **stretch** cipher. Why do you think it's called that?

Stretches the y-coordinates, changes the slope/amount of slant

Combination codes combine a shift and a stretch. Will your answers be different if you stretch before you shift versus shifting before you stretch?

Writing combination codes as linear functions:

Recall that in the coding process where c represents the coded value, p represents the position number, and s represents the amount of shift, $c = p + s$ is a linear function where c and p are variables and s is a constant. When we write combination codes, the amount of stretch is multiplied by p in the equation. For example, a stretch of 4 followed by a shift of 3 would yield the equation $y = 4x + 3$.

5. Example: Code the message KEEP IT ON THE DOWN LOW using a stretch of 2 followed by a shift of -4.

18 6 6 28 14 36 26 24 36 12 6 4 26 42 24 20 26 42

6. How will we decode a combination code if we have the key?

Add the opposite of the shift and divide by the stretch, enter the new linear equation into the STAT list

1. Decode the message: **7 40 37 ‘ 55 19 -2 61 10 -2**

4 40 64 , 7 58 7 10

The key is stretch 3 followed by a shift of -5.

8. Do you think combination codes are easy to crack? (without being given a key)? Explain.

9. How do combination codes compare to shift ciphers?

10. How can you tell by looking at a graph whether a stretch was used in the process?

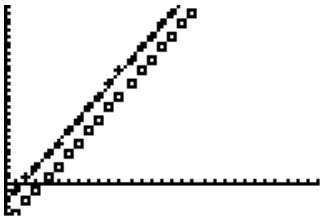
Let’s explore some graphs.

11. Consider the coding processes $y = 2x - 3$ and $y = 2x - 7$.

d. How are they similar? **Stretch of 2, same slope**

e. How are they different? **Different shifts, different y-intercepts**

f. Do these two processes code any letters the same?



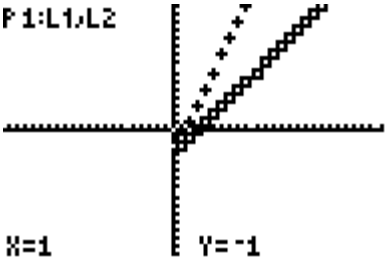
No two letters will have the same coded value because they don't intersect, discuss parallel lines have the same slope, etc.

Consider the coding processes $y = 2x - 3$ and $y = x - 3$.

How are they similar? **Same shift, same y-intercept**

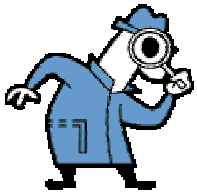
How are they different? **Different slopes, different stretch**

Do these two processes code any letters the same?



Students will be tempted to say yes because the y-intercepts are the same. Our original value of A is 1 and the point of intersection is (0, -3) so none of our letters will code the same, unless we shift our original values.

Day 3 Worksheet I: Combination Code:



Someone has been sending threatening letters to Mr. Flabs at his restaurant, The Flusty Flab. We have been hired by the FBI to decode messages from the nefarious Prankton, who is a suspect in the case.

MESSAGE 1:

33 89 33 45 45 13 57 49 33 53 1 77 17 77 29 17 89 57 69 45 13

(The FBI believes this to be a stretch of 4 and a shift of -3)

MESSAGE 2:

62 26 17 35 56 5 8 8 77 50 5 62 62 77 56 17 11 29 50 17

71 29 38 38 8 17 41 29 44 17

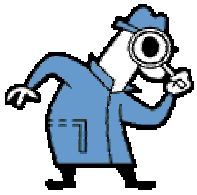
(The FBI believes this to be a stretch of 3 and a shift of +2. Can you confirm this?)

MESSAGE 3:

19 23 49 15 31 15 51 21 7 45 23 51 7 33 45 !

(The FBI believes this to be a stretch of 2 and a shift of +5)

Day 3 Worksheet I: Combination Code: Answer Key



Someone has been sending threatening letters to Mr. Flabs at his restaurant, The Flusty Flab. We have been hired by the FBI to decode messages from the nefarious Prankton, who is a suspect in the case.

MESSAGE 1:

33 89 33 45 45 13 57 49 33 53 1 77 17 77 29 17 89 57 69 45 13

(The FBI believes this to be a stretch of 4 and a shift of -3)

I WILL DOMINATE THE WORLD

MESSAGE 2:

62 26 17 35 56 5 8 8 77 50 5 62 62 77 56 17 11 29 50 17

71 29 38 38 8 17 41 29 44 17

(The FBI believes this to be a stretch of 3 and a shift of +2. Can you confirm this?)

THE KRABBY PATTY RECIPE WILL BE MINE

MESSAGE 3:

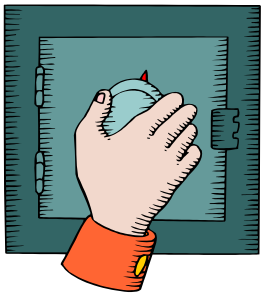
19 23 49 15 31 15 51 21 7 45 23 51 7 33 45 !

(The FBI believes this to be a stretch of 2 and a shift of +5)

GIVE ME WHAT I WANT!

Closure activity (Exit Pass)

The classroom has been hermetically sealed from the outside. The only way to free yourselves is to decode the password

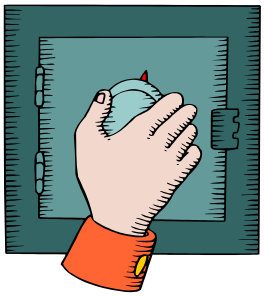


The Password is … (Key: Stretch 3 and shift +2)

14 17 62 17 56 41 29 44 5 62 29 47 44

Closure activity (Exit Pass) (KEY)

The classroom has been hermetically sealed from the outside. The only way to free yourselves is to decode the password



The Password is … (Key: Stretch 3 and shift +2)

14 17 62 17 56 41 29 44 5 62 29 47 44

(Solution: DETERMINATION)

Assessment

Name: _____ Period: _____

Teacher: _____

I. Code and decode the following messages using simple shift cipher.

1. Code the message in letters with a shift of -4: “Watermelon is delicious.”

---

2. Decode the following message that was coded with a shift of 7:

   15 8 23 23 16 21 12 26 26 11 12 23 12 21 11 26 28 23 22 21

   22 28 25 26 12 19 29 12 26

II. Function Representations in Coding.

1. Identify the shift cipher that is represented by the arrow diagram below:

_____

A 1 -2

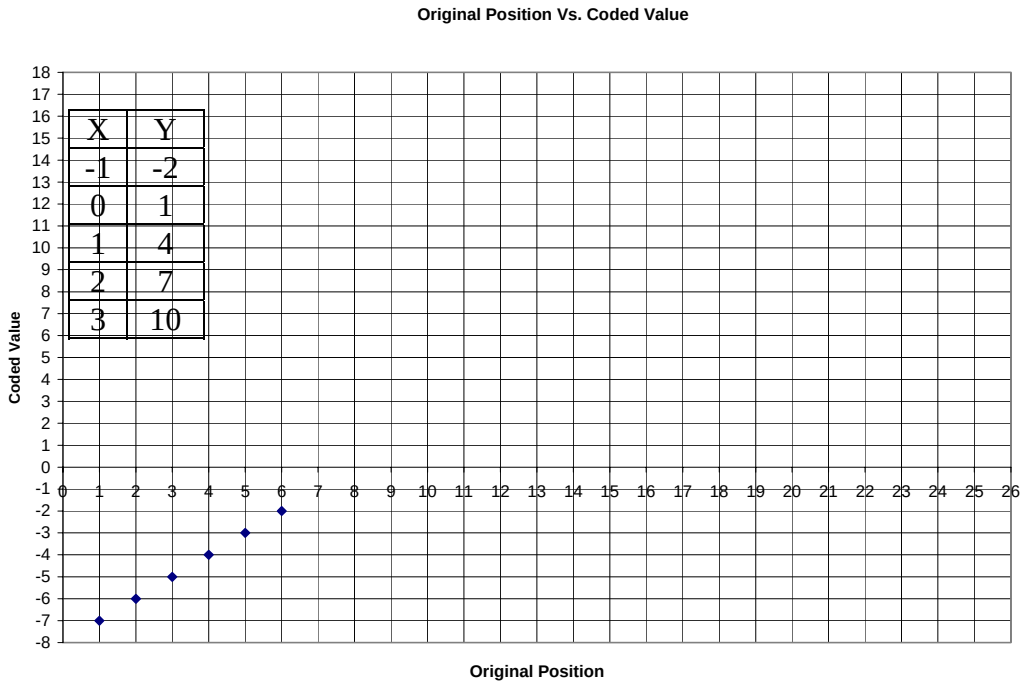
2. Identify the equation that represents the relationship between the original position (p) of a letter and its coded value (c):

a.

Original Position	Coded Value
1	12
2	13
3	14
4	15
5	16
6	17

Equation: _____

b.



Equation:

III. Stretch Cipher and Shift Cipher:

1. The table below shows the relationship between x and y. Which of these equations represents the relationship?

   a) $y = -3x + 1$

   b) $y = x - 3$

   c) $y = x + 3$

   d) $y = 3x + 1$

2. Code the following message numerically with the equation: $c = 2p + 7$: I ‘m a great student

3. Decode the following message that was coded with the following cipher: $c = 3p - 1$.

   8 44 68 56 2 53 14 20 53 2 41 11

Assessment (KEY)

Name: _____ Period: _____

Teacher: _____

IV. Code and decode the following messages using simple shift cipher.

3. Code the message in letters with a shift of -4: "Watermelon is delicious."

SWPANIAHKJ EO ZAHEYEKQO

4. Decode the following message that was coded with a shift of 7:

15 8 23 23 16 21 12 26 26 11 12 23 12 21 11 26 28 23 22 21

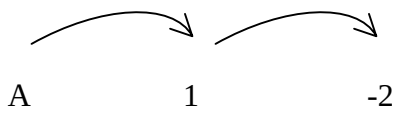
22 28 25 26 12 19 29 12 26

HAPPINESS DEPENDS UPON OURSELVES

V. Function Representations in Coding.

3. Identify the shift cipher that is represented by the arrow diagram below:

_____**SHIFT - 3**_____



4. Identify the equation that represents the relationship between the original position (p) of a letter and its coded value (c):

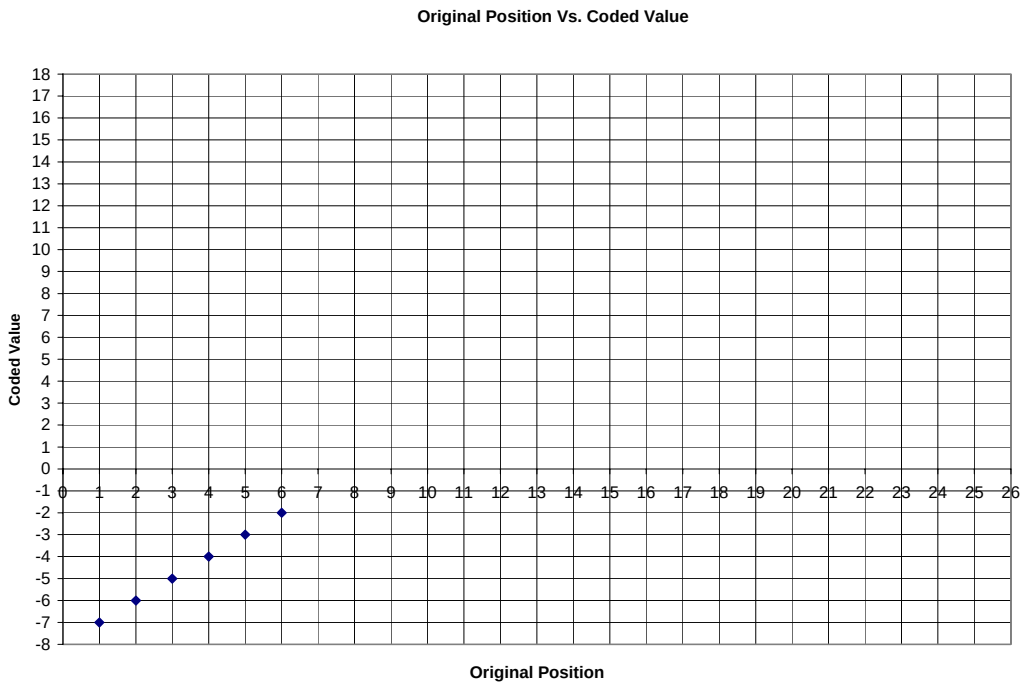
a.

Original Position	Coded Value
1	12

2	13
3	14
4	15
5	16
6	17

Equation: _____**C = P + 11** _____

b.



Equation: **C = P – 8**

VI. Stretch Cipher and Shift Cipher:

4. The table below shows the relationship between x and y. Which of these equations represents the relationship?

e) $y = -3x + 1$

f) $y = x - 3$

X	Y
-1	-2
0	1
1	4
2	7
3	10

g) $y = x + 3$

h) $y = 3x + 1$

5. Code the following message numerically with the equation: c = 2p + 7: I ‘m a great student

25’ 33 9 21 43 17 9 47 45 47 49 15 17 35 47

6. Decode the following message that was coded with the following cipher: c = 3p – 1.

8 44 68 56 2 53 14 20 53 2 41 11

COWS ARE GRAND