Title: Matrix Madness!!!!

Brief Overview:

This unit has the students multiplying matrices, finding matrix determinants and inverses, and several real world applications of these processes. Upon completion of unit, the students will be able to use the determinant of a matrix in order to find the area of a triangular region. They will also be able to use matrix multiplication to encode a message and decode the message using its inverse. It is assumed before starting this unit that students are able to multiply matrices with/without calculators depending on level.

NCTM Content Standard/National Science Education Standard:

- Understand meanings of operations and how they relate to one another – develop an understanding of properties of, and representations for, the addition and multiplication of vectors and matrices;
- Compute fluently and make reasonable estimates – develop fluency in operations with real numbers, vectors, and matrices, using mental computation or paper–and–pencil calculations for simple cases and technology for more–complicated cases.
- Specify locations and describe spatial relationships using coordinate geometry and other representational systems – use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations;

Grade/Level:

Algebra 2 grades 8–11

Duration/Length:

Three 45 – minute class periods

Student Outcomes:

Students will:

- Calculate the determinant of 2x2 and 3x3 matrices.
- Use the determinant and Cartesian coordinates to calculate the area of a triangular region.
- Find the inverse of 2x2 matrices.
- Use multiplication of matrices and inverses to encode and decode phrases

Materials and Resources:

- Graphing calculator
- Envelopes
- Rectangular prism
- Multiple small prizes
- Copies of worksheets and activities
  - Matrix Madness Pre–assessment
  - Area of a Triangle
  - Inside – Outside Circle
  - Area of a Triangle Determinant Style
  - Treacherous Triangles
  - Mystical Matrices
  - Matrix Inverses Class Notes
  - Finding the Inverse of a Matrix
  - Inverse Round Table 1–4
  - Decode the Encode

Development/Procedures:

Day 1

- Pre–assessment
  Pre–assess students' prior knowledge and skill level using the worksheet "Matrix Madness". Reteach skills as necessary before continuing with the unit.

- Exploration
  Distribute the worksheet "Area of a Triangle" and challenge the students to determine the area of each of the triangles. Recall the formula $A = \frac{1}{2}bh$ to determine the areas for examples 1 and 2 only if students show the need for assistance. Guide the students through determining the area of the third example, as necessary. Note the strategy of drawing a square around the triangle, and then determining the areas of each of the outer triangles and subtract from the area of the square.

  Forecast for the students that by the end of the lesson they will have a new way of determining the area of a triangle, which will facilitate finding areas of triangles like the one in example 3.

- Explanation
  Introduce the notation for *determinants*, $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$ or $\det\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

Define the formula for the determinant for a 2 x 2 *matrix*, $ad - bc$. Emphasize that the answer is a number rather than a matrix. Model two examples with the students, and then allow the students to practice using the Inside – Outside Circle Activity.

Illustrate the diagonal method for evaluating 3 x 3 determinants. Distinguish the direction of the diagonals using colored chalk or markers. Improve accuracy by inserting parentheses around the sums of the diagonal products as shown below.

$$\begin{vmatrix} -1 & 3 & -3 \\ 4 & -2 & -1 \\ 0 & -5 & 2 \end{vmatrix} \begin{matrix} -1 & 3 \\ 4 & -2 \\ 0 & -5 \end{matrix} = (4+0+60)-(0-5+24)$$

Work through two more examples with the students.

Return to the "Area of a Triangle" problems, which appear on the new worksheet "Area of a Triangle Determinant Style". Preface the activity by explain to the students that the determinant can be used to find the area of triangles. Have the students determine the coordinates of the vertices of triangle 1. Illustrate how to enter those vertices into a 3 x 3 matrix and elicit suggestions as to how to fill in the third column. Lead the students into noticing that filling in the third column with ones will keep the values the same when multiplying the diagonals. Post the formula below, which can be used to determine the area of a triangle.

Where the vertices of the triangle are (a, b), (c, d) and (e, f)

$$A = \frac{1}{2} \cdot \begin{vmatrix} a & b & 1 \\ c & d & 1 \\ e & f & 1 \end{vmatrix}$$

Ask, "Why is it necessary to take the absolute value of the determinant in the area formula?" Lead the students to the conclusion that this guarantees a nonnegative value for the area. Allocate time for the students to practice with triangles 2 and 3.

- Application
  Have the students read and complete the worksheet "Treacherous Triangles." Allow the students to check and compare their answer with a partner.

- Differentiation
  - Reteach
    Use the graphing calculator overhead projection view screen to demonstrate using the calculator to evaluate determinants.

- Enrich
  Challenge the students to determine the area of a parallelogram.

- Assessment
  Teacher should circulate the room and monitor students as they work practice problems and check answers with the class as appropriate.

Day 2

- Exploration
  Distribute the worksheet, " Mystical Matrices". Have the students work through the exercises paying particular attention to the arithmetic and commonalities of each exercise. Summarize the activity with the students, guiding them to focus on identities and inverses.

- Explanation
  Transition from the exploration to the class notes. Distribute the guided notes pages "Matrix Inverses Class Notes". Allow the students time to examine the given matrices and identify patterns, one set/rule at a time. Acknowledge student thoughts and guide them as appropriate to the desired key concepts. Complete the rules and demonstrate finding the inverse of a 2x2 matrix with a few guided practices using "Finding the Inverse of a Matrix."

- Application
  Divide the class into groups of four for a Round Table activity. Distribute worksheets "Matrix Inverse Roundtable 1–4" to each group. Circulate the room to observe and assist students as appropriate. Post final, complete, and correct answers around the room. Have additional practice prepared as necessary for time adjustment.

- Differentiation
  - Reteach
    Use the graphing calculator to find the inverse of any square matrix.

  - Enrich
    Introduce using inverse matrices to solve systems of equations. Challenge students to solve a complex system of three equations and three unknowns as an added challenge.

- Assessment
  Teacher should circulate the room and monitor students as they work practice problems and check answers with the class as appropriate.

Day 3

- Preparation
  Copy and cut enough "Decode the Encode Matching" cards for groups of three in the class prior to the start of class. Put the encodes (locks) and decodes (keys) in an envelope for each group. The clues are: "the prize is hidden","in a rectangular prism", "north", "west", "right angle". Hide a prize for each group in a rectangular prism (briefcase, treasure chest, tissue box…) and place the "rectangular prism" in the "North" "West" "corner" of the classroom.

- Exploration
  Distribute the worksheet, "Decode the Encode". Have the students translate the phrases at the top of the first page.

  3up3dn → baseball strike out
  RUOK→Are you okay?
  KIT→Keep in touch
  LBRT4AL →Liberty for all
  TRQLZR→tranquilizer
  10SNE1 → tennis anyone.

  Read the first two paragraphs and discuss the idea of writing things in other ways. Ask the students, "Do you know of any other ways besides codes and texting where we replace the original message with other words, symbols, or numbers?" Include such answers as: zip codes, UPC labels, state abbreviations, etc.

- Explanation
  Work through the Caesar cipher of 'Mathematics is fascinating' section of "Decode the Encode" with the students in order to demonstrate how ancient coding first worked. Have each student select a phrase to encode using a Caesar Cipher. Arrange students in pairs to decode each other's messages.

  Discuss with the students the recent uses of codes, particularly for government agencies during hostile situations. Reference the ones listed at the bottom of page 1 for the students.

- Application
  Demonstrate the process of using matrices and their inverses in order to encode and decode messages. Follow the steps set up on pages 2 and 3 of "Decode the Encode" to work as a class.

  Circulate the room as students attempt to encode and decode 'Pi day is great' at the bottom of page 3. Have the students share their results with

their partners. Show the solution to work after the students have had a chance to share. Work additional examples as needed.

Group the students in triples in order to match the correct locks and keys. Explain to the students that they will be given five encoded matrices and five keys. Have the students use inverse matrices and matrix multiplication in order to match the correct key with encoded matrix and then decode the matrix. Note that the decoded phrases will produce five clues to hidden prizes as described in teacher preparation.

Have the students show their work on a separate sheet of paper to be collected at the end of the lesson for assessment. Observe and assist students as needed.

Tell the groups as they are discovering clues and the possible location to the prizes that they are not to divulge the answers to any other groups. Caution the students to be secretive in the retrieval of the prizes.

Display the solutions if there are any groups who were unable to find the location of the prizes.

- Differentiation
  - Reteach
    Remind students how to multiply matrices and how to find the inverse of a matrix. Show students how to use the graphing calculator in order to assist with matrix operations and finding the inverse.

  - Enrich
    Have students encode two or three clues of their own to tell another student their favorite color or movie. Have the students trade clues to see if they can decode them.

Summative Assessment:

Have students complete the worksheet, "Join the Madness" in order to assess skills learned in this unit. Reteach or give additional practice for those skills not mastered.

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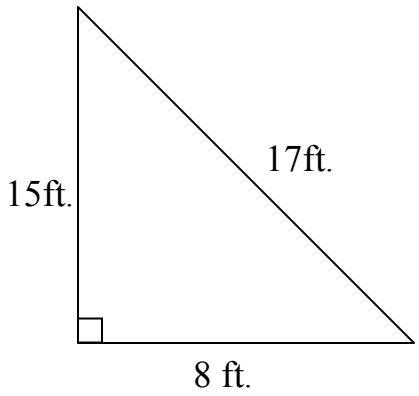
Matrix Madness Name: _____

Pre–assessment

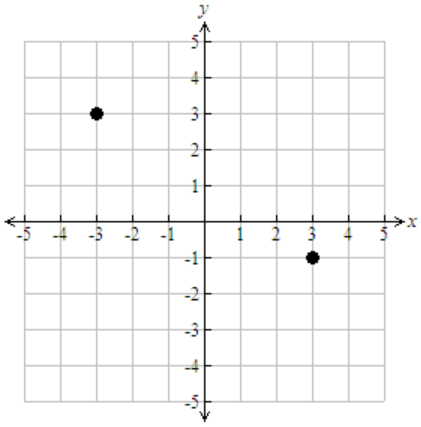
1. $5 \times \frac{3}{7} =$

2. $\frac{4}{15} \times \frac{9}{16} =$

3. Find the area of the triangle:



4. Identify the coordinates of the points plotted on the graph below.



5. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 0 & -2 \\ 3 & 1 \end{bmatrix} =$

Matrix Madness Name: __ANSWER KEY___

Pre–assessment

1. $5 \times \frac{3}{7} = \frac{15}{7}$

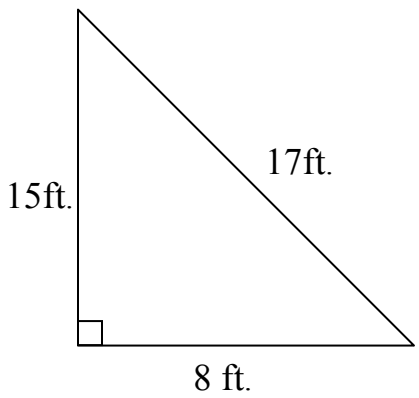
2. $\frac{4}{15} \times \frac{9}{16} = \frac{3}{20}$

291525

840

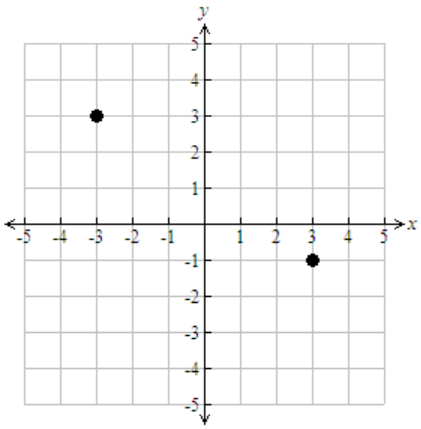
3. Find the area of the triangle:

60 square feet



4. Identify the coordinates of the points plotted on the graph below.

$(-3,3)$
$(3,-1)$

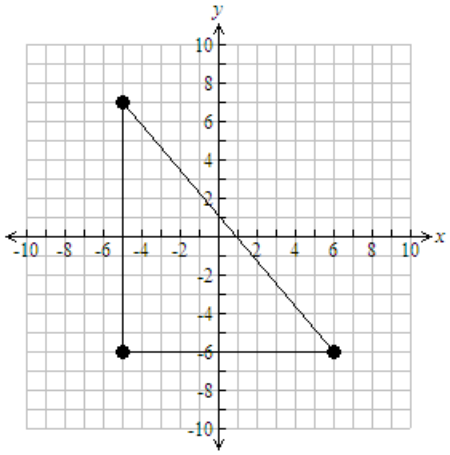


5. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 0 & -2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 12 & -2 \end{bmatrix}$

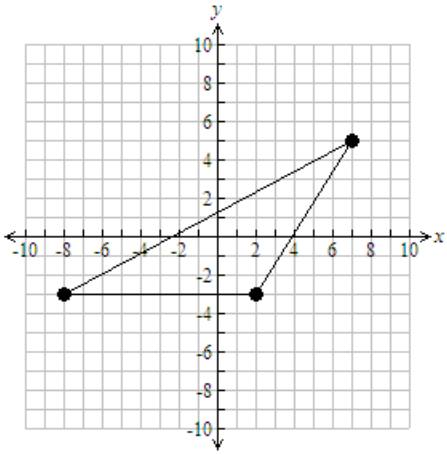
Area of a Triangle Name: _____

Determine the area of each of the triangles.

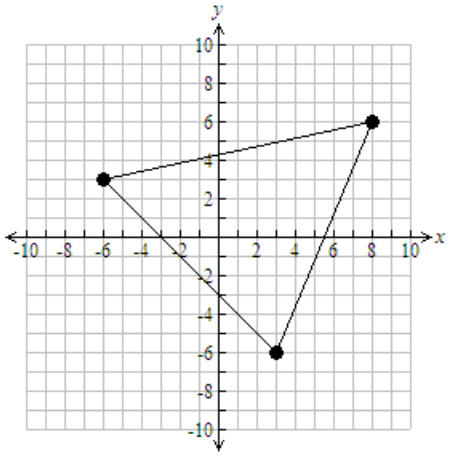
1.



2.



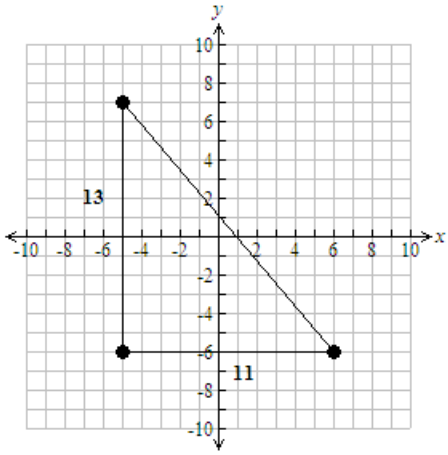
3.



Area of a Triangle Name: __ANSWER KEY___

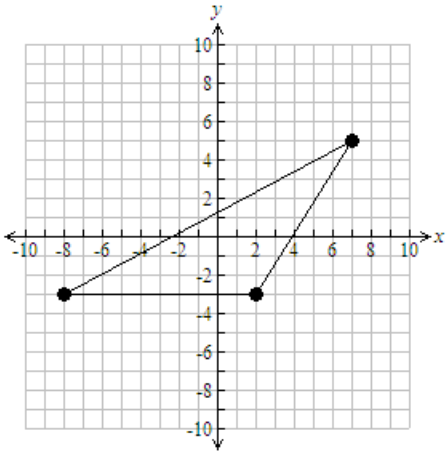
Determine the area of each of the triangles.

1.



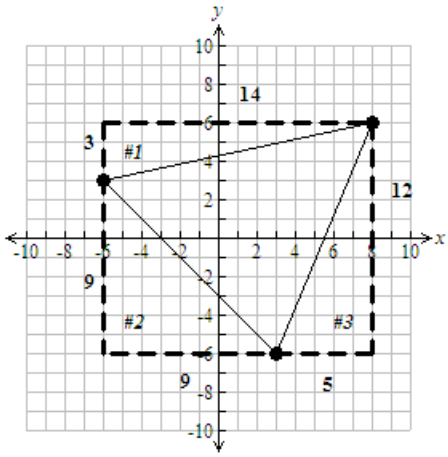
$$A = \frac{1}{2}(11)(13) = 71.5 \text{ square units}$$

2.



$$A = \frac{1}{2}(10)(8) = 40 \text{ square units}$$

3.



$$A_{\text{Triangle 1}} = \frac{1}{2}(14)(3) = 21 \text{ square units}$$

$$A_{\text{Triangle 2}} = \frac{1}{2}(5)(12) = 30 \text{ square units}$$

$$A_{\text{Triangle 3}} = \frac{1}{2}(9)(9) = 40.5 \text{ square units}$$

$$A_{\text{Rectangle}} = (12)(14) = 168 \text{ square units}$$

$$A = 168\text{-}21\text{-}30\text{-}40.5$$

$$A = 76.5 \text{ square units}$$

INSIDE – OUTSIDE CIRCLE
Directions

1. Have the students stand as a class in one large circle.

2. Have the students form pairs with one student moving inside the circle facing his/her partner on the outside forming two concentric circles.

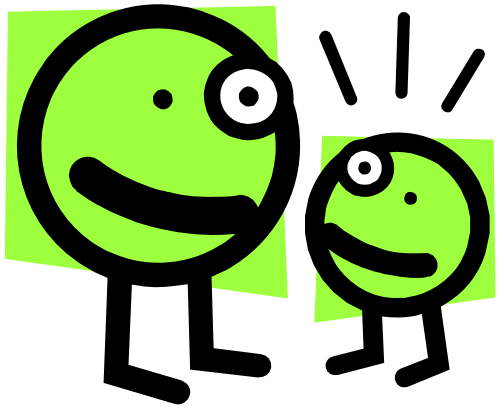
3. Provide each student with a question card (flashcard) with the answer on the back.

4. Explain that the inside partner asks the question from the flashcard and the outside partner answers with the inside partner praising or coaching as necessary.

5. Direct students to exchange roles after completing the question so that the outside partner asks the question and praises or coaches, as necessary.

6. Explain that the students are to rotate to a new partner according to the teacher's directions, such as "Inside circle move two people to the right."

7. Continue the process as necessary.



INSIDE – OUTSIDE CIRCLE
Cards

Question	Answer
$\begin{vmatrix} 7 & 4 \\ 3 & 2 \end{vmatrix}$	2
$\begin{vmatrix} 6 & 4 \\ -1 & 0 \end{vmatrix}$	4
$\begin{vmatrix} -3 & -6 \\ 4 & 8 \end{vmatrix}$	0
$\begin{vmatrix} 0 & 8 \\ 5 & 9 \end{vmatrix}$	-40
$\begin{vmatrix} 10 & 6 \\ 5 & 5 \end{vmatrix}$	20
$\begin{vmatrix} -8 & 3 \\ -2 & 1 \end{vmatrix}$	-2
$\begin{vmatrix} 4 & 7 \\ 5 & 9 \end{vmatrix}$	1

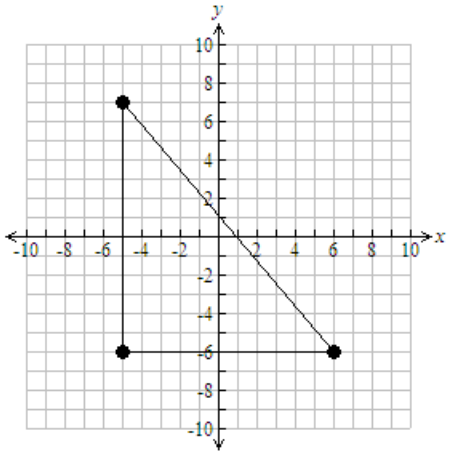
$\begin{vmatrix} -2 & 4 \\ 3 & -6 \end{vmatrix}$	0
$\begin{vmatrix} -6 & -3 \\ -4 & -1 \end{vmatrix}$	-6
$\begin{vmatrix} 1 & 6 \\ 2 & 7 \end{vmatrix}$	-5
$\begin{vmatrix} 4 & 1 \\ -2 & -5 \end{vmatrix}$	-18
$\begin{vmatrix} -3 & 7 \\ 2 & -6 \end{vmatrix}$	4
$\begin{vmatrix} 2 & -5 \\ 5 & -11 \end{vmatrix}$	3
$\begin{vmatrix} 1 & -5 \\ 3 & 4 \end{vmatrix}$	19

$\begin{vmatrix} 2 & -3 \\ -3 & -2 \end{vmatrix}$	-13
$\begin{vmatrix} 3 & -2 \\ 5 & 4 \end{vmatrix}$	22
$\begin{vmatrix} 3 & 1 \\ 5 & -2 \end{vmatrix}$	-11
$\begin{vmatrix} -3 & 1 \\ 4 & 1 \end{vmatrix}$	-7
$\begin{vmatrix} 9 & 6 \\ 3 & 2 \end{vmatrix}$	0
$\begin{vmatrix} 2 & -5 \\ 6 & 1 \end{vmatrix}$	32
$\begin{vmatrix} 8 & 5 \\ -3 & -2 \end{vmatrix}$	-1

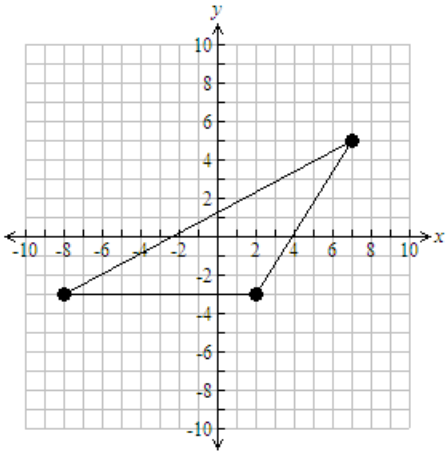
Area of a Triangle Determinant Style Name: _____

Determine the area of each of the triangles using determinants.

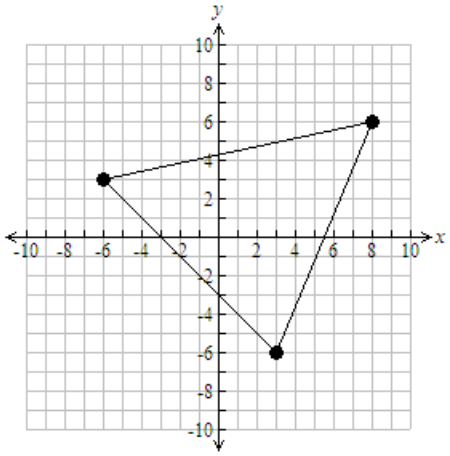
1.



2.



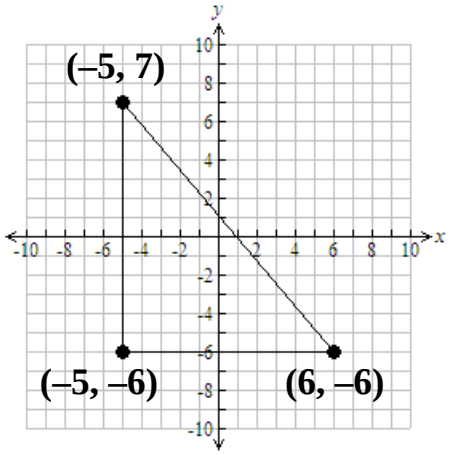
3.



Area of a Triangle Determinant Style Name: __ANSWER KEY___

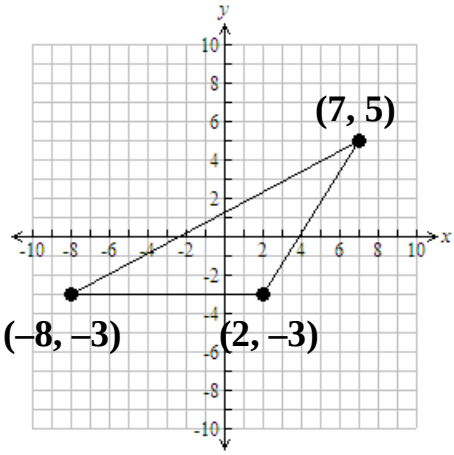
Determine the area of each of the triangles using determinants.

1.



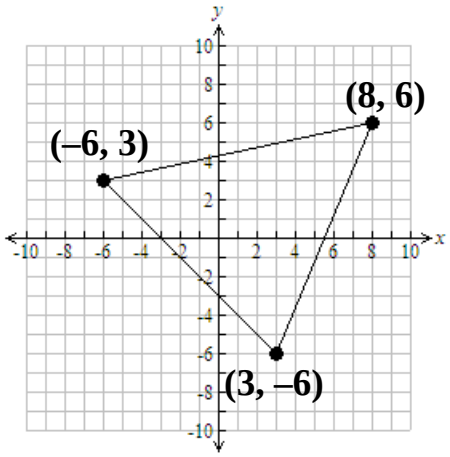
$$A = \frac{1}{2}\begin{vmatrix} -5 & 7 & 1 \\ 6 & -6 & 1 \\ -5 & -6 & 1 \end{vmatrix} = 71.5 \text{ square units}$$

2.



$$A = \frac{1}{2}\begin{vmatrix} 7 & 5 & 1 \\ 2 & -3 & 1 \\ -8 & -3 & 1 \end{vmatrix} = 40 \text{ square units}$$

3.

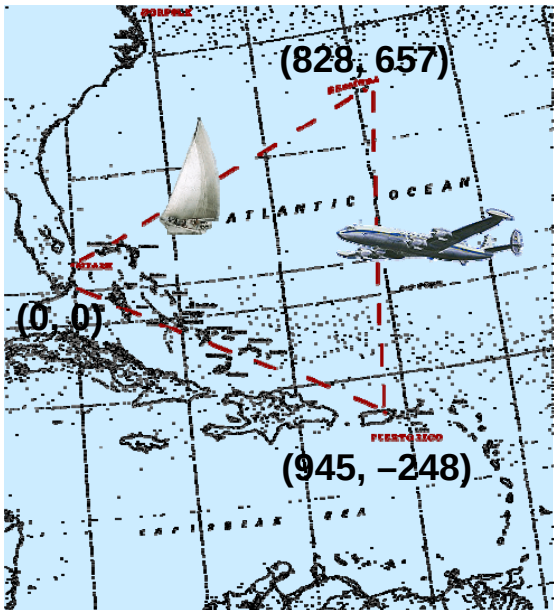


$$A = \frac{1}{2}\begin{vmatrix} -6 & 3 & 1 \\ 8 & 6 & 1 \\ 3 & -6 & 1 \end{vmatrix} = 76.5 \text{ square units}$$

Treacherous Triangles Name: _____

Throughout history ships and planes have been disappearing without a trace in two triangular regions of the world. One of these treacherous triangular regions is The Devil's Triangle, most commonly known as The Bermuda Triangle.

The boundary lines of The Bermuda triangle connect Miami, Florida, the island of Bermuda, and San Juan, Puerto Rico. The coordinates of The Bermuda Triangle are: 25.48°N, 80.18°W (*Miami*), 32.20°N, 64.45°W (*Bermuda*), and 18.50°N, 66.90°W (San Juan). The lesser known treacherous triangular region is The Dragon's Triangle. The two triangles directly align through the center of the earth and share the same latitude and longitude coordinates. The boundary lines of The Dragon's Triangle connect Tokyo, Japan, the island of Guam, and the island of Taiwan.



http://www.bermuda–triangle.org/

- Use the coordinates given in the figure to determine the approximate area (*in square miles*) of The Bermuda Triangle.

- Use mathematics to explain how you determine your answer. Use words, symbols, or both in your explanation.

Treacherous Triangles Name: ___ANSWER KEY__

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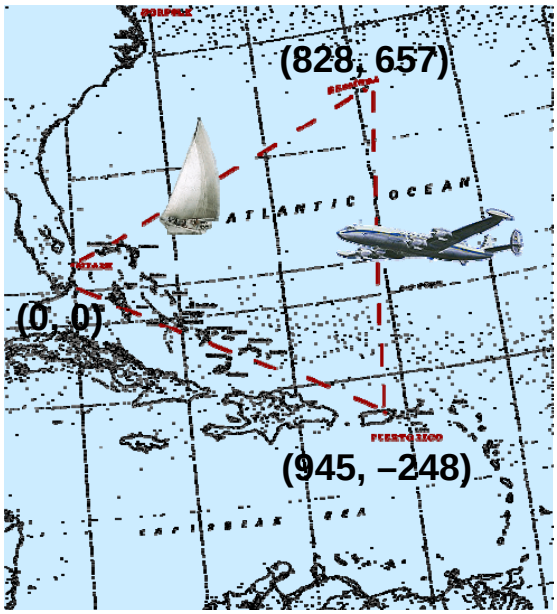


http://www.bermuda–triangle.org/

- Use the coordinates given in the figure to determine the approximate area (*in square miles*) of The Bermuda Triangle.

$$A = 413104.5 \text{ square miles}$$

- Use mathematics to explain how you determine your answer. Use words, symbols, or both in your explanation.

$$A = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 828 & 657 & 1 \\ 945 & -248 & 1 \end{vmatrix}$$

Mystical Matrices Name: _____

I. Solve each equation. Show the arithmetic in each step.

1. $x+5=17$

2. $x-11=24$

3. $7x=68$

4. $\frac{x}{5}=2$

II. Multiply the following matrices.

$$A=\begin{bmatrix}0 & 1\\ 1 & 1\end{bmatrix} \quad B=\begin{bmatrix}-1 & 1\\ 1 & 0\end{bmatrix} \quad C=\begin{bmatrix}-2 & -1/2\\ -5 & -1\end{bmatrix} \quad D=\begin{bmatrix}2 & -1\\ -10 & 4\end{bmatrix}$$

1. $A\times B$

2. $B\times A$

3. $C\times D$

4. $D\times C$

III. Examine all of the exercises carefully and identify what all of them have in common.

Summary of the Exploration

1. Zero is the _____

2. Addition and Subtraction are _____

3. One is the _____

4. Multiplication and Division are _____

5. $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ is the _____

6. Matrix *A* and *B* (and Matrix *C* and *D*) are _____

Mystical Matrices Name: __ANSWER KEY___

I. Solve each equation. Show the arithmetic in each step.

1. $x+5-5=17-5$
   $x=12$

2. $x-11+11=24+11$
   $x=35$

3. $\frac{7x}{7}=\frac{68}{7}$
   $x=\frac{68}{7}$

4. $5\cdot\frac{x}{5}=2\cdot 5$
   $x=10$

II. Multiply the following matrices.

$$A=\begin{bmatrix}0 & 1\\ 1 & 1\end{bmatrix}\quad B=\begin{bmatrix}-1 & 1\\ 1 & 0\end{bmatrix}\quad C=\begin{bmatrix}-1 & -1/2\\ -5 & -1\end{bmatrix}\quad D=\begin{bmatrix}2 & -1\\ -10 & 4\end{bmatrix}$$

1. $A\times B$
   $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

2. $B\times A$
   $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

3. $C\times D$
   $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

4. $D\times C$
   $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

III. Examine all of the exercises carefully and identify what all of them have in common.

Summary of the Exploration

1. Zero is the ___additive inverse_____

2. Addition and Subtraction are ___inverse operations_____

3. One is the __multiplicative inverse_____

4. Multiplication and Division are __inverse operations_____

5. $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ is the _2 x 2 matrix identity_____

6. Matrix *A* and *B* (and Matrix *C* and *D*) are _inverse matrices_____

Matrix Inverses Name: _____

Class Notes

Matrix Facts:

- Two matrices (A and B) are inverses of each other if the product of A x $B = B$ x $A = I$ where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ the identity matrix.

- Not every matrix has an inverse.

Consider the following…

Each of the following matrices <u>does not</u> have an inverse. Can you find what they have in common that will help us determine if a matrix has an inverse or not?

$$\begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix} \quad \begin{bmatrix} -5 & -10 \\ 2 & 4 \end{bmatrix} \quad \begin{bmatrix} 6 & 6 \\ 6 & 6 \end{bmatrix} \quad \begin{bmatrix} 3 & -3 \\ 2 & -2 \end{bmatrix} \quad \begin{bmatrix} -1 & 5 \\ 2 & -10 \end{bmatrix}$$

- Thoughts:

_____

_____

_____

- Rule:

_____

_____

_____

Consider the following…

Each of the following pairs of matrices are inverses of each other (trust me). Examine them carefully and identify patterns to help us define the rule for finding the inverse of a matrix.

$$\begin{bmatrix} 3 & 1 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 1/7 & -1/7 \\ 4/7 & 3/7 \end{bmatrix} \qquad \begin{bmatrix} 2 & -5 \\ -4 & 15 \end{bmatrix} \begin{bmatrix} 1.5 & .5 \\ .4 & .2 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 8 \\ 2 & 7 \end{bmatrix} \begin{bmatrix} 7/5 & -8/5 \\ -2/5 & 3/5 \end{bmatrix} \qquad \begin{bmatrix} -8 & 5 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} -2 & -5 \\ -3 & -8 \end{bmatrix}$$

- Thoughts:

_____

_____

_____

- Rule:

_____

_____

_____

Matrix Inverses Name: _ANSWER KEY____

Class Notes

Matrix Facts:

- Two matrices (A and B) are inverses of each other if the product of A x $B = B$ x $A = I$ where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ the identity matrix.

- Not every matrix has an inverse.

Consider the following…

Each of the following matrices <u>does not</u> have an inverse. Can you find what they have in common that will help us determine if a matrix has an inverse or not?

$$\begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix} \quad \begin{bmatrix} -5 & -10 \\ 2 & 4 \end{bmatrix} \quad \begin{bmatrix} 6 & 6 \\ 6 & 6 \end{bmatrix} \quad \begin{bmatrix} 3 & -3 \\ 2 & -2 \end{bmatrix} \quad \begin{bmatrix} -1 & 5 \\ 2 & -10 \end{bmatrix}$$

- Thoughts:

  Answers will vary. Teacher should acknowledge, record and guide student conjecture.

- Rule:

  If the determinant of the matrix $(ad - bc)$ equals zero then the matrix has no inverse

Consider the following…

Each of the following pairs of matrices are inverses of each other (trust me). Examine them carefully and identify patterns to help us define the rule for finding the inverse of a matrix.

$$\begin{bmatrix} 3 & 1 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 1/7 & -1/7 \\ 4/7 & 3/7 \end{bmatrix} \qquad \begin{bmatrix} 2 & -5 \\ -4 & 15 \end{bmatrix} \begin{bmatrix} 1.5 & .5 \\ .4 & .2 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 8 \\ 2 & 7 \end{bmatrix} \begin{bmatrix} 7/5 & -8/5 \\ -2/5 & 3/5 \end{bmatrix} \qquad \begin{bmatrix} -8 & 5 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} -2 & -5 \\ -3 & -8 \end{bmatrix}$$

- Thoughts:

  Answers will vary. Teacher should acknowledge, record and guide student conjecture.

- Rule:

  $$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -c \\ -b & a \end{bmatrix}$$

Finding the Inverse of a Matrix Name: _____

Find the Inverse of each Matrix.

1. $\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$

2. $\begin{bmatrix} 3 & 2 \\ 0 & -4 \end{bmatrix}$

3. $\begin{bmatrix} 3 & -6 \\ 2 & -4 \end{bmatrix}$

4. $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

Finding the Inverse of a Matrix Name: _____

Find the Inverse of each Matrix.

1. $\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$

2. $\begin{bmatrix} 3 & 2 \\ 0 & -4 \end{bmatrix}$

3. $\begin{bmatrix} 3 & -6 \\ 2 & -4 \end{bmatrix}$

4. $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

Finding the Inverse of a Matrix Name: __ANSWER KEY___

Find the Inverse of each Matrix.

1. $\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$

$\begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix}$

2. $\begin{bmatrix} 3 & 2 \\ 0 & -4 \end{bmatrix}$

$\begin{bmatrix} 1/3 & 1/6 \\ 0 & -1/4 \end{bmatrix}$

3. $\begin{bmatrix} 3 & -6 \\ 2 & -4 \end{bmatrix}$

no inverse exits

4. $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

Finding the Inverse of a Matrix Name: _____

Find the Inverse of each Matrix.

1. $\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$

2. $\begin{bmatrix} 3 & 2 \\ 0 & -4 \end{bmatrix}$

3. $\begin{bmatrix} 3 & -6 \\ 2 & -4 \end{bmatrix}$

4. $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

Finding the Inverse of Matrix

Names

_____

_____

_____

_____

Directions: Find the inverse of Matrix A

$$A = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$$

Person 1: Find the determinant of Matrix A

Person 1 check and initial:_____

Person 2: Find the inverse of Matrix A.

Matrix $B = A^{-1}$

Person 2 check and initial:_____

Person 3: Check/Verify that A x $B = I$

Person 3 check and initial:_____

Person 4: Check/Verify that B x $A = I$

Person 4 check and initial:_____

Finding the Inverse of Matrix

Names

_____

_____

_____

_____

Directions: Find the inverse of Matrix A

$$A = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$$

Person 1: Find the determinant of Matrix *A*

$$(-1 \cdot 4) - (3 \cdot 2) = -10$$

Person 1 check and initial:_____

Person 2: Find the inverse of Matrix *A*.

Matrix $B = A^{-1}$

$$\begin{bmatrix} -2/5 & 1/5 \\ 3/10 & 1/10 \end{bmatrix}$$

Person 2 check and initial:_____

Person 3: Check/Verify that *A* x *B* = *I*

Person 3 check and initial:_____

Person 4: Check/Verify that *B* x *A* = *I*

Person 4 check and initial:_____

Finding the Inverse of Matrix 2

Names

_____

_____

_____

_____

Directions: Find the inverse of Matrix A

$$A = \begin{bmatrix} -1 & -2 \\ 9 & -6 \end{bmatrix}$$

Person 1: Find the determinant of Matrix A Person 1 check and initial:_____
Person 2: Find the inverse of Matrix A. Matrix $B = A^{-1}$ Person 2 check and initial:_____
Person 3: Check/Verify that A x $B = I$ Person 3 check and initial:_____
Person 4: Check/Verify that B x $A = I$ Person 4 check and initial:_____

Finding the Inverse of Matrix 2

Names

_____

_____

_____

_____

Directions: Find the inverse of Matrix A

$$A = \begin{bmatrix} -1 & -2 \\ 9 & -6 \end{bmatrix}$$

Person 1: Find the determinant of Matrix A

$(-1 \cdot -6) - (9 \cdot -2) = 24$

Person 1 check and initial:_____

Person 2: Find the inverse of Matrix A.

Matrix $B = A^{-1}$

$$\begin{bmatrix} -1/4 & 1/12 \\ -9/24 & -1/24 \end{bmatrix}$$

Person 2 check and initial:_____

Person 3: Check/Verify that A x $B = I$

Person 3 check and initial:_____

Person 4: Check/Verify that B x $A = I$

Person 4 check and initial:_____

Finding the Inverse of Matrix 3

Names

_____

_____

_____

_____

Directions: Find the inverse of Matrix A

$$A = \begin{bmatrix} 2 & -5 \\ 3 & 0 \end{bmatrix}$$

Person 1: Find the determinant of Matrix A

Person 1 check and initial:_____

Person 2: Write the inverse of matrix A.

Matrix $B = A^{-1}$

Person 2 check and initial:_____

Person 3: Check/Verify that A x $B = I$

Person 3 check and initial:_____

Person 4: Check/Verify that B x $A = I$

Person 4 check and initial:_____

Finding the Inverse of Matrix 3

Names

_____

_____

_____

_____

Directions: Find the inverse of Matrix A

$$A = \begin{bmatrix} 2 & -5 \\ 3 & 0 \end{bmatrix}$$

Person 1: Find the determinant of Matrix A

$$(2 \cdot 0) - (3 \cdot -5) = 15$$

Person 1 check and initial:_____

Person 2: Write the inverse of matrix A.

Matrix $B = A^{-1}$

$$\begin{bmatrix} 0 & 1/3 \\ -1/5 & 2/15 \end{bmatrix}$$

Person 2 check and initial:_____

Person 3: Check/Verify that A x $B = I$

Person 3 check and initial:_____

Person 4: Check/Verify that B x $A = I$

Person 4 check and initial:_____

Finding the Inverse of Matrix 4

Names

_____

_____

_____

_____

Directions: Find the inverse of Matrix A

$$A = \begin{bmatrix} 1 & 2 \\ 9 & 19 \end{bmatrix}$$

Person 1: Find the determinant of Matrix A Person 1 check and initial:_____
Person 2: Write the inverse of matrix A. Matrix $B = A^{-1}$ Person 2 check and initial:_____
Person 3: Check/Verify that A x $B = I$ Person 3 check and initial:_____
Person 4: Check/Verify that B x $A = I$ Person 4 check and initial:_____

Finding the Inverse of Matrix 4

Names

_____

_____

_____

_____

Directions: Find the inverse of Matrix A

$$A = \begin{bmatrix} 1 & 2 \\ 9 & 19 \end{bmatrix}$$

Person 1: Find the determinant of Matrix A

$(1 \cdot 19) - (9 \cdot 2) = 1$

Person 1 check and initial:_____

Person 2: Write the inverse of matrix A.

Matrix $B = A^{-1}$

$$\begin{bmatrix} 19 & -2 \\ -9 & 1 \end{bmatrix}$$

Person 2 check and initial:_____

Person 3: Check/Verify that A x $B = I$

Person 3 check and initial:_____

Person 4: Check/Verify that B x $A = I$

Person 4 check and initial:_____

Decode the Encode Name: _____

Try to decode the following phrases:

3up3dn	RUOK	KIT
LBRT4AL	TRQLZR	10SNE1

These are all ways to say things in a different way!!! All of the above have been encoded in some way. You use this on a daily basis when text messaging. Governments have been using codes for hundreds of years to keep the wrong people from getting certain information.

One simple cipher or code is the Caesar Cipher which translates the alphabet a certain amount of spaces. This allows the writer of the information to use different letters to write their information (encoding) and as long as the recipient knows how many spaces the alphabet was shifted (the key), they can decode the information.

See the example below on how to encode and decode the sentence (shift 5):

Mathematics is fascinating

A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**	**L**	**M**	**N**	**O**	**P**	**Q**	**R**	**S**	**T**	**U**	**V**	**W**	**X**	**Y**	**Z**
V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U

"Mathematics is fascinating" encoded would be: hvoczhvodxn dn avnxdivodib. To decode this phrase back to the original one would need to work backwards.

You try. Encode a phrase or sentence:

_____

See if someone else can use the key above to decode what you have written!!

Can you come up with a shift that someone can't figure out? If you can, you might want to think about working for the NSA or CIA. Shifts are codes easily broken. so over the years more sophisticated encoding devices have been created, thus requiring more and more mathematicians to encode and decode for government and non–government agencies. Thomas Jefferson used codes to send messages with Lewis and Clark (wrapped around cigars). There have been codes involving words from the Navajo Indian Language and a German machine called the Enigma.

Today we will look at encoding information using matrices and inverse matrices. First, assign numbers to your alphabet – we'll use the normal number assignments.

_	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**	**L**	**M**	**N**	**O**	**P**	**Q**	**R**	**S**	**T**	**U**	**V**	**W**	**X**	**Y**	**Z**
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

Your directions to encode.

1. Use "**Mathematics is fascinating**", and assign a number to each letter and space.

m	**a**	**t**	**h**	**e**	**m**	**a**	**t**	**i**	**c**	**s**	**_**	**i**	**s**	**_**	**f**	**a**	**s**	**c**	**i**	**n**	**a**	**t**	**i**	**n**	**g**
13	1	20	8	5	13	1	20	9	3	19	0	9	19	0	6	1	19	3	9	14	1	20	9	14	7

2. Write the message into a __ x 2 matrix form being sure to add in a space at the end or beginning to have an even amount of letters.

$$\begin{bmatrix} 13 & 19 \\ 1 & 0 \\ 20 & 6 \\ 8 & 1 \\ 5 & 19 \\ 13 & 3 \\ 1 & 9 \\ 20 & 14 \\ 9 & 1 \\ 3 & 20 \\ 19 & 9 \\ 0 & 14 \\ 9 & 7 \end{bmatrix}$$

3. Select a matrix (one that can have an inverse and has the right dimensions) to multiply by this matrix.

4. Now multiply the two matrices together to get your encoded matrix to send to a friend.

$$\begin{bmatrix} 13 & 19 \\ 1 & 0 \\ 20 & 6 \\ 8 & 1 \\ 5 & 19 \\ 13 & 3 \\ 1 & 9 \\ 20 & 14 \\ 9 & 1 \\ 3 & 20 \\ 19 & 9 \\ 0 & 14 \\ 9 & 7 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 70 & 102 \\ 1 & 2 \\ 38 & 64 \\ 11 & 20 \\ 62 & 86 \\ 31 & 38 \\ 28 & 38 \\ 62 & 96 \\ 12 & 22 \\ 63 & 86 \\ 46 & 74 \\ 42 & 56 \\ 30 & 46 \end{bmatrix}$$

Assuming you have sent the key in a separate note to a friend, they can decode your matrix by multiplying the decoded matrix by the inverse of your key matrix.

Friend's directions to decode.

1. Find inverse of key matrix: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^{-1} = \frac{1}{-2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$

2. Now use your inverse to decode the matrix:

$$\begin{bmatrix} 70 & 102 \\ 1 & 2 \\ 38 & 64 \\ 11 & 20 \\ 62 & 86 \\ 31 & 38 \\ 28 & 38 \\ 62 & 96 \\ 12 & 22 \\ 63 & 86 \\ 46 & 74 \\ 42 & 56 \\ 30 & 46 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} 13 & 19 \\ 1 & 0 \\ 20 & 6 \\ 8 & 1 \\ 5 & 19 \\ 13 & 3 \\ 1 & 9 \\ 20 & 14 \\ 9 & 1 \\ 3 & 20 \\ 19 & 9 \\ 0 & 14 \\ 9 & 7 \end{bmatrix}$$

Multiply the decoded matrix by the inverse. Take the numbers from the answer and convert to letters using letter–number assignments.

3. Take the decoded matrix and convert back to letters and your friend should have the original message!!!!

4. Now try encoding and decoding "Pi day is great." Use your key matrix as $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Decode the Encode Matching Cards

$\begin{bmatrix} 46 & 9 \\ 112 & 14 \\ 48 & 0 \\ 6 & 1 \\ 72 & 0 \\ 36 & 18 \\ 74 & 5 \\ 78 & 3 \\ 76 & 20 \\ 78 & 1 \\ 80 & 14 \end{bmatrix}$	$\begin{bmatrix} 2 & 1 \\ 4 & 0 \end{bmatrix}$
$\begin{bmatrix} 14 & 128 \\ 15 & 70 \\ 18 & 36 \end{bmatrix}$	$\begin{bmatrix} 1 & 2 \\ 0 & 5 \end{bmatrix}$
$\begin{bmatrix} 107 & 99 \\ 55 & 85 \end{bmatrix}$	$\begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$

$\begin{bmatrix} 20 & 29 \\ 8 & 27 \\ 5 & 5 \\ 0 & 8 \\ 16 & 25 \\ 18 & 22 \\ 9 & 18 \\ 26 & 40 \\ 5 & 12 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$
$\begin{bmatrix} 14 & 22 \\ 36 & 54 \\ 10 & 29 \\ 10 & 24 \\ 0 & 14 \\ 2 & 8 \\ 28 & 62 \\ 14 & 46 \\ 24 & 36 \\ 10 & 19 \\ 0 & 5 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$

Join the Madness!!!!! Name: _____

1. Calculate the determinant:

$$\begin{bmatrix} 3 & -2 \\ 5 & 4 \end{bmatrix}$$

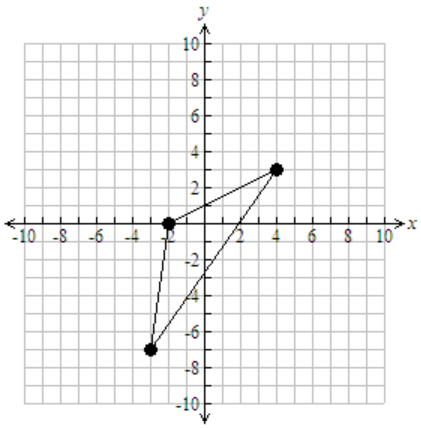
a. 0 b. 2 c. 22 d. 24

2. Find the inverse:

$$\begin{bmatrix} 5 & 3 \\ 0 & 2 \end{bmatrix}$$

a. $\begin{bmatrix} .5 & -.3 \\ 0 & .5 \end{bmatrix}$ b. $\begin{bmatrix} .5 & -.3 \\ 0 & .2 \end{bmatrix}$ c. $\begin{bmatrix} .5 & 0 \\ -.3 & .2 \end{bmatrix}$ d. $\begin{bmatrix} .2 & -.3 \\ 0 & .5 \end{bmatrix}$

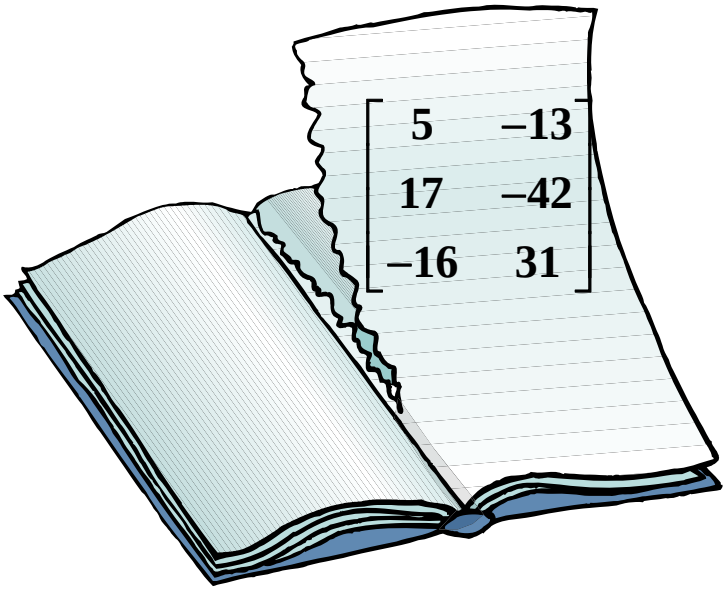
3. Find the area of the triangular region:

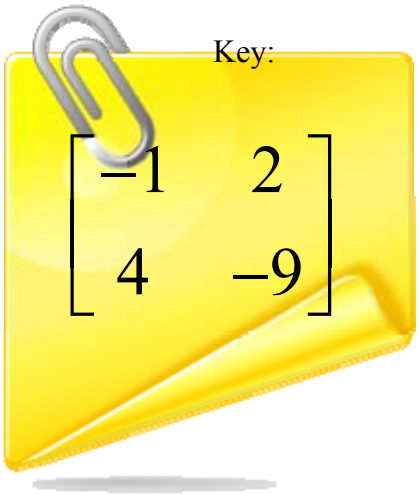


a. 19.5 b. 42 c. 39 d. 24

BCR

The FBI has intercepted an encoded message sent from the NSA to the CIA and a random piece of paper with a matrix on it. They want to be sure those agencies are not doing something on their turf. Here is what the two items looked like:





- Find the inverse of the key matrix: _____

- Encode the message using your knowledge of matrices and inverses. Explain how you arrived at your answer using words, symbols, or both.

Join the Madness!!!!! Name: __ANSWER KEY___

1. Calculate the determinant:

$$\begin{bmatrix} 3 & -2 \\ 5 & 4 \end{bmatrix}$$

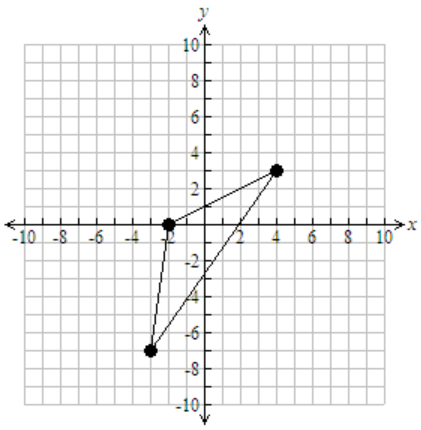
a. 0 b. 2 $\boxed{\text{c.}}$ 22 d. 24

2. Find the inverse:

$$\begin{bmatrix} 5 & 3 \\ 0 & 2 \end{bmatrix}$$

a. $\begin{bmatrix} .5 & -.3 \\ 0 & .5 \end{bmatrix}$ b. $\begin{bmatrix} .5 & -.3 \\ 0 & .2 \end{bmatrix}$ c. $\begin{bmatrix} .5 & 0 \\ -.3 & .2 \end{bmatrix}$ $\boxed{\text{d.}}$ $\begin{bmatrix} .2 & -.3 \\ 0 & .5 \end{bmatrix}$

3. Find the area of the triangular region:



$\boxed{\text{a.}}$ 19.5 b. 42 c. 39 d. 24

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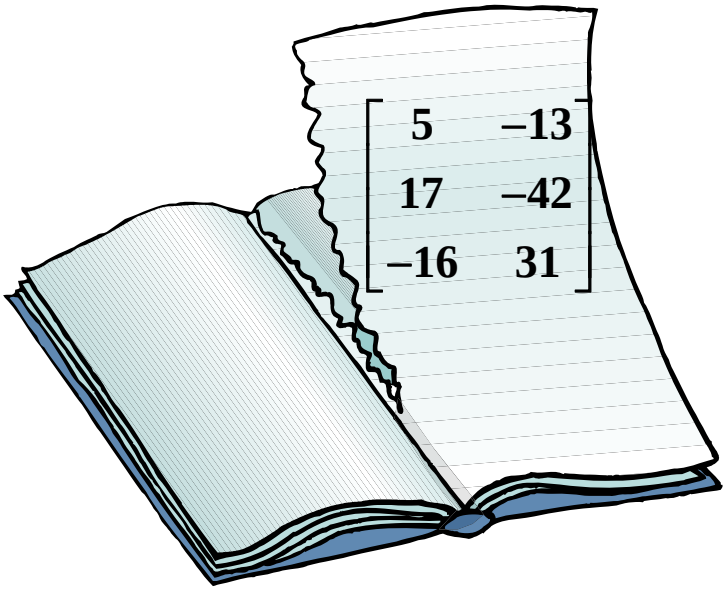

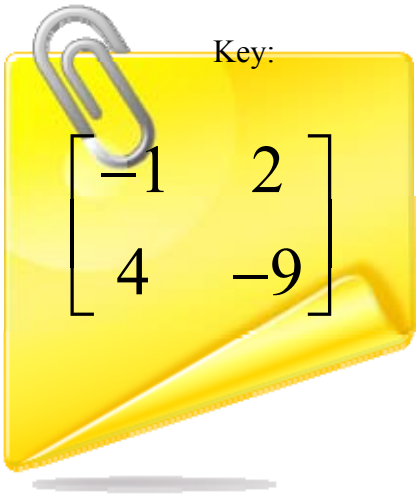

- Find the inverse of the key matrix: _____

$$\begin{bmatrix} -9 & -2 \\ -4 & -1 \end{bmatrix}$$

- Encode the message using your knowledge of matrices and inverses. Explain how you arrived at your answer using words, symbols, or both.

Decoded message: Gotcha!