Title: Catapult Trajectories: Don’t Let Parabolas Throw You

Brief Overview:
- Students will use a trajectory as a means of learning about a quadratic function.
- Students will model a parabolic path and find the equation of a parabola from given points by solving a system of equations and by using quadratic regression.
- Students will study the effects of the constants $a$, $b$, and $c$ through an interactive graphing calculator application and learn to find the coordinates of the vertex.

NCTM Content Standard/National Science Education Standard:
Algebra
- Students will understand patterns, relations, and functions.
- Students will represent and analyze mathematical situations and structures using algebraic symbols.
- Students will use mathematical models to represent and understand quantitative relationships.
- Students will analyze changes in various contexts.

Geometry
- Students will analyze characteristics and properties of two-and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
- Students will specify locations and describe spatial relationships using coordinate geometry and other representational systems.
- Students will apply transformations and use symmetry to analyze mathematical situations.

Data Analysis and Probability
- Students will formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them.
- Students will develop and evaluate inferences and predictions that are based on data.

Communication
- Students will communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- Students will analyze and evaluate the mathematical thinking and strategies of others.
- Student will use the language of mathematics to express mathematical ideas precisely.
Connections
- Students will recognize and use connections among mathematical ideas.

Representation
- Students will create and use representations to organize, record, and communicate mathematical ideas.
- Students will use representations to model and interpret physical, social, and mathematical phenomena.

Grade/Level:
Algebra I and II, grades 9-11

Duration/Length:
Three or four class periods, each approximately 50 minutes in length.

Student Outcomes:
The students will be able to:
- Collect trajectory data and use technology to predict a quadratic curve of best fit.
- Identify key characteristics of the behavior of the graph of a quadratic equation.
- Use the equation of a parabolic path to investigate the horizontal and vertical position over an interval.
- Solve a system of linear equations (using three points) to determine the exact equation of a parabolic graph.

Materials and Resources:
- Plastic spoons (1 per group)
- Small and medium Styrofoam balls (1 per group)
- Wooden block stand (1cm deep)
- Easel size graph paper (1 inch grid)
- Colored markers
- Tape

Development/Procedures:
Lesson 1 Preassessment – The teacher will introduce the term parabola, describe the characteristics of a parabola, and the general equation yielding a parabolic graph. The teacher will define terms, such as maximum and minimum, which students will use in the Launch section. The teacher will also distribute the Introduction to Parabolas Worksheet (WS
#1. This worksheet will apply prior knowledge of graphing on the xy-plane, using ordered pair notation, to parabolas.

Launch - To engage students, the teacher will illustrate the shape of a parabola and its maximum/minimum value (the vertex) using a bowl, rope, and/or umbrella. The teacher will introduce the vocabulary keywords and concepts.

Teacher Facilitation – We will discuss characteristics of quadratics key terms:

- Line/linear
- Parabola
- Quadratic equation
- Leading coefficient
- Coordinates
- Vertex
- Maximum/minimum
- Zeros/roots/x-intercepts
- Table of values

The teacher will explain polynomial equation progression from linear to quadratic, comparing and contrasting shape of the graph, degree of x, number of terms, slopes, etc. In addition, the students will see that while two points can be used to find the equation of a line, three points are needed for a parabola. The standard form will be used for the basic lessons (in alignment with typical Algebra I curricula). The vertex form could be as an extension. The teacher will ask the students to give an example of an object that is roughly parabolic and opens up. (Does it have a minimum or maximum value? Similarly, does a parabolic-shaped object opening down have a minimum or maximum?)

Student Application – Students will be presented with the Graphing Parabolas Worksheet (WS #2). Students will individually create table of values for quadratic equations. Students will identify key parts of the graph (where it intersects the x- and y- axes and the vertex).

Embedded Assessment – This is included in the Graphing Parabolas Worksheet (WS #2).

Reteaching/Extension –

- To further understanding of the shape of the quadratic function, have students identify classroom and home objects containing that shape (i.e., rainbow, St. Louis arch, suspension bridge).
- To prepare for solving a $3 \times 3$ system of equations at the end of Lesson 2, students will solve the following system and state the method used:
Lesson 2

Preassessment – Students will now know how to create a table of values for a quadratic equation. (What causes the graph to go up or down?) The teacher will distribute the Behavior of Parabolas Worksheet (WS #3). To complete this worksheet, the student uses the TI – list and graphing utilities with data generated from quadratic equations.

Launch – The teacher will link the behavior of parabolas to a trajectory caused by catapulting an object.

Teacher Facilitation - The teacher will ask questions to tie Lesson 1’s material to the current lesson. (Why does a projected object take the path of a parabola? Which upward or downward does this path might have?) In addition, student pairs will discuss how key concepts of vertex and x-intercepts might have practical meaning in a trajectory situation.

Student Application - For tactile and visual learning, groups of four students will investigate the trajectories of Styrofoam balls (of two sizes) and record the paths of the catapulted balls. The teacher will distribute the Catapult Lab Investigation Worksheet (WS #4) and model the procedure described in the worksheet.

Embedded Assessment - This is included in the analysis section of the Catapult Lab Investigation Worksheet (WS #4).

Reteaching/Extension – Students will complete the Post Lab Activity Worksheet (WS #5) and the Finding the Exact Quadratic Worksheet (WS #6).

Lesson 3

Preassessment – From the first two days’ work, students will recognize catapult paths as tracing downward-opening quadratic equation graphs. They will be able to identify coefficients of quadratic equations (i.e., A, B, and C) and special parts of the graphs (e.g., x- and y-intercepts, maximum). These concepts will be reviewed during the launch.

Launch – Students will be asked: How could catapults apply in real life? Why would accuracy of the projected path be important? The teacher will link the path of a catapult to coefficients changes in quadratic formulas.

Teacher Facilitation - The teacher will demonstrate TI Transform to show how the parent function parabola \( y = ax^2 \) changes as a changes, then how the parabola \( y = ax^2 + bx + c \) changes when coefficients a, b, and c
changes. Students will complete the Using the TI Transform Program with Parabolas (WS #7).

Student Application - Given Target Practice (WS #8), students will use TI Transform in pairs or groups of three to discover parabolas that trace conditions such as projectiles fired from a catapult to hit a target a given distance away, reaching a certain height, etc. They will identify quadratic equations that represent catapult paths.

Embedded Assessment - This is included in the Target Practice (WS #8).

Reteaching/Extension – This day’s lesson is reteaching and providing an extension of the previous two days. As a homework assignment, each student will describe a situation demonstrating parabolic motion, give specific dimensions for the situation, graph a parabola that fits the situation, and determine the quadratic equation for the situation.

Summative Assessment:
See Summative Assessment Worksheet (WS #9).

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Identify the marked coordinates of the following parabola in ordered pair notation.

1).

<table>
<thead>
<tr>
<th>point</th>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2).

<table>
<thead>
<tr>
<th>point</th>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3). If a parabola opens down (like a lampshade), does it have a maximum or minimum?

4). If a parabola opens up (like a tulip), does it have a maximum or minimum?
Identify the marked coordinates of the following parabola in ordered pair notation.

1).

![Graph of a parabola with coordinates labeled A through G and a table of points]

<table>
<thead>
<tr>
<th>point</th>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-3</td>
<td>9</td>
</tr>
<tr>
<td>B</td>
<td>-2</td>
<td>4</td>
</tr>
<tr>
<td>C</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>E</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>F</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>G</td>
<td>3</td>
<td>9</td>
</tr>
</tbody>
</table>

2).

![Graph of a parabola with coordinates labeled A through D and a table of points]

<table>
<thead>
<tr>
<th>point</th>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-3</td>
<td>-5</td>
</tr>
<tr>
<td>B</td>
<td>-1</td>
<td>3</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>D</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

3). If a parabola opens down (like a lampshade), does it have a **maximum** or minimum?

4). If a parabola opens up (like a tulip), does it have a maximum or **minimum**?
Graphing Parabolas

Use the given quadratic equation to create a table of values, graph the function, and circle the vertex (maximum/minimum) and the zeros (x-intercepts) on the graph.

1). \( y = 2x^2 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>2</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Use the graph to find the vertex and the zeros.
2). \( y = -x^2 + 6x \)

<table>
<thead>
<tr>
<th>x</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

![Graph of the function]

The graph shows the function \( y = -x^2 + 6x \) plotted on a coordinate plane with x and y axes ranging from -10 to 10.
3. \( y = -x^2 + 4x + 12 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Using the given quadratic equation to create a table of values, graph and circle the vertex (maximum/minimum) and the zeros (x-intercepts) on the graph.

1). \( y = 2x^2 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y  )</td>
<td>8</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>8</td>
</tr>
</tbody>
</table>

![Graph of the parabola with vertex and zeros labeled.]
2). $y = -x^2 + 6x$

<table>
<thead>
<tr>
<th>$x$</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>-7</td>
<td>0</td>
<td>5</td>
<td>8</td>
<td>0</td>
</tr>
</tbody>
</table>

maximum zeros
3. $y = -x^2 + 4x + 6$

<table>
<thead>
<tr>
<th>$x$</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>-6</td>
<td>1</td>
<td>6</td>
<td>9</td>
<td>10</td>
<td>9</td>
<td>6</td>
<td>1</td>
<td>-6</td>
</tr>
</tbody>
</table>

Maximum zeros

Graph of $y = -x^2 + 4x + 6$
Behavior of Parabolas

A). Investigating \( y = 1x^2 + x + 1 \)
Select and record three integers between -6 and -1, three integers between 1 and 6, and zero as \( x \)-values for the table below.

\[
\begin{array}{|c|c|}
\hline
x & y \\
\hline
\hline
& \\
\hline
0 & \\
\hline
\end{array}
\]

Enter these values under the \( L1 \) list on the graphing calculator.
Under \( L2 \), enter \( 1*L1^2 + 1*L1 + 1 \).
Create a scatter plot for \( L1 \) and \( L2 \).
Enter the function in \( Y1 \) to see the graph.
\textbf{Describe the graph of the data.}

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

B). Investigating \( y = -1x^2 + x + 1 \)
Select and record three integers between -6 and -1, three integers between 1 and 6, and zero as \( x \)-values for the table below.

\[
\begin{array}{|c|c|}
\hline
x & y \\
\hline
\hline
& \\
\hline
0 & \\
\hline
\end{array}
\]

Enter these values under the \( L1 \) list on the graphing calculator.
Under \( L2 \), enter \( -1*L1^2 + 1*L1 + 1 \).
Create a scatter plot for \( L1 \) and \( L2 \).
Enter the function in $Y_1$ to see the graph.

**Describe the graph of the data.**

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

Compare/contrast the two quadratic equations investigated. What might have caused the differences in the shape of the data for each example?

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

Without graphing, describe the graph of the following curves:

A). $y = -2x^2 + 3x + 1$
B). $y = 3x^2 + 4x - 4$

________________________________________________________________________
________________________________________________________________________
Behavior of Parabolas – Answer Key

A). Investigating \( y = 1x^2 + x + 1 \)
Select and record three integers between -6 and -1, three integers between 1 and 6, and zero as \( x \)-values for the table below. \((answers may vary)\)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td>21</td>
</tr>
<tr>
<td>-3</td>
<td>7</td>
</tr>
<tr>
<td>-2</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>6</td>
<td>43</td>
</tr>
</tbody>
</table>

Enter these values under the \( L1 \) list on the graphing calculator.
Under \( L2 \), enter \( 1*L1^2 + 1*L1 + 1 \).
Create a scatter plot for \( L1 \) and \( L2 \).
Enter the function in \( Y1 \) to see the graph.

Describe the graph of the data.
The data is in the shape of an upward parabola (concave up). The data goes down, and the minimum value occurs close to zero (-.5 exactly) and then goes back up.
The curve that connects the data does not cross the \( x \)-axis.

B). Investigating \( y = -1x^2 + x + 1 \)
Select and record three integers between -6 and -1, three integers between 1 and 6, and zero as \( x \)-values for the table below. \((answers may vary)\)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6</td>
<td>-41</td>
</tr>
<tr>
<td>-4</td>
<td>-19</td>
</tr>
<tr>
<td>-2</td>
<td>-5</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>-5</td>
</tr>
<tr>
<td>6</td>
<td>-29</td>
</tr>
</tbody>
</table>

Enter these values under the \( L1 \) list on the graphing calculator.
Under \( L2 \), enter \(-1*L1^2 + 1*L1 + 1\).
Create a scatter plot for L1 and L2.
Enter the function in Y1 to see the graph.

Describe the graph of the data.
The data is in the shape of a downward parabola (concave down.) The data goes up, the maximum value occurs close to zero (.5 exactly) and then goes back down. The curve that connects the data crosses the x-axis twice (once to the left of the y-axis and once to the right of the y-axis.)

Compare/contrast the two quadratic equations investigated. What might have caused the differences in the shape of the data for each example?
The quadratic equation with the positive leading coefficient (+1) results in a upward parabola, whereas the quadratic equation with the negative leading coefficient (-1) results in a downward parabola.

Without graphing, describe the graph of the following curves:
A). \( y = -2x^2 + 3x + 1 \)
B). \( y = 3x^2 + 4x - 4 \)
Curve A will result in a downward parabola while Curve B will result in an upward parabola.
**Catapult Lab Investigation**

**Purpose**
To find the equation of the curve of best fit formed by a projectile from an algebraic approach and using quadratic regression. This lab will compare forward distance and height by recording data at specified forward distance intervals.

**Materials**
1 plastic spoon
1 small Styrofoam ball
1 large Styrofoam ball
tape
colored markers
1 (1cm deep) wooden block stand
easel-sized graph paper (1 inch grid)

**Pre Lab Questions**
A). Based on the purpose, what will be the:

Independent variable ________________________________
Dependent variable ________________________________

B). What type of function do we expect this trajectory to model? What will the sign of the leading coefficient of the equation of this trajectory be … positive … negative … zero?

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
Setup

NOTE: Each group should set up the lab station in the same manner so class averages can be calculated. All spoons should be taped the same distance from the edge of the block.

The vertical scale is height in inches.

---

**Figure 1**

<table>
<thead>
<tr>
<th>3 inches</th>
<th>6 inches</th>
<th>9 inches</th>
</tr>
</thead>
</table>

---

**Procedure**

1. Orient the paper vertically.
2. Make the origin the lower left corner.
3. Draw three vertical lines on the grid at 3, 6, and 9 inches, each in a different color.
4. Hang and tape graph paper from the desk edge.
5. Tape spoon so that the tip of the handle is 4 inches from the edge of the wood block.
6. Put catapult (with spoon attached) in front of graph as shown.
7. Place large ball in spoon.
8. Release the ball from spoon and observe the projectile path. Each of the other three students must mark the spot where the foam ball crosses a given colored line.
9. Record the heights in data table.
10. Repeat steps 8 and 9 twice more for a total of three trials.
11. Repeat steps 7 to 10 using small Styrofoam ball.
### Data : Large Ball

<table>
<thead>
<tr>
<th>Trials</th>
<th>at $x = 0$</th>
<th>at $x = 3$</th>
<th>at $x = 6$</th>
<th>at $x = 9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trial 1</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trial 2</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trial 3</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Group Average</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Data : Small Ball

<table>
<thead>
<tr>
<th>Trials</th>
<th>at $x = 0$</th>
<th>at $x = 3$</th>
<th>at $x = 6$</th>
<th>at $x = 9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trial 1</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trial 2</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trial 3</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Group Average</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Analysis:
1). Enter the four data points for the forward distance under $L_1$ and the corresponding four data points for the group average data for vertical distance under $L_2$ using the small ball data.
2). Enter the corresponding four data points for the class average data for vertical distance under $L_3$.
3). Graph $L_1$ and $L_2$ as a scatter plot and create a parabolic curve of best fit using QuadReg on the graphing calculator. Confirm your curve with the data points. Record your equation in the appropriate cell below (to the nearest tenth).
4). Repeat this procedure with $L_1$ and $L_3$.
5). Repeat this entire process for the large ball and record the curve equation.

<table>
<thead>
<tr>
<th>Ball Size</th>
<th>Averaged Data</th>
<th>Fitted Curve Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>Group</td>
<td></td>
</tr>
<tr>
<td>Large</td>
<td>Group</td>
<td></td>
</tr>
</tbody>
</table>
Purpose
To find the equation of the curve of best fit formed by a projectile from an algebraic approach and using quadratic regression. This lab will compare forward distance and height by recording data at specified forward distance intervals.

Materials
1 plastic spoon
1 small Styrofoam ball
1 medium Styrofoam ball
tape
colored markers
1 (1cm deep) wooden block stand
easel-sized graph paper (1 inch grid)

Pre Lab Questions
A). Based on the purpose, what will be the:

Independent variable ____________ forward distance ______________________________
Dependent variable ____________ vertical distance _______________________________

B). What type of function do we expect this trajectory to model? What will the sign of the leading coefficient of the equation of this trajectory be … positive … negative … zero?

One would expect the trajectory to be modeled as a parabolic path opening down. Therefore, the leading coefficient must be negative.
**Setup**

NOTE: Each group should set up the lab station in the same manner. Any deviation should be discouraged if class averages will be calculated. All spoons should be taped the same distance from the edge of the block.

In Figure 1 the dots represent the students’ observations as the ball crosses the vertical lines. There are three students who each observe the location of the ball as it passes one of the vertical lines. The dotted line represents the actual path of the ball (only three points of which are recorded).

The vertical scale is height in inches.

![Figure 1](image)

**Procedure**

1. Orient the paper vertically.
2. Make the origin the lower left corner.
3. Draw three vertical lines on the grid at 3, 6, and 9 inches, each in a different color.
4. Hang and tape graph paper from the desk edge.
5. Tape spoon so that the tip of the handle is 4 inches from the edge of the wood block.
6. Put catapult (with spoon attached) in front of graph as shown.
7. Place large ball in spoon.
8. Release the ball from spoon and observe the projectile path. Each of the other three students must mark the spot where the foam ball crosses a given colored line.
9. Record the heights in data table.
10. Repeat steps 8 and 9 twice more for a total of three trials.
11. Repeat steps 7 to 10 using small Styrofoam ball.
### Data: Large Ball

<table>
<thead>
<tr>
<th>Trials</th>
<th>at $x=0$</th>
<th>at $x=3$</th>
<th>at $x=6$</th>
<th>at $x=9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trial 1</td>
<td>0</td>
<td>5</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>Trial 2</td>
<td>0</td>
<td>6</td>
<td>8</td>
<td>8.5</td>
</tr>
<tr>
<td>Trial 3</td>
<td>0</td>
<td>4</td>
<td>5</td>
<td>5.5</td>
</tr>
<tr>
<td>Group Average</td>
<td>0</td>
<td>5</td>
<td>6.5</td>
<td>7</td>
</tr>
</tbody>
</table>

### Data: Small Ball

<table>
<thead>
<tr>
<th>Trials</th>
<th>at $x=0$</th>
<th>at $x=3$</th>
<th>at $x=6$</th>
<th>at $x=9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trial 1</td>
<td>0</td>
<td>10.5</td>
<td>14</td>
<td>14.5</td>
</tr>
<tr>
<td>Trial 2</td>
<td>0</td>
<td>12.5</td>
<td>16.5</td>
<td>17.5</td>
</tr>
<tr>
<td>Trial 3</td>
<td>0</td>
<td>15</td>
<td>18</td>
<td>11</td>
</tr>
<tr>
<td>Group Average</td>
<td>0</td>
<td>12.7</td>
<td>16.2</td>
<td>14.3</td>
</tr>
</tbody>
</table>
Analysis:

1). Enter the four data points for the forward distance under $L_1$ and the corresponding four data points for the group average data for vertical distance under $L_2$ using the small ball data.

2). Graph $L_1$ and $L_2$ as a scatter plot and create a parabolic curve of best fit using QuadReg on the graphing calculator. Confirm your curve with the data points. Record your equation in the appropriate cell below (to the nearest tenth).

3). Repeat this entire process for the large ball and record the curve equation.

<table>
<thead>
<tr>
<th>Ball Size</th>
<th>Averaged Data</th>
<th>Fitted Curve Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>Group</td>
<td>$y = -.41x^2 + 5.2x + .19$</td>
</tr>
<tr>
<td>Large</td>
<td>Group</td>
<td>$y = -.13x^2 + 1.9x + .13$</td>
</tr>
</tbody>
</table>

![Graph of Small and Large Ball Data](image-url)
Post Lab Activity

Answer the following questions:
1). Using your group’s curve of best fit equations, predict the height of the ball when the forward distance is 5 inches for both balls.

2). Using your group fitted curve of best fit equations, predict the forward distance of the balls when the projectiles hits the ground.

3). Elton and Carol used the same catapult to launch Styrofoam balls. Carol’s ball was twice as large as Elton’s. In this graph, label which path would most likely represent Carol’s ball. Explain your answer.
Answer the following questions:

1). Using your group’s curve of best fit equations, predict the height of the ball when the forward distance is 5 inches for both balls. (answers will vary)

\[ y = f(x) = -0.41x^2 + 5.2x + 0.19 \]
\[ f(5) = -0.41(5)^2 + 5.2(5) + 0.19 = 15.94 \]

At a forward distance of 5 inches, the vertical distance is 15.94 inches for the small ball.

\[ y = g(x) = -0.13x^2 + 1.9x + 0.13 \]
\[ g(5) = -0.13(5)^2 + 1.9(5) + 0.13 = 6.38 \]

At a forward distance of 5 inches, the vertical distance is 6.38 inches for the large ball.

2). Using your group fitted curve of best fit equations, predict the forward distance of the ball when the projectile hits the ground.

\[ x \approx 12.72 \]

The forward distance is 12.72 inches when the projectile hits the ground for the small ball.

\[ x \approx 14.68 \]

The forward distance is 14.68 inches when the projectile hits the ground for the large ball.

3). Elton and Carol used the same catapult to launch Styrofoam balls. Carol’s ball was twice as large as Elton’s. In this graph, label which path would most likely represent Carol’s ball. Explain your answer.

The smaller ball will go higher. Therefore, the solid curve represents the path of the smaller ball while the dotted curve represents the path of the larger ball.
Finding the Exact Quadratic

Background: If you know three points on a trajectory (path of a projectile), you can find the exact quadratic equation using a system of equations. That is, three points determine a particular parabola that can be expressed as a quadratic equation of the form:

$$y = ax^2 + bx + c.$$ 

Suppose you have the following three points:

$$(0, 0); (4, 6); (10, 3)$$

Find $a$, $b$, and $c$ in the standard form of the quadratic equation from above by solving this system:

$$0 = a(0)^2 + b(0) + c$$
$$6 = a(4)^2 + b(4) + c$$
$$3 = a(10)^2 + b(10) + c$$

1). Use substitution or linear combination to find $a$, $b$, and $c$. (Hint: solve the first equation first.)

2). State the equation of the path of the ball in the standard quadratic form.

3). Use your equation to find the height of the ball when the horizontal distance is 12 feet.
Background: If you know three points on a trajectory (path of a projectile), you can find the exact quadratic equation using a system of equations. That is, three points determine a particular parabola that can be expressed as a quadratic equation of the form:

\[y = ax^2 + bx + c.\]

Suppose you have the following three points:

(0, 0); (4, 6); (10, 3)

Find \(a\), \(b\), and \(c\) in the standard form of the quadratic equation from above by solving this system:

\[
\begin{align*}
0 &= a(0)^2 + b(0) + c \\
6 &= a(4)^2 + b(4) + c \\
3 &= a(10)^2 + b(10) + c \\
\end{align*}
\]

1). Use substitution or linear combination to find \(a\), \(b\), and \(c\). (Hint: solve the first equation first.)

\[
\begin{align*}
0 &= a(0)^2 + b(0) + c \Rightarrow c = 0 \text{ (from equation 1)} \\
Now plugging 0 for c in equations 2 and 3, we get
6 &= a(4)^2 + b(4) \\
3 &= a(10)^2 + b(10) \\
\end{align*}
\]

After some simplifying …

\[
\begin{align*}
16a + 4b &= 6 \\
100a + 10b &= 3 \\
-5(16a + 4b = 6) &\rightarrow 80a + 20b = 30 \\
-2(100a + 10b = 3) &\rightarrow 200a - 20b = -6 \\
-120a &= 24 \\
a &= -0.2 \\
16(-0.2) + 4b &= 6 \\
-3.2 + 4b &= 6 \\
4b &= 9.2 \\
b &= 2.3 \\
a = -0.2, b = 2.3, c = 0
\end{align*}
\]

2). State the equation of the path of the ball in the standard quadratic form.

The equation is \(y = -0.2x^2 + 2.3x.\)
3). Use your equation to find the height of the ball when the horizontal distance is 12 feet.

Let \( y = f(x) = -0.2x^2 + 2.3x \). \( f(12) = -0.2(12)^2 + 2.3(12) = -1.2 \) feet. Therefore

when the horizontal distance is 12 ft, the vertical distance is 1.2 feet below ground zero.
Using TI Transform with Parabolas

1. Check that this program is on your calculator.
   a. Press APPS
   b. Look for Transform (new name) or Interact (same program, old name)
   c. If not there, get help from teacher to copy onto your calculator. Otherwise, go to step 2.
2. Clear all Y= and STAT PLOTS.
3. Push APPS> Transform > Continue
   NOTE: If you don’t see a play sign, you have turned off the program. Repeat step 3.
4. Go to Y=, Enter Ax² + Bx + C
5. Go to WINDOWS ^ > SETTINGS > ENTER
   Turn on play/pause by selecting: >||
   Set A=-1, B=1, C=1, Step = .25
6. Set the following parameters on your calculator
   • MODE>FLOAT>2>ENTER (this will round answers to nearest hundredth)
   • MODE>CONNECTED>ENTER
   • 2nd ZOOM (FORMAT) > AxesOn >ENTER
   • 2nd ZOOM (FORMAT) > GridOn >ENTER
7. Go to ZOOM 6, ENTER
8. Sketch the view on your screen.

9. Highlight \( A = -1 \) > type 2 >ENTER. This will change the value of \( A \) so \( A = -2 \). Use a dotted line to sketch the new view on the screen above.

10. Change the value of \( A \) so \( A = -3 \). Use a dashed line to sketch the new view on the screen above.

11. Describe what happens when you press the cursor right.

12. Describe what happens when you press the cursor left.
13. Change the value of $A$ back to $A=-1$. Sketch the view on your screen.

14. Highlight $B=1 >$ type 2 > ENTER. This will change the value of $B$ so $B=2$. Use a dotted line to sketch the new view on the screen above.

15. Change the value of $B$ so $B=3$. Use a dashed line to sketch the new view on the screen above.

16. Describe what happens when you press the cursor right.

17. Describe what happens when you press the cursor left.
18. Change the value of $B$ back to $B=1$. Sketch the view on your screen.

19. Highlight $C=1 >$ type 2 > ENTER. This will change the value of $C$ so $C=2$. Use a dotted line to sketch the new view on the screen above.

20. Change the value of $C$ so $C=3$. Use a dashed line to sketch the new view on the screen above.

21. Describe what happens when you press the cursor right.

22. Describe what happens when you press the cursor left.
Using TI Transform with Parabolas – Answer Key

1. Check that this program is on your calculator.
   d. Press APPS
   e. Look for Transform (new name) or Interact (same program, old name)
   f. If not there, get help from teacher to copy onto your calculator. Otherwise, go to step 2.
2. Clear all Y= and STAT PLOTS.
3. Push APPS > Transform > Continue
4. Go to Y=, Enter \(Ax^2 + Bx + C\)
5. Go to WINDOWS > SETTINGS > ENTER
   - Turn on play/pause by selecting: >||
   - Set \(A=-1, B=1, C=1, Step = .25\)
6. Set the following parameters on your calculator
   - MODE>FLOAT>2>ENTER (this will round answers to nearest hundredth)
   - MODE>CONNECTED>ENTER
   - 2nd ZOOM (FORMAT) > AxesOn >ENTER
   - 2nd ZOOM (FORMAT) > GridOn >ENTER
7. Go to ZOOM 6, ENTER
8. This should be the view on your screen.

Changing \(A\) in \(y=Ax^2+Bx+C\)
9. Highlight \( A = -1 \) > type 2 > ENTER. This will change the value of \( A \) so \( A = -2 \). Use a dotted line to sketch the new view on the screen above.

10. Change the value of \( A \) so \( A = -3 \). Use a dashed line to sketch the new view on the screen above.

11. Describe what happens when you press the cursor right.

| As \( A \) increases from -1 toward 0; the downward opening parabola widens, and the vertex moves diagonally upward and rightward. At \( A = 0 \), there is a straight line. As \( A \) increases from 0; the upward opening parabola narrows, and the vertex moves diagonally upward and rightward. |

12. Describe what happens when you press the cursor left.

| As \( A \) decreases from a positive value toward 0; the upward opening parabola widens, and the vertex moves diagonally downward and leftward. At \( A = 0 \), there is a straight line. As \( A \) decreases below 0; the downward opening parabola narrows, and the vertex moves diagonally downward and leftward. |

13. Change the value of \( A \) back to \( A = -1 \).

This should be the view on your screen.

14. Highlight \( B = 1 \) > type 2 > ENTER. This will change the value of \( B \) so \( B = 2 \). Use a dotted line to sketch the new view on the screen above.

15. Change the value of \( B \) so \( B = 3 \). Use a dashed line to sketch the new view on the screen above.

16. Describe what happens when you press the cursor right.

As \( B \) increases from 1, the vertex of the parabola moves diagonally upward and
17. Describe what happens when you press the cursor left.

As positive $B$ decreases toward 0, the vertex of the parabola moves diagonally downward and leftward. At $B=0$ the parabola is at its lowest. As $B$ decreases below 0 the parabola moves diagonally upward and leftward.


This should be the view on your screen.

19. Highlight $C=1$ > type 2 > ENTER. This will change the value of $C$ so $C=2$. Use a dotted line to sketch the new view on the screen above.

20. Change the value of $C$ so $C=3$. Use a dashed line to sketch the new view on the screen above.

21. Describe what happens when you press the cursor right.

As $C$ increases the parabola moves up.

22. Describe what happens when you press the cursor left.

As $C$ decreases the parabola moves down.
Directions: Use the TI Transform program to answer questions below.

Set-Up
1). Turn off Stat plots and Y=.
2). Set the following parameters on your calculator.
   - Y=Ax^2+Bx+C
   - Window [-1, 10, 1; -1, 10, 1] (this is just the starting window; it will change as you work)
   - Window>Settings>: ||; A=-1; B=0; C=0; Step=.01
   - MODE>FLOAT>2>ENTER (this will round answers to nearest hundredth)
   - MODE>CONNECTED>ENTER
   - 2nd ZOOM (FORMAT) > AxesOn >ENTER
   - 2nd ZOOM (FORMAT) > GridOn >ENTER
3). The vertex of your parabola should now be at (0, 0).
4). If you use TRACE, in order to return to the TRANSFORM screen push GRAPH. This will show the A, B, and C values.

Directions
Use TI Transform to identify a parabola (there may be more than one possible) to fit each condition in the left column below. In some cases coefficients of the quadratic equation are given. In other cases, conditions are described in terms of a catapult.

- On the appropriate graph, sketch the parabola (aka catapult path) you select.
- Write the parabola’s equation.

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Graph</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>If A=-1 and B=6, find C so that the parabola ascends through (0, 0) and descends through (6, 0).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>If C=0, B=6, find A so that the catapult has a maximum point (3, 9).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>If A=-.5 and C=0, find B so that the catapult has a maximum point of (4, 8).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A catapult begins at (0, 0), and there is a wall 4.5 units high exactly 8 units from the launch site. Find a parabola that goes over the wall and lands as close as possible to the far side of the</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Target Practice – Answer Key

Directions: Use the TI Transform program to answer questions below.

Set-Up
1. Turn off Stat plots and Y=.
2. Set the following parameters on your calculator.
   - Y=Ax²+Bx+C
   - Window [-1, 10, 1; -1, 10, 1] (this is just the starting window; it will change as you work)
   - Window>Settings> >||; A=-1; B=0; C=0; Step=.01
   - MODE>FLOAT>2>ENTER (this will round answers to nearest hundredth)
   - 2nd ZOOM (FORMAT) > AxesOn >ENTER
   - 2nd ZOOM (FORMAT) > GridOn >ENTER
3. The vertex of your parabola should now be at (0, 0).
4. If you use TRACE, in order to return to the TRANSFORM screen, enter GRAPH. This will show the A, B, and C values.

Directions
Use TI Transform to identify a parabola (there may be more than one possible) to fit each condition in the left column below. In some cases coefficients of the quadratic equation are given. In other cases, conditions are described in terms of a catapult.
- On the appropriate graph, sketch the parabola (aka catapult path) you select.
- Write the parabola’s equation.

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<tbody>
<tr>
<td>If A=-1 and B=6, find C so that the parabola ascends through (0, 0) and descends through (6, 0).</td>
<td>C=0</td>
<td>y=-x²+6x</td>
</tr>
<tr>
<td>If C=0, B=6, find A so that the catapult has a maximum point (3, 9).</td>
<td>A=-1</td>
<td>y=-x²+6x</td>
</tr>
<tr>
<td>If A=-.5 and C=0, find B so that the catapult has a maximum point of (4, 8).</td>
<td>B=4</td>
<td>y=-.5x²+4x</td>
</tr>
<tr>
<td>A catapult begins at (0, 0), and there is a wall 4.5 units high exactly 8 units from the launch site. Find a parabola that goes over the wall and lands as close as possible to the far side of the wall.</td>
<td>Sample A&lt;br&gt;( A=-1 )&lt;br&gt;( B=8.6 )&lt;br&gt;( C=0 )&lt;br&gt;( \text{max of } (8, 4.8) )&lt;br&gt;Sample B&lt;br&gt;( A=-1 )&lt;br&gt;( B=8.57 )&lt;br&gt;( C=0 )&lt;br&gt;( \text{max of } (8, 4.56) )</td>
<td>( y=-x²+8.6x )&lt;br&gt;(A) &lt;br&gt;( y=-x²+8.57x )&lt;br&gt;(B)</td>
</tr>
</tbody>
</table>
1). The jump rope held by Alice and Bonita has its lowest point closer to Alice. (How could this be?)

   A. Make a sketch labeling the position of the girls, the shape of the jump rope and indicate the lowest point.

   B. Where would the lowest point be if the students were of equal heights?

2). Which of the following is a possible equation for the graph on the left?

   a) $y = 2x$
   b) $y = -x^2$
   c) $y = 1 + x$
   d) $y = 1 + x^2$
3). Evaluate \( y = f(x) = -x^2 + 8x + 2 \) when \( x = 3 \).

4). Graph the following quadratic equations of the form \( y = ax^2 + bx + c \) using the TI-Transform/TI-Interact Application.

A). Set the graphing calculator using the following window \([-7, 7; -10, 10]\). Use the default values of the coefficients:

\[
A=1; \quad B=0; \quad C=0; \quad y = x^2.
\]

This is the parent function.

Sketch the parent function on the right.

Describe the change when only the single modification is made to the parent function.

- \( A=4 \)
- \( A=-2 \)
- \( A=1/3 \)
- \( A=-1/5 \)
- \( B=3 \)
- \( B=-4 \)
- \( C=2 \)
- \( C=-4 \)

BONUS: Find any \( A, B, \) and \( C \) such that the vertex of the graph of \( y = ax^2 + bx + c \) falling in quadrant and the graph is inverted.
1). The jump rope held by Alice and Bonita has its lowest point closer to Alice. (How could this be?)

A. Make a sketch labeling the position of the girls, the shape of the jump rope and indicate the lowest point.

B. Where would the lowest point be if the students were of equal heights?

The lowest point would be exactly in the middle of the two students.

2). Which of the following is a possible equation for the graph on the left?

a) $y = 2x$

b) $y = -x^2$

c) $y = 1 + x$

d) $y = 1 + x^2$
3). Evaluate \( y = f(x) = -x^2 + 8x + 2 \) when \( x = 3 \).

\[
f(3) = -(3)^2 + 8(3) + 2 = 17
\]

4). Graph the following quadratic equations of the form \( y = ax^2 + bx + c \) using the TI-Transform/TI-Interact Application.

A). Set the graphing calculator using the following window \([-7, 7, 1; -10, 10,1]\). Use the default values of the coefficients: A=1; B=0; C=0; \( y = x^2 \). This is the parent function.

Sketch the parent function on the right.

Describe the change when only the single modification is made to the parent function.

- \( A=4 \) The parabola narrows.
- \( A=-2 \) The parabola inverts and narrows.
- \( A=1/3 \) The parabola widens.
- \( A=-1/5 \) The parabola inverts and widens.
- \( B=3 \) The parabola shifts to the left and down.
- \( B=-4 \) The parabola shifts to the right and down.
- \( C=2 \) The parabola shifts up two units.
- \( C=-4 \) The parabola shifts down four units.

BONUS: Find any \( A, B, \) and \( C \) such that the vertex of the graph of \( y = ax^2 + bx + c \) falling in quadrant and the graph is inverted.

**sample answer**

\[
A = -1 \\
B = 6 \\
C = -3
\]