Title: Systems of Equations and Inequalities

Brief Overview:

Students will use symbolic algebra to represent, solve, and graph inequalities and systems of inequalities, a skill learned toward the end of Algebra 1.

The students should have previous knowledge of solving systems of equations by graphing, algebraically using substitution, and algebraically using elimination. Students have graphed systems of equations and inequalities. This unit includes three lessons focusing on interpreting mathematical sentences, graphing inequalities, and determines the feasible region in order to find the maximum and minimum values of a system.

NCTM Content Standard/National Science Education Standard:

- identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships;
- use symbolic expressions, including iterative and recursive forms, to represent relationships arising from various contexts;
- draw reasonable conclusions about a situation being modeled.

Grade/Level:

9th – 12th grade/Algebra 2, Algebra 2 Honors/GT.

Duration/Length:

Three 90 – minute class periods

Student Outcomes:

Students will:

- Use mathematics in order to model real world situations.
- Graph inequalities in order to identify the vertices of the feasible region.
- find the maximum and minimum values of a function over a region in order to solve real–world problems using linear programming

Materials and Resources:

- Envelopes
- Sticky notes
- Poster paper
- Markers (permanent and wet-erase)
- Colored pencils
• Beads
• Zip lock bags
• Sentence strips
• Graph chart paper
• String
• Graph paper
• Self-stick easel pad
• Copies of worksheets/transparencies
  o Matching
  o Writing Mathematical Sentences
  o Review Writing Equations for Sentences
  o Fan and Pick cards
  o Working on Writing
  o Sticky Note Activity
  o Review Writing Equations for Inequalities
  o Practice Writing Equations for Inequalities
  o Linear Programming
  o Round Table
  o Systems of Equations Drill
  o Beads for Profit
  o Graphs and Linear Programming
  o A Programming Problem of Profit to Ponder Prolifically
  o Linear Programming True/False
  o Story Group Problems
  o Steppin’ Up
  o Linear Programming Drill
  o Graphing Inequalities using TI–84
  o Linear Programming with Three Inequalities Guided Notes
  o Linear Programming with Three Inequalities Practice 1
  o Linear Programming with Three Inequalities Practice 2
  o System of Equations and Inequalities Assessment

Development/Procedures:

Day 1
  o Pre–assessment
    Cut up the cards from “Matching” prior to the start of class and place them in an envelope. Make enough envelopes to give one envelope per student pair. Divide the class into pairs at the start of class and have the students match the cards from the envelopes.

    Use the matching activity to transition into a brainstorming session, asking the students, “Can you think of other words that mean the same thing?” Distribute the “Writing Mathematical Sentences” chart and have the students add additional words they come up with during the brainstorming
session. Create a large version of this chart to hang in the room for reference throughout the unit.

- **Explanation**
  Give out copies of “Review Writing Equations for Sentences”, and model solving the first exercise using a transparency of this same worksheet. Invite a student to the overhead to underline key parts of the sentences and to circle key vocabulary. Lead the class in translating the words into the mathematical equation. Elicit a volunteer to lead the second exercise. Encourage the other students to explain to the volunteer what to write underline and circle. Allow the students to independently complete the remaining exercises. Invite selected students to the board to show their answers and demonstrate their thinking to the class.

- **Exploration**
  Divide the class into groups of three or four. Cut out the cards from “Fan and Pick Cards” prior to the start of class. Give a set of the twelve cards to each of the student groups. Tell the students to arrange the cards face down on the desk. Have one student in each group randomly choose a card and read it to the other group members. Have these students write the answer to the card on a piece of paper. Ask the reader to share the answer, and the students check their work. Rotate the role of ‘reader’ to the next student in the group, and continue until all cards have been read.

- **Application**
  Assign the worksheet, “Working on Writing” for the students to complete individually and to be collected for a grade.

- **Exploration**
  Arrange a set of sticky notes on the board that have different terms for the symbols $>$, $<$, $\geq$, and $\leq$. Draw a large box divided into four sections, each labeled with each of the four inequality symbols, either on the front board or a large piece of chart paper. Invite students to come to the board and place the sticky notes the appropriate section. Have the other students completing a copy of “Sticky Note Activity” individually, as well as correcting mistakes made on the board.

- **Explanation**
  Distribute copies of “Review Writing Equations for Inequalities”, and model solving the first exercise using a transparency of this same worksheet. Invite a student to the overhead to underline key parts of the sentences and to circle key vocabulary. Lead the class in translating the words into the mathematical inequality and the steps to arrive at the solution set. Elicit a volunteer to lead the second exercise. Encourage the other students to explain to the volunteer what to write, underline and circle. Allow the students to independently complete the remaining
exercises. Invite selected students to the board to show their answers and demonstrate their thinking to the class.

- **Application**
  Assign the worksheet, “Practice Writing Equations for Inequalities” for the students to complete individually and to be collected for a grade.

- **Explanation**
  Distribute the worksheet, “Linear Programming”. Present the first exercise and emphasize underlining key parts of sentences and circling key vocabulary.

- **Exploration**
  Divide the class into groups of four for a “Round Table activity”. Give each student one problem and have the students each work on the first step. Set an appropriate amount of time, and then instruct the students pass their problem to the student to their right. Explain that this next student is to check the work and make corrections to the first student’s work, and then complete step two. Have the students pass the problem again, check the previous students’ work, and do the next step. End the “Round Table” activity when the students have their original problem back.

- **Exploration**
  Divide the class into the four corners of the room according to the problem they received during the “Round Table” activity. Place in each corner an overhead transparency of the problem and a wet-erase marker. Have the students check each other’s answers to see if they all arrived at the same solution. Have the students prepare a presentation of the individual problems, assigning rolls as follows:

  - **Recorder**– student copies the correct solution on the overhead.
  - **Presenter 1** – Reads the problem
  - **Presenter 2**– underlines key information, circles key vocabulary.
  - **Presenter 3**– defines the variables.
  - **Presenter 4**– writes the inequalities
  - **Presenter 5**– writes the function used to maximize or minimize.
  - **Presenter 6**– Answers student questions about the problem.

- **Differentiation**
  Throughout the first lessons ESOL students will be provided dictionaries where they can translate the key words identified in the charts into their own language.
Round table groups are assigned to have two stronger and two weaker students at each table. This grouping can occur by student self selection. Students are asked to line up against the classroom wall standing toward the left if they don’t understand what we have been learning, and to the right if they do, and more toward the middle if they feel they somewhat understand. Then the teacher partners the two students lined up or the far right with the two students from the far left.

- Assessment
  Assign the exit ticket, “Writing Inequalities and Profit Functions” for the students to complete on their own.

Day 2
- Pre-assessment
  Assess the students’ prior knowledge of solving systems of equations by graphing with the opening activity, “Systems of Equations Drill.”

- Exploration
  Distribute the “Beads For Profit Activity”, an exploration activity in which the students will discover the maximum of a linear programming problem. Provide a bag containing twenty fancy beads and twenty-four standard beads for each student to use as a manipulative to assist them as they work through the problem. Provide guidance as needed, but this activity is intended for students to work independently using problem solving skills.

  Summarize the bead activity by reviewing the answers to the last questions. Explain that the maximum and minimum values in the situation will always lie at one of the vertices of the feasible region.

- Explanation
  Transfer the “Graphs and Linear Programming” graph to a large piece of graph chart paper. Place the vocabulary terms and definitions on separate sentence strips. Give various students the sentence strips. Ask the students who are given the vocabulary terms to tape them on the chart paper at the appropriate locations. Invite the students with the definition sentence strips to match the definition to the vocabulary term at the graph, taping it underneath the vocabulary term. Ask the students to go back to the Bead activity and label their graph with this new vocabulary.

  Model a linear programming example problem, “A Programming Problem of Profit to Ponder Prolifically”, from start to finish with the students. Reiterate the fact that not every situation requires finding the maximum. Emphasize the fact that when substituting the vertices into the constraints, the maximum/minimum function can come to the same conclusion.
Distribute copies of “Linear Programming True/False” and seven sticky notes to each student. Have the students decide if each statement is true or false, and use the sticky notes to cover the incorrect answer. Use this visual activity to provide a quick check for student understanding, and clarify any misconceptions as a class.

- **Exploration**
  Divide the class into six groups, or more. Give each group one of the exercises from “Story Group Problems”. Provide each group with a piece of large chart paper, a piece of blank “Graph Paper” with no axis, and markers. Have the students complete and label each of steps of linear programming, including circling and underlining of the problem itself. Have the students also label graph with vocabulary. Finalize the group activity by having the students display their work around the room and perform a gallery walk.

- **Differentiation**
  For the group presentations problems of varying degree of difficulty are offered. Students are assigned an appropriately leveled assignment.

- **Assessment**
  Transfer the steps written on the worksheet, “Steppin’ Up” to sentence strips. Distribute these sentence strips to various students, while providing a copy of the worksheet to the remaining students. Instruct the students to number the steps in the order in which the students would take to solve a linear programming problem. Invite the students with the sentence strips to the board to arrange them in the correct order, allowing the remaining students to correct the order if necessary.

**Day 3**

- **Drill (15 minute) time passed 15 minutes.**
  The teacher will hand out the drill (Linear Programming Drill work sheet) as the students enter the classroom. The students will solve the system of equations by substitution or elimination. After solving the system, the students are to graph the system of equations on the grid at the bottom of the page. Students will discuss their findings with a partner. The group of two will present their findings on post-it easel paper. One student will write the solution on the easel paper and the other will present their findings.

- **Exploration (15 minutes) time passed 30 minutes.**
  Students will stay in their groups of two. The drill has been modified by turning the equalities into inequalities (Linear Programming of Two Inequalities work sheet). The group will graph the inequalities and interpret what the intersection of the system represents. The teacher will
select a group to present their findings on the overhead transparency of the Linear Programming of Two Inequalities.

The teacher will used the Linear Programming of Two Inequalities present the vocabulary needed for Linear Programming. The teacher will use the previous student work from the exploration activity for the overhead transparency to list the vocabulary terms (constraint, bounded, feasible region, vertices). Students will the label their work sheets accordingly.

The teacher will hand out the Graphing Inequalities work sheet. Allow the students five minutes to graph the system inequalities and find the vertices. Select three or four students to write their responses on the overhead transparency. Teacher will ask following the questions:

- What were the problems you have graphing the inequalities?
- What were the problems you had finding the vertices of the feasible region?

Have the students list these problems on the overhead transparency.

○ Explanation

Hand out the “Graphing Inequalities using the TI–84” work sheet. Select a volunteer to operate the overhead calculator screen. Walk through the instructions on solving a system of inequalities using the TI–84 as the students follow along.

Model the concept of solving a linear programming problem using the calculator with the worksheet “Linear Programming with Three Inequalities Guided Notes”. Use the guided notes work sheet to record the steps for solving linear programming problem, and, between each step, allow the students to perform the step as it is described.

Hand out the “Linear Programming with Three Inequalities Practice 1” work sheet. Give the students five minutes to set up the problem, which involves writing five inequalities. Guide the students through solving the linear programming problem using the TI–84. Emphasize the fact that the problem can be solved without the use of the calculator, but it the calculator simplifies the process. Have the students refer back to the “Graphing Inequalities using TI–84” work sheet for reference.

○ Application

Provide individual practice with “Linear Programming with Three Inequalities Practice 2”. Divide the class into groups after a certain amount of time, and allow the students to share answers and complete the problem. Give each group a piece of self-stick easel paper to write their
answers on. Instruct the students to post their easel papers, and then the groups will do a gallery walk to view the solution.

- Differentiation:
  During calculator activities students are paired by calculator ability. A student more fluent in calculator use is paired next to someone who has more difficulty finding the correct key strokes.

**Summative Assessment:**
Assign “System of Equations and Inequalities Assessment” for the students to complete individually. Use the assessment to evaluate student understanding.

**Authors:**

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Parkville High School      Lansdowne High School
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<table>
<thead>
<tr>
<th>Matching</th>
<th>Name: ____________________</th>
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<tbody>
<tr>
<td>Opening Pre-Assessment</td>
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<td><strong>Increased by</strong></td>
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<td><strong>No more than</strong></td>
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<td><strong>The difference of</strong> x and y</td>
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<td><strong>Unknown</strong></td>
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<td><strong>x more than y</strong></td>
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<td><strong>Totals</strong></td>
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<td><strong>Quotient</strong></td>
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<td><strong>At least</strong></td>
<td>n</td>
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<td><strong>Take away</strong></td>
<td>x - y</td>
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<td><strong>x less than y</strong></td>
<td>y - x</td>
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<td><strong>Greater than</strong></td>
<td>y + x</td>
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<tr>
<td>Operation</td>
<td>Symbols</td>
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<td>Increased by</td>
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<td>The difference of ( x ) and ( y )</td>
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<td>( y + x ) &gt;</td>
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<td>Take away</td>
<td>- ( x - y )</td>
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<td>( x ) less than ( y )</td>
<td>( y - x ) ( y - x )</td>
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<tr>
<td>Greater than</td>
<td>&gt; ( y + x )</td>
</tr>
</tbody>
</table>
Writing Mathematical Sentences

Name: __________________

ADD

SUBTRACT

EQUAL

VARIABLE

MULTIPLY

DIVIDE

Linear Programming
Writing Mathematical Sentences
(Sample Responses)

**ADD**

Plus
\[ x \text{ plus } y = x + y \]
Increased by
\[ x \text{ increased by } y = x + y \]
The Sum of
\[ \text{The sum of } x \text{ and } y = x + y \]
The Total of
\[ \text{The total of } x \text{ and } y = x + y \]
More than
\[ x \text{ more than } y = y + x \]

**SUBTRACT**

Minus
\[ x \text{ minus } y = x - y \]
Decreased by
\[ x \text{ decreased by } y = x - y \]
The Difference of
\[ \text{The difference of } x \text{ and } y = x - y \]
Take Away
\[ x \text{ take away } y = x - y \]
Less Than
\[ x \text{ less than } y = y - x \]

**EQUAL**

totals
yields
Is the same as
Is equivalent to
Is identical to

**MULTIPLY**

Twice
\[ \text{Twice } x = 2x \]
Triple
\[ \text{Triple } x = 3x \]
Percent of
\[ 9\% \text{ of } x = .09x \]
Per
\[ \$7 \text{ per hour } = 7h \]
Times
\[ 12 \text{ times } x = 12x \]
The Product of
\[ \text{The product of } 6 \text{ and } x = 6x \]

**VARIABLE**

Follows per
\[ \text{A number “}n\text{”} \]
The unknown

**DIVIDE**

Half of
\[ \frac{x}{2} \]
One third of
\[ \frac{x}{3} \]
Out of
\[ x \text{ out of } 7 = \frac{x}{7} \]
Is broken into
\[ x \text{ is broken into } 4 \text{ parts } = \frac{x}{4} \]
Is separated into
\[ x \text{ is separated into } 9 \text{ parts } = \frac{x}{9} \]
The quotient of
\[ \text{The quotient of } x \text{ and } 5 = \frac{x}{5} \]

Name: __________________
Review Writing Equations for Sentences

Name: ____________________

Directions: Underline key parts of the sentence and circle key vocabulary terms.

For exercises 1 – 3, choose the letter whose equation represents the given situation. For exercises 4 – 6, write the equation that represents the given situation.

1. Russ earned $3,600 during the month of July. Russ earned $40 per hour plus $100 each month. Which of these equations represents this situation?

   a. \( 3600 = 40 + 100m \)
   b. \( 40 = 100m + 3600 \)
   c. \( 3600 = 100 + 40h \)
   d. \( 100 = 40h + 3600 \)

2. A fish tank empties at a rate of 8 in\(^3\) per minute. The tank when filled with water holds 630 in\(^3\). Which equation describes the volume \((v)\) of water in the tank as a function of time \((t)\)?

   a. \( v = -8t + 622 \)
   b. \( v = -8t + 630 \)
   c. \( v = 8t + 622 \)
   d. \( v = 8t + 630 \)

3. A parachutist is 500 feet above the ground. After she opens her parachute, she falls at a constant rate of 5 feet per second. Which of these equations gives her height \((h)\) above the ground in feet after \((t)\) seconds?

   a. \( h = 5 - 500t \)
   b. \( h = -500 + 5t \)
   c. \( h = 5 + 500t \)
   d. \( h = 500 - 5t \)

4. Cell America charges an hourly call rate of $2 plus an initial phone set up fee of $30. Write an equation to show the cost \((y)\) per hour \((x)\).

5. Martha’s Movie House charges $32 for a membership plus $3 per movie rental. Write an equation to show the total cost \((C)\) of renting movies for \((R)\) rentals.

6. A hot air balloon at a height of 150 feet descends at a rate of 5 feet per second. Write an equation to show the balloon’s height \((y)\) after so many seconds \((x)\).
Review Writing Equations for Sentences

Name: ___ANSWER KEY___

Directions: Underline key parts of the sentence and circle key vocabulary terms.

For exercises 1 – 3, choose the letter whose equation represents the given situation. For exercises 4 – 6, write the equation that represents the given situation.

1. Russ earned $3,600 during the month of July. Russ earned $40 per hour plus $100 each month. Which of these equations represents this situation?
   a.  \( 3600 = 40 + 100m \)
   b.  \( 40m + 3600 \)
   c.  \( 3600 = 100 + 40h \)
   d.  \( 100 = 40h + 3600 \)

2. A fish tank empties at a rate of 8 in³ per minute. The tank when filled with water holds 630 in³. Which equation describes the volume (v) of water in the tank as a function of time (t)?
   a.  \( v = -8t + 622 \)
   b.  \( v = -8t + 630 \)
   c.  \( v = 8t + 622 \)
   d.  \( v = 8t + 630 \)

3. A parachutist is 500 feet above the ground. After she opens her parachute, she falls at a constant rate of 5 feet per second. Which of these equations gives her height (h) above the ground in feet after (t) seconds?
   a.  \( h = 5 - 500t \)
   b.  \( h = -500 + 5t \)
   c.  \( h = 5 + 500t \)
   d.  \( h = 500 - 5t \)

4. Cell America charges an hourly call rate of $2 plus an initial phone set up fee of $30. Write an equation to show the cost (y) per hour (x).
   \( y = 2x + 30 \)

5. Martha’s Movie House charges $32 for a membership plus $3 per movie rental. Write an equation to show the total cost (C) of renting movies for (R) rentals.
   \( C = 32 + 3R \)

6. A hot air balloon at a height of 150 feet descends at a rate of 5 feet per second. Write an equation to show the balloons height (y) after so many seconds (x).
   \( y = 150 - 5x \)
**Fan and Pick Cards**

<table>
<thead>
<tr>
<th>Problem</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dominic rents a car for a trip. He pays $300 plus $0.20 per mile. Write an equation to show the cost (y) per mile (x).</td>
<td>[ y = 300 + 0.20x ]</td>
</tr>
<tr>
<td>A group of students began a camping trip with 252 pounds of food. They plan to eat 12 pounds of food a day. Write an equation to show the amount of food in pounds (F) after (D) days.</td>
<td>[ F = 252 - 12D ]</td>
</tr>
<tr>
<td>A local gym charges $4 per week for a student membership plus an initial startup fee of $80. Write an equation to determine the cost of a membership (y) if they used the gym for (x) weeks.</td>
<td>[ y = 4x + 80 ]</td>
</tr>
<tr>
<td>The depth of a lake is 26 meters. Melting snow causes the lake to rise 0.05 meters each day. Write an equation to show the depth of the lake in (M) meters for (D) days.</td>
<td>[ M = 26 + 0.05D ]</td>
</tr>
<tr>
<td>Lydia has $200 in her bank account at the beginning of the year. Each month, she deposits $40 into her account. She does not withdraw any money from her account, and the account pays no interest. Write and equation to find the total amount (T) in her bank account at the end of (m) months?</td>
<td>[ T = 200 + 40m ]</td>
</tr>
<tr>
<td>Dylan’s swimming pool contains 20,000 gallons of water. He drains the pool at a rate of 5 gallons per minute. Which of these equations represents the number of gallons of water (g) remaining in the pool after (m) minutes.</td>
<td>[ g = 20000 - 5g ]</td>
</tr>
<tr>
<td>Equation</td>
<td>Answer</td>
</tr>
<tr>
<td>----------------------------------------------</td>
<td>---------------------------------------------</td>
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<tr>
<td>Two hundred minus three times a number is equal to nine.</td>
<td>(200 - 3n = 9)</td>
</tr>
<tr>
<td>Half the sum of nine and (p) is the same as (p) minus three.</td>
<td>(\frac{1}{2}(9 + p) = p - 3)</td>
</tr>
<tr>
<td>To put on the game it will cost $90 for the lights, plus $2 per ticketed person for cleanup and other costs. Write and equation to represent the cost ((c)) for each ticketed person ((p)).</td>
<td>(c = 90 + 2p)</td>
</tr>
</tbody>
</table>
1. A library is trying to decrease the number of overdue books by increasing the fines. They plan to charge $2.25 for the first day a book is late and $0.10 for each additional day. Write an equation to show the amount of fines \( F \) for each day \( d \) a book is late.

   a. \( F = -2.25d + 0.10 \)
   b. \( F = -0.10d + 2.25 \)
   c. \( F = 2.25d + 0.10 \)
   d. \( F = 0.10d + 2.25 \)

**ECR** – Justify which equation you chose and explain why the others are incorrect.

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2. The number of people \( n \) who will attend a dance depends on the admission price \( p \), in dollars. This relationship is represented by the equation shown below.

\[ n = 800 - 50p \]

Which of these is a correct interpretation of this equation?

   a. The number of people attending the dance will increased by 50 for every dollar the admission price increases
   b. The number of people attending the dance will decrease by 50 for every dollar the admission price increases.
   c. The minimum number of people attending the dance will be 800.
   d. The maximum number of tickets that can be sold is 50.

**ECR** – Justify which equation you chose and explain why the others are incorrect.

________________________________________________________________________

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a. \( F = 2.25d + 0.10 \)
b. \( F = 0.10d + 2.25 \)
c. \( F = 2.25d + 0.10 \)
d. \( F = 0.10d + 2.25 \)

**ECR** – Justify which equation you chose and explain why the others are incorrect. Answers will vary.

The librarian charges $2.25 fee, which will be a constant, positive charge. Thus (a) and (c) are incorrect. The additional charge of $0.10 for each day late is the rate of change and is positive. (b) cannot be the answer. The answer is (d).

2. The number of people \( n \) who will attend a dance depends on the admission price \( p \), in dollars. This relationship is represented by the equation shown below.

\[
 n = 800 - 50p
\]

Which of these is a correct interpretation of this equation?

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c. The minimum number of people attending the dance will be 800.

d. The maximum number of tickets that can be sold is 50.

**ECR** – Justify which equation you chose and explain why the others are incorrect. Answers will vary.

800 is the \( y \)-intercept and -50 is the slope (rate of change). The rate of change is negative, showing a decrease. So, (a) is incorrect. The maximum number of people attending is 800, so (c) and (d) are wrong.
<table>
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<th>&gt;</th>
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<tbody>
<tr>
<td>Greater than</td>
<td>Less than</td>
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<td>More than</td>
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<td>Larger than</td>
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<tr>
<td>Greater than or equal to</td>
<td>Less than or equal to</td>
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<tr>
<td>At least/ the least</td>
<td>At most/ the most</td>
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<tr>
<td>No less than</td>
<td>No more than</td>
</tr>
<tr>
<td>A maximum of/ The maximum is</td>
<td>A minimum of/ The minimum is</td>
</tr>
<tr>
<td>Does not exceed</td>
<td></td>
</tr>
</tbody>
</table>

Linear Programming
1. Tickets for the school play cost $5 per student and $7 per adult. The school wants to earn at least $5400 on each performance.
   a. Write an inequality that represents this situation.
   
      b. If 500 adult tickets are sold, what is the minimum number of student tickets that must be sold?

2. An auto parts company can produce 525 four–cylinder engines or 270 V–6 engines per day. It wants to produce up to 300,000 engines per year.
   a. Write an inequality that represents this situation.
   
      b. Are there restrictions on the domain or range?

3. A fast food restaurant charges $3 for a hamburger and $1.50 for a soda The Math Club has no more than $60 to spend on hamburgers and sodas for their next meeting.
   a. Write an inequality that represents this situation.
   
      b. Club members want to know if they can purchase 12 hamburgers and 12 sodas.

4. A moving van has an interior height of 7 feet (84 inches). You have boxes in 12 inch and 15 inch heights, and want to stack them as high as possible to fit. Write an inequality that represents this situation.

5. The grocery store has grapes that sell for $2.25 a pound and oranges that sell for $1.90 a pound. Write a linear inequality that represents how much of each type of fruit can be bought with no more than $20.

6. A wholesaler has $80,000 to spend on certain models of mattress sets and bed frames. If the mattress sets may be obtained at $200 each and the bed frames at $100 each, write an inequality that restricts the purchase on x mattress sets and y bed frames.
Review Writing Equations for Inequalities

Name: __ANSWER KEY___

1. Tickets for the school play cost $5 per student and $7 per adult. The school wants to earn at least $5400 on each performance.
   a. Write an inequality that represents this situation.
      \[5s + 7a \geq 5400\]
   b. If 500 adult tickets are sold, what is the minimum number of student tickets that must be sold?
      \[5s + 7(500) \geq 5400\]
      \[s \geq 380\]
      The minimum number of student tickets is 380.

2. An auto parts company can produce 525 four-cylinder engines or 270 V-6 engines per day. It wants to produce up to 300,000 engines per year.
   a. Write an inequality that represents this situation.
      \[525c + 270v \leq 300000\]
   b. Are there restrictions on the domain or range?
      \[c \geq 0\] and \[v \geq 0\]. There cannot be a negative number of engines produces.

3. A fast food restaurant charges $3 for a hamburger and $1.50 for a soda. The Math Club has no more than $60 to spend on hamburgers and sodas for their next meeting.
   a. Write an inequality that represents this situation.
      \[3h + 1.5s \leq 60\]
   b. Club members want to know if they can purchase 12 hamburgers and 12 sodas.
      \[3(12) + 1.5(12) \leq 60\]
      12 hamburgers and 12 sodas can be purchased
      \[54 \leq 60\]

4. A moving van has an interior height of 7 feet (84 inches). You have boxes in 12 inch and 15 inch heights, and want to stack them as high as possible to fit. Write an inequality that represents this situation.
   \[12s + 15l \leq 84\]

5. The grocery store has grapes that sell for $2.25 a pound and oranges that sell for $1.90 a pound. Write a linear inequality that represents how much of each type of fruit can be bought with no more than $20.
   \[2.25g + 1.90n \leq 20\]

6. A wholesaler has $80,000 to spend on certain models of mattress sets and bed frames. If the mattress sets may be obtained at $200 each and the bed frames at $100 each, write an inequality that restricts the purchase on x mattress sets and y bed frames.
   \[200x + 100y \leq 80000\]
1. The senior class is producing a play to raise money for graduation activities. They would like to earn at least $500. They intend to charge $2 for each student ticket, and $5 for each adult ticket. Write an inequality that represents this situation.

2. Several times a week a student runs part of the way and walks part of the way on a trail that is less than 4 miles long. The student’s running speed is 6 miles per hour and her walking speed is 4 miles per hour. Write an inequality that represents this situation.

3. Satchi found a used bookstore that sells pre-owned videos and CDs. Videos cost $9 each, and CDs cost $7 each. Satchi can spend no more than $35.
   a. Write an inequality that represents this situation.
   b. Does Satchi have enough money to buy 2 videos and 3 CDs?

4. The perimeter of a rectangular lot is less than 800 feet. Write an inequality that represents the amount of fencing that will enclose the lot.

5. At a grocery store the price of a lemon is $0.50 and the price of a lime is $0.25. Write an inequality to model the relationship between the number of lemons and the number of limes that can be purchased for less than $5.00.

6. You have $4000 to buy stock and have decided on The Clothes Store (TCS) and United Computers (UC). TCS sells for $20 per share and UC sells for $15 per share. Write an inequality which restricts the purchase of x shares of TCS and y shares of UC.
Practice Writing Equations for Inequalities

1. The senior class is producing a play to raise money for graduation activities. They would like to earn at least $500. They intend to charge $2 for each student ticket, and $5 for each adult ticket. Write an inequality that represents this situation.
   
   \[ 2s + 5a > 500 \]

2. Several times a week a student runs part of the way and walks part of the way on a trail that is less than 4 miles long. The student’s running speed is 6 miles per hour and her walking speed is 4 miles per hour. Write an inequality that represents this situation.
   
   \[ 6x + 4w < 4 \]

3. Satchi found a used bookstore that sells pre-owned videos and CDs. Videos cost $9 each, and CDs cost $7 each. Satchi can spend no more than $35.
   
   a. Write an inequality that represents this situation.
      
      \[ 9v + 7c \leq 35 \]
   
   b. Does Satchi have enough money to buy 2 videos and 3 CDs?
      
      \[ 9(2) + 7(3) \leq 35 \]
      
      \[ 39 \leq 35 \]
      
      Satchi cannot purchase 2 videos and 3 CDs

4. The perimeter of a rectangular lot is less than 800 feet. Write an inequality that represents the amount of fencing that will enclose the lot.
   
   \[ 2w + 2l < 800 \]

5. At a grocery store the price of a lemon is $0.50 and the price of a lime is $0.25. Write an inequality to model the relationship between the number of lemons and the number of limes that can be purchased for less than $5.00.
   
   \[ 0.50e + 0.25i < 5.00 \]

6. You have $4000 to buy stock and have decided on The Clothes Store (TCS) and United Computers (UC). TCS sells for $20 per share and UC sells for $15 per share. Write an inequality which restricts the purchase of x shares of TCS and y shares of UC.
   
   \[ 20x + 15y \leq 4000 \]
Linear programming

Name: __________________

Examples

1. A furniture manufacturer makes a profit of $15 on each chair and $8 on each table. To meet dealer demand, daily production of chairs should be at least 30 and no more than 80, whereas the number of tables should be at least 10 and no more than 30. In order to maintain high quality, the total number of chairs and tables should not exceed 80 per day. Write a system of inequalities, be sure to identify and define variables. Write the function to be used to maximize profit.

a. Circle key vocabulary terms and underline key information in the problem.

Check your answer with the one below…

A furniture manufacturer makes a profit of $15 on each chair and $8 on each table. To meet dealer demand, daily production of chairs should be at least 30 and no more than 80, whereas the number of tables should be at least 10 and no more than 30. In order to maintain high quality, the total number of chairs and tables should not exceed 80 per day.

b. Write a system of inequalities to represent the given situation. Be sure to define the variables used.

c. Determine the function used to maximize profit.
2. Molly is planning to invest up to $20,000 in stocks and bonds. The minimum she can invest in stocks is $3000, and she does not want to invest more than $11,000 in stocks or more than $15,000 in bonds. The annual simple interest is 6% on stocks and 4.5% on bonds.

   a) Identify the variables.

   b) Write the systems of inequalities from the description in the problem.

   c) Determine the function used to maximize the values on the feasible region.

3. In a certain furniture making plant, it takes 2 hours of carpentry and 3 hours of finishing to make one chair. To make one cabinet, it takes 3 hours of carpentry and 1 hour of finishing. The plant manager knows that with the employees in the plant today she has 130 hours of carpentry labor and 90 hours of finishing labor available. You make a $50 profit for each chair assembled. You make a $55 profit for each cabinet assembled.

   a) Identify the variables.

   b) Write the systems of inequalities from the description in the problem.

   c) Determine the function used to maximize the values on the feasible region.

   d) Find the maximum profit.
Linear programming

Examples

1. A furniture manufacturer makes a profit of $15 on each chair and $8 on each table. To meet dealer demand, daily production of chairs should be at least 30 and no more than 80, whereas the number of tables should be at least 10 and no more than 30. In order to maintain high quality, the total number of chairs and tables should not exceed 80 per day. Write a system of inequalities, be sure to identify and define variables. Write the function to be used to maximize profit.

a) Circle key vocabulary terms and underline key information in the problem.

Check your answer with the one below…

A furniture manufacturer makes a profit of $15 on each chair and $8 on each table. To meet dealer demand, daily production of chairs should be at least 30 and no more than 80, whereas the number of tables should be at least 10 and no more than 30. In order to maintain high quality, the total number of chairs and tables should not exceed 80 per day.

b) Write a system of inequalities to represent the given situation. Be sure to define the variables used.

\[ c \geq 30 \quad c \leq 80 \]
\[ t \geq 10 \quad t \leq 30 \]
\[ c + t \leq 80 \]

These inequalities represent the constraints on the number of chairs and tables.

c) Determine the function used to maximize profit.

\[ f(c, t) = 15c + 8t \]
2. Molly is planning to invest up to $20,000 in stocks and bonds. The minimum she can invest in stocks is $3000, and she does not want to invest more than $11,000 in stocks or more than $15,000 in bonds. The annual simple interest is 6% on stocks and 4.5% on bonds.

a) Identify the variables.
\(s\): the amount invested in stocks
\(b\): the amount invested in bonds

b) Write the systems of inequalities from the description in the problem.
\[
\begin{align*}
s & \geq 3000 && s \leq 11000 \\
b & \geq 0 && b \leq 11000 \\
s + b & \leq 20000
\end{align*}
\]

c) Determine the function used to maximize the values on the feasible region.
\[
\begin{align*}
f(s, b) & = 0.06s + 0.045b
\end{align*}
\]

3. In a certain furniture making plant, it takes 2 hours of carpentry and 3 hours of finishing making one chair. To make one cabinet, it takes 3 hours of carpentry and 1 hour of finishing. The plant manager knows that with the employees in the plant today she has 130 hours of carpentry labor and 90 hours of finishing labor available. You make a $50 profit for each chair assembled. You make a $55 profit for each cabinet assembled.

a) Identify the variables.
\(h\): the number of chairs manufactured
\(b\): the number of cabinets manufactured

b) Write the systems of inequalities from the description in the problem.
(Making a table helps students organize data)
\[
\begin{array}{|c|c|c|}
\hline
& \text{Chairs (}h\text{)} & \text{Cabinets (}b\text{)} \\
\hline
\text{Carpentry} & 2 & 3 & 130 \\
\text{Finishing} & 3 & 1 & 90 \\
\hline
\end{array}
\]
\[
\begin{align*}
2h + 3b & < 130 \\
3h + 1b & < 90 \\
h & > 0 \\
b & > 0
\end{align*}
\]

c) Determine the function used to maximize the values on the feasible region.
\[
\begin{align*}
f(h, b) & = 50h + 55b
\end{align*}
\]
A local street vendor sells hotdogs and pretzels. To make a profit, the street vendor must sell at least 30 hotdogs but cannot prepare more than 70. The street vendor must also sell at least 10 pretzels but cannot prepare more than 40. The street vendor cannot prepare more than a total of 90 hotdogs and pretzels altogether. The profit is $0.48 on a hotdog and $0.25 on a pretzel. Write a system of inequalities, be sure to identify and define variables. Write an equation to be used to maximize profit.

<table>
<thead>
<tr>
<th>1. Underline key sentences and circle key vocabulary in the paragraph above.</th>
<th>2. Identify the variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>3. Write the systems of inequalities from the description in the problem.</td>
<td>4. Determine the function used to maximize or minimize.</td>
</tr>
</tbody>
</table>
You are starting a new workout routine. You want to burn the most calories. You burn 25 calories for each set you do on a nautilus machine. You burn 35 calories on each set you do on the free weights. You want to do at least 5 of the possible 10 nautilus machines. You want to do at least 3 of the possible 10 free weights. In total, you can do no more than 18 weights. Write a system of inequalities, be sure to identify and define variables. Write an equation to be used to maximize the number of calories burned.

1. Underline key sentences and circle key vocabulary in the paragraph above.  
2. Identify the variables

| 3. Write the systems of inequalities from the description in the problem. | 4. Determine the function used to maximize or minimize. |
**Round Table**

<table>
<thead>
<tr>
<th>Student Names</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
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<tr>
<td></td>
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<tr>
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</tr>
</tbody>
</table>

Manufacturing Chairs and Tables: You make a $50 profit for each chair assembled. A chair takes 1 unskilled labor hours, 3 machine hours, and 1 hour of skilled time. The table requires 6 unskilled labor hours, 1 hour on machine, and 1 hour skilled labor. Unskilled hours are no more than 2700 hours. Machine time cannot exceed 1500 hours. Skilled labor cannot exceed 700 hours. Write a system of inequalities, be sure to identify and define variables. Write an equation to be used to maximize the profit.

<table>
<thead>
<tr>
<th>1. Underline key sentences and circle key vocabulary in the paragraph above.</th>
<th>2. Identify the variables</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tbody>
</table>

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<th>3. Write the systems of inequalities from the description in the problem.</th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Your school is planning to make toques and mitts to sell at the winter festival as a fundraiser. The school’s sewing classes divide into two groups – one group can make toques, the other group knows how to make mitts. The sewing teachers are also willing to help out. Considering the number of people available and time constraints due to classes, only 150 toques and 120 pairs of mitts can be made each week. Enough material is delivered to the school every Monday morning to make a total of 200 items per week. Because the material is being donated by community members, each toque sold makes a profit of $2 and each pair of mitts sold makes a profit of $5. Write a system of inequalities, be sure to identify and define variables. Write an equation to be used to maximize the profit.

1. Underline key sentences and circle key vocabulary in the paragraph above.
2. Identify the variables
3. Write the systems of inequalities from the description in the problem.
4. Determine the function used to maximize or minimize.
A local street vendor sells hotdogs and pretzels. To make a profit, the street vendor must sell at least 30 hotdogs but cannot prepare more than 70. The street vendor must also sell at least 10 pretzels but cannot prepare more than 40. The street vendor cannot prepare more than a total of 90 hotdogs and pretzels altogether. The profit is $0.48 on a hotdog and $0.25 on a pretzel. Write a system of inequalities, be sure to identify and define variables. Write an equation to be used to maximize profit.

1. Underline key sentences and circle key vocabulary in the paragraph above.

2. Identify the variables
   \[ h \text{: number of hotdogs sold} \]
   \[ p \text{: number of pretzels sold} \]

3. Write the systems of inequalities from the description in the problem.
   \[ h > 30 \quad h < 70 \]
   \[ p > 10 \quad p < 40 \]
   \[ h + p < 90 \]

4. Determine the function used to maximize or minimize.
   \[ f(h, p) = 0.48h + 0.25p \]
You are starting a new workout routine. You want to burn the most calories. You burn 25 calories for each set you do on a nautilus machine. You burn 35 calories on each set you do on the free weights. You want to do at least 5 of the possible 10 nautilus machines. You want to do at least 3 of the possible 10 free weights. In total, you can do no more than 18 weights. Write a system of inequalities, be sure to identify and define variables. Write an equation to be used to maximize the number of calories burned.

1. Underline key sentences and circle key vocabulary in the paragraph above.
2. Identify the variables:
   - \( n \): number of nautilus machines
   - \( f \): number of free weights

3. Write the systems of inequalities from the description in the problem.
   - \( n > 5 \)
   - \( n < 10 \)
   - \( f > 3 \)
   - \( f < 10 \)
   - \( n + f < 18 \)

4. Determine the function used to maximize or minimize.
   - \( f(n, f) = 25n + 35f \)
Manufacturing Chairs and Tables: You make a $50 profit for each chair assembled. A chair takes 1 unskilled labor hours, 3 machine hours, and 1 hour of skilled time. The table requires 6 unskilled labor hours, 1 hour on machine, and 1 hour skilled labor. Unskilled hours are no more than 2700 hours. Machine time cannot exceed 1500 hours. Skilled labor cannot exceed 700 hours. Write a system of inequalities, be sure to identify and define variables. Write an equation to be used to maximize the profit.

1. Underline key sentences and circle key vocabulary in the paragraph above.

2. Identify the variables
   - \( c \): number of chairs
   - \( t \): number of tables

3. Write the systems of inequalities from the description in the problem.

   \[
   \begin{array}{c|c|c|c}
   & \text{Chairs (c)} & \text{Tables (t)} & \text{Limit} \\
   \hline
   \text{Unskilled} & 1 & 6 & 2700 \\
   \text{Machine} & 3 & 1 & 1500 \\
   \text{Skilled} & 1 & 1 & 700 \\
   \end{array}
   \]

   \( 1c + 6t < 2700 \)
   \( 3c + 1t < 1500 \)
   \( 1c + 1t < 700 \)

4. Determine the function used to maximize or minimize.
   \[ f(c, t) = 50c + 35t \]
Your school is planning to make toques and mitts to sell at the winter festival as a fundraiser. The school’s sewing classes divide into two groups – one group can make toques, the other group knows how to make mitts. The sewing teachers are also willing to help out. Considering the number of people available and time constraints due to classes, only 150 toques and 120 pairs of mitts can be made each week. Enough material is delivered to the school every Monday morning to make a total of 200 items per week. Because the material is being donated by community members, each toque sold makes a profit of $2 and each pair of mitts sold makes a profit of $5. Write a system of inequalities, be sure to identify and define variables. Write an equation to be used to maximize the profit.

1. Underline key sentences and circle key vocabulary in the paragraph above.
2. Identify the variables
   - \( h \): number of hats
   - \( m \): number of mittens
3. Write the systems of inequalities from the description in the problem.
   - \( h < 150 \)
   - \( m < 120 \)
   - \( h + m < 200 \)
4. Determine the function used to maximize or minimize.
   - \( f(h, m) = 2h + 5m \)
Writing Inequalities and Profit Functions

1. You manufacture cell phone accessories: cases and clips. You have to produce at least 20 clips each day and no more than 50 clips due to demand limitations. You need to produce at least 40 cases each day. Due to time constraints, we can produce no more than 80 items total in a day. You make a profit of $6 for each case sold and $4 for each clip sold. Write a system of inequalities, be sure to identify and define variables. Write an equation to be used to maximize the profit.

a) Identify the variables

b) Write the systems of inequalities from the description in the problem

c) Determine the function used to maximize or minimize the values on the feasible region

2. A Wii manufacture is making Wii remotes and Wii numchucks. For each Wii remote sold, they make $20 profit. For each Wii numchuck sold, they make $12 profit. The Wii remotes take 4 hours to prepare the parts and 1 hour to assemble. The Wii numchuck takes 2.5 hours to prepare the parts and 2.5 hours to assemble. The maximum preparation time available is 16 hours. The maximum assembly time available is 10 hours. Write a system of inequalities, be sure to identify and define variables. Write an equation to be used to maximize the profit.

a) Identify the variables

b) Write the systems of inequalities from the description in the problem

c) Determine the function used to maximize or minimize the values on the feasible region
Writing Inequalities and Profit Functions

1. You manufacture cell phone accessories: cases and clips. You have to produce at least 20 clips each day and no more than 50 clips due to demand limitations. You need to produce at least 40 cases each day. Due to time constraints we can produce no more than 80 items total in a day. You make a profit of $6 for each case sold and $4 for each clip sold. Write a system of inequalities, be sure to identify and define variables. Write an equation to be used to maximize the profit.

   a) Identify the variables
      \[ c: \text{number of cases} \]
      \[ p: \text{number of clips} \]

   b) Write the systems of inequalities from the description in the problem
      \[ p \geq 20 \quad p \leq 50 \]
      \[ c \geq 40 \quad p + c \leq 80 \]

   c) Determine the function used to maximize or minimize the values on the feasible region
      \[ f(c, p) = 6c + 4p \]

2. A wii manufacture is making wii remotes and wii numchucks. For each wii remote sold they make $20 profit. For each wii numchuck sold they make $12 profit. The wii remotes take 4 hours to prepare the parts and 1 hour to assemble. The wii numchuck takes 2.5 hours to prepare the parts and 2.5 hours to assemble. The maximum preparation time available is 16 hours. The maximum assembly time available is 10 hours. Write a system of inequalities, be sure to identify and define variables. Write an equation to be used to maximize the profit.

   a) Identify the variables
      \[ r: \text{number of wii remotes} \]
      \[ n: \text{number of wii numchucks} \]

   b) Write the systems of inequalities from the description in the problem
      \[
      \begin{array}{|c|c|c|c|c|}
      \hline
      & \text{Remotes (r)} & \text{Numchucks (n)} & \text{Preparation} \leq 16 & \text{Assembly} \leq 10 \\
      \hline
      \text{Preparation} & 4 & 2.5 & 16 \\
      \hline
      \text{Assembly} & 1 & 2.5 & 10 \\
      \hline
      \end{array}
      \]
      \[ 4r + 2.5n < 16 \]
      \[ r + 2.5n < 10 \]
      \[ r > 0, n > 0 \]

   c) Determine the function used to maximize or minimize the values on the feasible region
      \[ f(r, n) = 20r + 12n \]
Systems of Equations Drill

Solve the following system of equations by graphing.

\[
\begin{align*}
4x + 2.5y &= 16 \\
x + 2.5 &= 10
\end{align*}
\]
Solve the following system of equations by graphing.

\[
\begin{align*}
4x + 2.5y &= 16 \\
x + 2.5y &= 10
\end{align*}
\]
You want to have the after prom party at the ESPN zone. For a fundraiser you decided to sell bracelets. Each pack has 24 regular beads and 20 fancy beads. You decide to make two types of bracelets. A standard bracelet (shown below) is made with four regular beads and two fancy beads. A deluxe bracelet (shown below) is made with two regular beads and five fancy beads. If you make a profit of $2.50 on each standard bracelet and $3.00 on each deluxe bracelet, with one packet how many of each type of bracelet should you make to earn the maximum profit?

To complete this problem, each of you have been given a bag of beads containing 20 fancy beads and 24 regular beads.

1. Identify the variables.

2. Write a system of inequalities to represent this situation where \((d)\) is the number of deluxe bracelets and \((s)\) is the number of standard bracelets.

3. What is our equation for profit?
**Exploration:** Find possible combinations of standard bracelets and deluxe bracelets you can make with the given beads and then record the profit made.

<table>
<thead>
<tr>
<th>Deluxe Bracelets ($d$)</th>
<th>Standard Bracelets ($s$)</th>
<th>Total Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. What was the maximum profit?

2. How many of each bracelet do you need to sell in order to make the maximum profit?

3. Graph the system of inequalities below.

4. Plot the points from the table created in the exploration piece. What do you notice about the point that represents the maximum profit?
You want to have the after prom party at the ESPN zone. For a fundraiser you decided to sell bracelets. Each pack has 24 regular beads and 20 fancy beads. You decide to make two types of bracelets. A standard bracelet (shown below) is made with four regular beads and two fancy beads. A deluxe bracelet (shown below) is made with two regular beads and five fancy beads. If you make a profit of $2.50 on each standard bracelet and $3.00 on each deluxe bracelet, with one packet how many of each type of bracelet should you make to earn the maximum profit?

To complete this problem, each of you have been given a bag of beads containing 20 fancy beads and 24 regular beads.

1. Identify the variables.
   
   \[ d = \text{number of deluxe bracelets} \]
   \[ s = \text{number of standard bracelets}. \]

2. Write a system of inequalities to represent this situation where \((d)\) is the number of deluxe bracelets and \((s)\) is the number of standard bracelets.

   \[
   \begin{array}{c|c|c|c}
   \text{Regular} & \text{Deluxe} & \text{Standard} & \text{Total} \\
   \hline
   \text{Regular} & 2 & 4 & 24 \\
   \text{Fancy} & 5 & 2 & 20 \\
   \hline
   \end{array}
   \]

   \[
   \begin{cases}
   2d + 4s < 24 \\
   5d + 2s < 20 \\
   \end{cases}
   \]

3. What is your equation for profit?

   \[ f(d,s) = 3d + 2.5s \]
**Exploration:** Find possible combinations of standard bracelets and deluxe bracelets you can make with the given beads and then record the profit made.

<table>
<thead>
<tr>
<th>Deluxe Bracelets ($d$)</th>
<th>Standard Bracelets ($s$)</th>
<th>Total Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6</td>
<td>15</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>15.5</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>18.5</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>14</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>12</td>
</tr>
</tbody>
</table>

**Profit** = $3d + 2.5s$

4. What was the maximum profit?  
   $18.50$

5. How many of each bracelet do you need to sell in order to make the maximum profit?  
   You need to sell 2 deluxe bracelets and 5 standard.

6. Graph the system of inequalities below.

7. Plot the points from the table created in the exploration piece. What do you notice about the point that represents the maximum profit?  
   This point lies at the intersection of the boundary lines formed by the inequalities.
Graphs and Linear Programming

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Points of Intersection (POI)</td>
<td>Points where the lines in the system of equations intersect</td>
</tr>
<tr>
<td>Feasible Region</td>
<td>The enclosed polygonal region which represents the solutions of the system</td>
</tr>
<tr>
<td>Vertices</td>
<td>The points of intersection which define the feasible region</td>
</tr>
<tr>
<td>Constraints</td>
<td>The inequalities which contain the feasible region</td>
</tr>
</tbody>
</table>
A Programming Problem of Profit to Ponder Prolifically  Name: ______________

1. A furniture manufacturer makes a profit of $15 on each chair and $8 on each table. To meet dealer demand, daily production of chairs should be at least 30 and no more than 80, whereas the number of tables should be at least 10 and no more than 30. In order to maintain high quality, the total number of chairs and tables should not exceed 80 per day. Write a system of inequalities, be sure to identify and define variables. Write the function to be used to maximize profit.

   **Step 1:** Identify the variables

   **Step 2:** Write the systems of inequalities from the description in the problem

   **Step 3:** Graph the system of inequalities.

   ![Graph of inequalities]

   **Step 4:** Locate the vertices of the feasible region

   **Step 5:** Determine the function used to maximize or minimize the values on the feasible region.

   **Step 6:** Determine the vertex which provides the optimal value
A Programming Problem of Profit to Ponder Prolifically

1. A furniture manufacturer makes a profit of $15 on each chair and $8 on each table. To meet dealer demand, daily production of chairs should be at least 30 and no more than 80, whereas the number of tables should be at least 10 and no more than 30. In order to maintain high quality, the total number of chairs and tables should not exceed 80 per day. Write a system of inequalities, be sure to identify and define variables. Write the function to be used to maximize profit.

**Step 1:** Identify the variables
- \( c \): the number of chairs
- \( t \): the number of tables

**Step 2:** Write the systems of inequalities from the description in the problem
- \( c \geq 30 \)
- \( c \leq 80 \)
- \( t \geq 10 \)
- \( t \leq 30 \)
- \( c + t \leq 80 \)

**Step 3:** Graph the system of inequalities.

**Step 4:** Locate the vertices of the feasible region
- (30, 10)
- (30, 30)
- (50, 30)
- (70, 10)

**Step 5:** Determine the function used to maximize or minimize the values on the feasible region.
- \( f(c, t) = 15c + 8t \)

**Step 6:** Determine the vertex which provides the optimal value
- \( f(7, 1) = 113 \)
<table>
<thead>
<tr>
<th>Statement</th>
<th>True/False</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.. The graph of a linear inequality consists of a line and only some of the points on one side of the line.</td>
<td>TRUE FALSE</td>
</tr>
<tr>
<td>2. The graph of a linear inequality consists of a line and some points on both sides of the line.</td>
<td>TRUE FALSE</td>
</tr>
<tr>
<td>3. The graph of a linear inequality consists of a line and all of the points on one side of the line.</td>
<td>TRUE FALSE</td>
</tr>
<tr>
<td>4. If a linear programming problem has a solution at all, it will have a solution at some vertex of the feasible region.</td>
<td>TRUE FALSE</td>
</tr>
<tr>
<td>5. No point other than a vertex of the feasible region can be a solution to a linear programming problem.</td>
<td>TRUE FALSE</td>
</tr>
<tr>
<td>6. Every linear programming problem with a bounded nonempty feasible region has a solution.</td>
<td>TRUE FALSE</td>
</tr>
<tr>
<td>7. Every linear programming problem has a solution.</td>
<td>TRUE FALSE</td>
</tr>
<tr>
<td>Statement</td>
<td>True/False</td>
</tr>
<tr>
<td>--------------------------------------------------------------------------</td>
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</tr>
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<td>1. The graph of a linear inequality consists of a line and only some of</td>
<td>FALSE</td>
</tr>
<tr>
<td>the points on one side of the line.</td>
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<td>region has a solution.</td>
<td></td>
</tr>
<tr>
<td>7. Every linear programming problem has a solution.</td>
<td>FALSE</td>
</tr>
</tbody>
</table>
GROUP 1

Molly is planning to invest up to $20,000 in stocks and bonds. The minimum she can invest in stocks is $3000, and she does not want to invest more than $11,000 in stocks or more than $15,000 in bonds. The annual simple interest is 6% on stocks and 4.5% on bonds. Write a system of inequalities, be sure to identify and define variables. Write an equation to be used to maximize her income.
GROUP 2

In a certain furniture making plant, it takes 2 hours of carpentry and 3 hours of finishing to make one chair. To make one cabinet, it takes 3 hours of carpentry and 1 hour of finishing. The plant manager knows that with the employees in the plant today she has 130 hours of carpentry labor and 90 hours of finishing labor available. You make a $50 profit for each chair assembled. You make a $55 profit for each table assembled. Find the maximum profit. Write a system of inequalities, be sure to identify and define variables. Write an equation to be used to maximize the profit.
GROUP 3

A local street vendor sells hotdogs and pretzels. To make a profit, the street vendor must sell at least 30 hotdogs but cannot prepare more than 70. The street vendor must also sell at least 10 pretzels but cannot prepare more than 40. The street vendor cannot prepare more than a total of 90 hotdogs and pretzels altogether. The profit is $0.48 on a hotdog and $0.25 on a pretzel. Write a system of inequalities, be sure to identify and define variables. Write an equation to be used to maximize profit.
GROUP 4

You are starting a new workout routine. You want to burn the most calories. You burn 25 calories for each set you do on a nautilus machine. You burn 35 calories on each set you do on the free weights. You want to do at least 5 of the possible 10 nautilus machines. You want to do at least 3 of the possible 10 free weights. In total, you can do no more than 18 weights. Write a system of inequalities, be sure to identify and define variables. Write an equation to be used to maximize the number of calories burned.
GROUP 5

Manufacturing Chairs and Tables: You make a $50 profit for each chair assembled. You make a $30 profit for each table assembled. A chair takes 1 unskilled labor hours, 3 machine hours, and 1 hour of skilled time. The table requires 6 unskilled labor hours, 1 hour on machine, and 1 hour skilled labor. Unskilled hours are no more than 2700 hours. Machine time cannot exceed 1500 hours. Skilled labor cannot exceed 700 hours. Write a system of inequalities, be sure to identify and define variables. Write an equation to be used to maximize the profit.
GROUP 6

Your school is planning to make toques and mitts to sell at the winter festival as a fundraiser. The school’s sewing classes divide into two groups – one group can make toques, the other group knows how to make mitts. The sewing teachers are also willing to help out. Considering the number of people available and time constraints due to classes, only 150 toques and 120 pairs of mitts can be made each week. Enough material is delivered to the school every Monday morning to make a total of 200 items per week. Because the material is being donated by community members, each toque sold makes a profit of $2 and each pair of mitts sold makes a profit of $5. Write a system of inequalities, be sure to identify and define variables. Write an equation to be used to maximize the profit.
GROUP 1
Molly is planning to invest up to $20,000 in stocks and bonds. The minimum she can
invest in stocks is $3000, and she does not want to invest more than $11,000 in stocks or
more than $15,000 in bonds. The annual simple interest is 6% on stocks and 4.5% on
bonds. Write a system of inequalities, be sure to identify and define variables. Write an
equation to be used to maximize her income.

Step 1: Identify the variables.
\( s \): the amount invested in stocks
\( b \): the amount invested in bonds

Step 2: Write the systems of inequalities from the description in the problem.
\[
\begin{align*}
    s & \geq 3000 & s & \leq 11000 \\
    b & \geq 0 & b & \leq 11000 \\
    s + b & \leq 20000
\end{align*}
\]

Step 3: Graph the system of inequalities.

Step 4: Locate the Vertices of the Feasible Region.
(3000, 11000) (4000, 11000) (11000, 9000) (110000, 0)

Step 5: Determine the function used to maximize/minimize values on the feasible
region.
\[
f(s, b) = 0.06s + 0.045b
\]

Step 6: Determine the vertex which provides the optimal value.
\[
f(4000, 11000) = 5190
\]
GROUP 2
In a certain furniture making plant, it takes 2 hours of carpentry and 3 hours of finishing to make one chair. To make one cabinet, it takes 3 hours of carpentry and 1 hour of finishing. The plant manager knows that with the employees in the plant today she has 130 hours of carpentry labor and 90 hours of finishing labor available. You make a $50 profit for each chair assembled. You make a $55 profit for each table assembled. Find the maximum profit. Write a system of inequalities, be sure to identify and define variables. Write an equation to be used to maximize the profit.

Step 1: Identify the variables.

- $h$: the number of chairs manufactured
- $b$: the number of cabinets manufactured

Step 2: Write the systems of inequalities from the description in the problem.

\[
\begin{align*}
\begin{array}{c}
\text{Chairs (h)} \\
\text{Cabinets (b)}
\end{array}
\end{align*}
\]

\[
\begin{align*}
2h + 3b &< 130 \\
3h + 1b &< 90
\end{align*}
\]

Step 3: Graph the system of inequalities.

Step 4: Locate the Vertices of the Feasible Region.

(30, 0) (20, 30) (0, 43.3) (0, 0)

Step 5: Determine the function used to maximize/minimize values on the feasible region.

\[
f(h,b) = 50h + 55b
\]

Step 6: Determine the vertex which provides the optimal value.

\[
f(20,30) = 2650
\]
GROUP 3
A local street vendor sells hotdogs and pretzels. To make a profit, the street vendor must sell at least 30 hotdogs but cannot prepare more than 70. The street vendor must also sell at least 10 pretzels but cannot prepare more than 40. The street vendor cannot prepare more than a total of 90 hotdogs and pretzels altogether. The profit is $0.48 on a hotdog and $0.25 on a pretzel. Write a system of inequalities, be sure to identify and define variables. Write an equation to be used to maximize profit.

Step 1: Identify the variables.
- \( h \): number of hotdogs sold
- \( p \): number of pretzels sold

Step 2: Write the systems of inequalities from the description in the problem.

\[
\begin{align*}
    h &> 30 \quad h < 70 \\
p &> 10 \quad p < 40 \\
h + p &< 90
\end{align*}
\]

Step 3: Graph the system of inequalities.

![Graph of Vendor Sales](image)

Step 4: Locate the Vertices of the Feasible Region.

(30, 10) (30, 40) (50, 40) (70, 20)

Step 5: Determine the function used to maximize/minimize values on the feasible region.

\[
f(h, p) = 0.48h + 0.25p
\]

Step 6: Determine the vertex which provides the optimal value.

\[
f(70, 20) = 38.6
\]
GROUP 4
You are starting a new workout routine. You want to burn the most calories. You burn 25 calories for each set you do on a nautilus machine. You burn 35 calories on each set you do on the free weights. You want to do at least 5 of the possible 10 nautilus machines. You want to do at least 3 of the possible 10 free weights. In total, you can do no more than 18 weights. Write a system of inequalities, be sure to identify and define variables. Write an equation to be used to maximize the number of calories burned.

Step 1: Identify the variables.
- $n$: number of nautilus machines
- $f$: number of free weights

Step 2: Write the systems of inequalities from the description in the problem.

\[
\begin{align*}
    n &> 5 \quad n < 10 \\
    f &> 3 \quad f < 10 \\
    n + f &< 18
\end{align*}
\]

Step 3: Graph the system of inequalities.

Step 4: Locate the Vertices of the Feasible Region.
(5, 3) (5, 10) (8, 10) (10, 8)

Step 5: Determine the function used to maximize/minimize values on the feasible region.

\[f(n, f) = 25n + 35f\]

Step 6: Determine the vertex which provides the optimal value.

\[f(8, 10) = 550\]
GROUP 5

Manufacturing Chairs and Tables: You make a $50 profit for each chair assembled. You make a $30 profit for each table assembled. A chair takes 1 unskilled labor hours, 3 machine hours, and 1 hour of skilled time. The table requires 6 unskilled labor hours, 1 hour on machine, and 1 hour skilled labor. Unskilled hours are no more than 2700 hours. Machine time cannot exceed 1500 hours. Skilled labor cannot exceed 700 hours. Write a system of inequalities, be sure to identify and define variables. Write an equation to be used to maximize the profit.

**Step 1:** Identify the variables.
- \( c \): number of chairs
- \( t \): number of tables

**Step 2:** Write the systems of inequalities from the description in the problem.

\[
\begin{align*}
&c > 0 \quad t > 0 \\
&1c + 6t < 2700 \quad 3c + t < 1500 \quad c + t < 700
\end{align*}
\]

<table>
<thead>
<tr>
<th></th>
<th>Chairs ((c))</th>
<th>Tables ((t))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unskilled</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>Machine</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Skilled</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

**Step 3:** Graph the system of inequalities.

**Step 4:** Locate the Vertices of the Feasible Region.
- (0, 450) (300, 400) (400, 300) (500, 0)

**Step 5:** Determine the function used to maximize/minimize values on the feasible region.
\[
f(c,t) = 50c + 35t
\]

**Step 6:** Determine the vertex which provides the optimal value.
\[
f(8,10) = 550
\]

Linear Programming 61
GROUP 6
Your school is planning to make toques and mitts to sell at the winter festival as a fundraiser. The school’s sewing classes divide into two groups – one group can make toques, the other group knows how to make mitts. The sewing teachers are also willing to help out. Considering the number of people available and time constraints due to classes, only 150 toques and 120 pairs of mitts can be made each week. Enough material is delivered to the school every Monday morning to make a total of 200 items per week. Because the material is being donated by community members, each toque sold makes a profit of $2 and each pair of mitts sold makes a profit of $5. Write a system of inequalities, be sure to identify and define variables. Write an equation to be used to maximize the profit.

Step 1: Identify the variables.
  \( h \): number of hats
  \( m \): number of mittens

Step 2: Write the systems of inequalities from the description in the problem.

\[
\begin{align*}
  &h > 0 \quad h < 150 \\
  &m > 0 \quad m < 120 \\
  &h + m < 200
\end{align*}
\]

Step 3: Graph the system of inequalities.

Step 4: Locate the Vertices of the Feasible Region.
(0, 0) (0, 120) (80, 120) (150, 50) (150, 0)

Step 5: Determine the function used to maximize/minimize values on the feasible region.
\( f(h,m) = 2h + 5m \)

Step 6: Determine the vertex which provides the optimal value.
\( f(80,120) = 760 \)
**Steppin' Up**

**Name:** __________________

**Directions:** Number the steps in the order in which they would be completed when solving a linear programming problem.

<table>
<thead>
<tr>
<th>STEP NUMBER</th>
<th>STEP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Locate the Vertices of the Feasible Region</td>
</tr>
<tr>
<td></td>
<td>Determine the vertex which provides the optimal value</td>
</tr>
<tr>
<td></td>
<td>Determine the function used to maximize or minimize the values on the feasible region.</td>
</tr>
<tr>
<td></td>
<td>Write the systems of inequalities from the description in the problem</td>
</tr>
<tr>
<td></td>
<td>Identify the variables</td>
</tr>
<tr>
<td></td>
<td>Graph the system of inequalities.</td>
</tr>
</tbody>
</table>
**Directions:** Number the steps in the order in which they would be completed when solving a linear programming problem.

<table>
<thead>
<tr>
<th>STEP NUMBER</th>
<th>STEP</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Locate the Vertices of the Feasible Region</td>
</tr>
<tr>
<td>6</td>
<td>Determine the vertex which provides the optimal value</td>
</tr>
<tr>
<td>5</td>
<td>Determine the function used to maximize or minimize the values on the feasible region.</td>
</tr>
<tr>
<td>2</td>
<td>Write the systems of inequalities from the description in the problem</td>
</tr>
<tr>
<td>1</td>
<td>Identify the variables</td>
</tr>
<tr>
<td>3</td>
<td>Graph the system of inequalities.</td>
</tr>
</tbody>
</table>
I’m thinking of two whole numbers whose sum equals 12 and whose difference is equal to 2. What could the numbers be?

\[
\begin{align*}
    x + y &= 12 \\
    y - x &= 2
\end{align*}
\]
I’m thinking of two whole numbers whose sum equals 12 and whose difference is equal to 2. What could the numbers be?

\[
\begin{align*}
  x + y &= 12 \\
  y - x &= 2
\end{align*}
\]
I’m thinking of two whole numbers whose sum is less than 12 and whose difference is greater than 2. What could the numbers be?

\[
\begin{cases}
  x + y < 12 \\
  y - x > 2
\end{cases}
\]
I’m thinking of two whole numbers whose sum is less than 12 and whose difference is greater than 2. What could the numbers be?

\[
\begin{align*}
x + y &< 12 \\
y - x &> 2
\end{align*}
\]
I’m thinking of two rational numbers described by the following rules:

- The sum of one third of one number and another is less than 12.
- The difference is less than 5.

What could the numbers be?

\[
\left( \frac{1}{3} \right) x + y < 12 \\
y - x < 5
\]

1. Press the <APPS> key. The applications window will appear on the screen.
2. Scroll down (using the down arrow key) to locate the application “Inequalz”.
3. Highlight the character in front of “Inequalz” and press <ENTER>.
4. Press <ENTER> to start the program.
5. To change the equal sign to an inequality:
   a. Highlight the equal sign via the arrow keys.
   b. Press the <ALPHIA> KEY.
   c. Press the F key to obtain the appropriate inequality.

Before change:         After change:
\[ \begin{array}{c}
\text{Before change:} \\
Y = 0 \\
Y = 1 \\
Y = 2 \\
Y = 3 \\
Y = 4 \\
Y = 5 \\
Y = 6 \\
F1 \quad F2 \quad F3 \quad F4 \quad F5 \\
\end{array} \]

6. Move the cursor to the right using the arrow keys. The inequalities will disappear at the bottom of the screen. You are now ready to type in an expression.

7. Type in the expression. When you have completed this, press the <ENTER> key.

8. Repeat steps 5 through 7 to enter in the next inequality. Continue to do this until all inequalities have been entered.

9. Press the <ZOOM> key.
10. Scroll down (using the down arrow key) to locate “ZoomFit” and press <ENTER>. The calculator will graph the inequalities on the screen.

Zoom Window:

Final Graph:

11. To show the intersection of the inequalities:
   a. Press the <ALPHIA> KEY.
   b. Press the F1 key to obtain the Shades menu.
   c. Use the arrow keys to highlight “1. Ineq Intersection” and press the <ENTER> key. The calculator will now graph only where the inequalities intersect.

Before change:

After change:

12. To find the vertices of the intersection of the inequalities:
   a. Press the <ALPHIA> KEY.
   b. Press the F3 key to obtain the “PoI–Trace” menu. The calculator will locate the vertices of the intersection region and list its xy coordinates.

Before finding vertices:

After finding vertices:
Veteran Medicine: As a receptionist for a veterinarian, one of Dolores Alvarez’s tasks is to schedule appointments. She allots 20 minutes for a routine office and 40 minutes for a surgery. The veterinarian cannot do more than 6 surgeries per day. The office has 7 hours available for appointments. If an office visit cost $55 and most surgeries cost $125, find a combination of office visits and surgeries that will maximize the income the veterinarian practice receives per day.

Step 1:

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

\[ V = \]

\[ S = \]

Step 2:

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________
Step 3:

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Step 4:

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Step 5:

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Step 6:

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<table>
<thead>
<tr>
<th>(v, s)</th>
<th>Plug Vertices into Formula</th>
<th>f(v, s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</table>

Step 7:

________________________________________________________________________

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________________________________________________________________________
Veteran Medicine: As a receptionist for a veterinarian, one of Dolores Alvarez’s tasks is to schedule appointments. She allot 20 minutes for a routine office and 40 minutes for a surgery. The veterinarian cannot do more than 6 surgeries per day. The office has 7 hours available for appointments. If an office visit cost $55 and most surgeries cost $125, find a combination of office visits and surgeries that will maximize the income the veterinarian practice receives per day.

Step 1:

_Determine the variables._

\[ V = \text{number of office visits} \]
\[ S = \text{number of surgeries} \]

Step 2:

_Write the systems of inequalities from the descriptions in the problem._

\[
\begin{array}{c|c|c|c}
\text{minutes} & V & S & \text{total} \\
\hline
\text{total} & < 7 \times 60 = 420 \\
\text{surgeries} & S & < 7 \\
\end{array}
\]

\[ 20v + 40s < 420 \]
\[ s < 7 \]
Step 3:

Graph the system of inequalities.

Step 4:

Locate the vertices of the feasible region.

The polygon formed is a quadrilateral with vertices at (0, 7), (7, 7), and (21, 0).
Step 5:

Determine the function used to maximize or minimize the values on the feasible region.

\[ f(v, s) = 75v + 125s \]

Step 6:

Evaluate the function at each vertex.

<table>
<thead>
<tr>
<th>(v, s)</th>
<th>Plug Vertices into Formula</th>
<th>( f(v, s) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 7)</td>
<td>55(0) + 125(7)</td>
<td>$875.00</td>
</tr>
<tr>
<td>(7, 7)</td>
<td>55(7) + 125(7)</td>
<td>$1260.00</td>
</tr>
<tr>
<td>(21, 0)</td>
<td>55(21) + 125(0)</td>
<td>$1155.00</td>
</tr>
</tbody>
</table>

Step 7:

Determine the vertex that provides the optimal value.

If the doctor has 7 office visits and performs 7 surgeries, he will make $1260.00.
A concession stand makes a profit of $0.80 on each regular hot dog and $1.30 on each super dog. On a typical day, the concession stand sells at least 25 but no more than 40 regular hot dogs and at least 30 but no more than 50 super dogs. The total sales have never exceeded 80 hot dogs. How many of each type of hot dog should be prepared to maximize profit?

Step 1:

Step 2:
Step 3:

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Step 4:

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Step 5:

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Step 6:

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Step 7:

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________________________________________________________________________
Manufacturing: Machine A can produce 30 steering wheels per hour at a cost of $16 per hour; Machine B can produce 40 steering wheels per hour at a cost of $22 per hour. At least 280 steering wheels must be made in each 8–hour shift. What is the least cost involved in making 280 steering wheels in one shift?

Step 1:

Step 2:
Step 3:

Step 4:
Step 5:

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Step 6:

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Step 7:

__________________________________________________________________________

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__________________________________________________________________________
Business: Ingrid is planning to start a business. She will be baking decorated cakes and specialty pies. She estimates that a decorated cake will take 75 minutes to prepare and a specialty pie will take 30 minutes to prepare. She plans to work no more than 40 hours per week and does not want to make more than 60 pies in any one week. If she plans to charge $34 for a cake and $16 for a pie, find a combination of cakes and pies that will maximize her income for a week.

Step 1:

Step 2:
Step 3:

Step 4:
Step 5:

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Step 6:

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</tr>
</tbody>
</table>

Step 7:

________________________________________________________________________

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________________________________________________________________________
A concession stand makes a profit of $0.80 on each regular hot dog and $1.30 on each super dog. On a typical day, the concession stand sells at least 25 but no more than 40 regular hot dogs and at least 30 but no more than 50 super dogs. The total sales have never exceeded 80 hot dogs. How many of each type of hot dog should be prepared to maximize profit?

Step 1:

Determine the variables.

- \( h \): hot dogs
- \( s \): super dogs

Step 2:

Write the systems of inequalities from the descriptions in the problem.

<table>
<thead>
<tr>
<th></th>
<th>( h )</th>
<th>( s )</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>hot dogs</td>
<td>( h )</td>
<td></td>
<td>&gt; 25</td>
</tr>
<tr>
<td>hot dogs</td>
<td>( h )</td>
<td></td>
<td>≤ 40</td>
</tr>
<tr>
<td>super dogs</td>
<td></td>
<td>( s )</td>
<td>&gt; 30</td>
</tr>
<tr>
<td>Super dogs</td>
<td></td>
<td>( s )</td>
<td>≤ 50</td>
</tr>
<tr>
<td>sales</td>
<td>( h )</td>
<td>( s )</td>
<td>&lt; 80</td>
</tr>
</tbody>
</table>

- \( h \geq 25 \)
- \( h \leq 40 \)
- \( s \geq 30 \)
- \( s \leq 50 \)
- \( h + s \leq 80 \)
Step 3:

Graph the system of inequalities.

Step 4:

Locate the vertices of the feasible region.

The polygon formed is a quadrilateral with vertices at (25, 30), (40, 30), (40, 40), (30, 50), and (25, 50).
Step 5:

Determine the function used to maximize or minimize the values on the feasible region.

\[ f(h, s) = 0.80h + 1.30s \]

Step 6:

Evaluate the function at each vertex.

<table>
<thead>
<tr>
<th></th>
<th>Plug Vertices into Formula</th>
<th>( f(h, s) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(25, 30)</td>
<td>0.80(25) + 1.30(30)</td>
<td>$59.00</td>
</tr>
<tr>
<td>(40, 30)</td>
<td>0.80(40) + 1.30(30)</td>
<td>$71.00</td>
</tr>
<tr>
<td>(40, 40)</td>
<td>0.80(40) + 1.30(40)</td>
<td>$84.00</td>
</tr>
<tr>
<td>(30,50)</td>
<td>0.80(30) + 1.30(50)</td>
<td>$89.00</td>
</tr>
<tr>
<td>(25,50)</td>
<td>0.80(25) + 1.30(50)</td>
<td>$85.00</td>
</tr>
</tbody>
</table>

Step 7:

Determine the vertex that provides the optimal value.

If the vendor sells 30 hot dogs and 50 super dogs, he will make $89.00.
Linear Programming with Three Inequalities

Manufacturing: Machine A can produce 30 steering wheels per hour at a cost of $16 per hour; Machine B can produce 40 steering wheels per hour at a cost of $22 per hour. At least 280 steering wheels must be made in each 8-hour shift. What is the least cost involved in making 280 steering wheels in one shift?

Step 1:

**Determine the variables.**

\(a\): number hours on Machine A.
\(b\): number hours on Machine B.

---

Step 2:

**Write the systems of inequalities from the descriptions in the problem.**

<table>
<thead>
<tr>
<th></th>
<th>(a)</th>
<th>(b)</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steering wheels</td>
<td>30</td>
<td>40</td>
<td>(&lt; 280)</td>
</tr>
<tr>
<td>hours</td>
<td>(a)</td>
<td>(b)</td>
<td>(&lt; 8)</td>
</tr>
</tbody>
</table>

\[30a + 40b < 280 \quad a + b < 8\]
Step 3:

Graph the system of inequalities.

Step 4:

Locate the vertices of the feasible region

The polygon formed is a quadrilateral with vertices at (0, 7.5), (2, 6), and (8, 0).
Step 5:

Determine the function used to maximize or minimize the values on the feasible region.

\[ f(a,b) = 16a + 22b \]

Step 6:

Evaluate the function at each vertex.

\[
\begin{array}{|c|c|c|}
\hline
(a, b) & \text{Plug Vertices into Formula} & f(a, b) \\
\hline
(0, 7.5) & 16(0) + 22(7.5) & 165.00 \\
(2, 6) & 16(2) + 22(6) & 164.00 \\
(8, 0) & 16(8) + 22(0) & 128.00 \\
\hline
\end{array}
\]

Step 7:

Determine the vertex that provides the optimal value.

If the run Machine A for 8 hours and Machine B for 0 hours, you will incur a cost of $128.00
Business: Ingrid is planning to start a business. She will be baking decorated cakes and specialty pies. She estimates that a decorated cake will take 75 minutes to prepare and a specialty pie will take 30 minutes to prepare. She plans to work no more than 40 hours per week and does not want to make more than 60 pies in any one week. If she plans to charge $34 for a cake and $16 for a pie, find a combination of cakes and pies that will maximize her income for a week.

Step 1:

Determine the variables.

- \( c \): number of cakes
- \( p \): number of pies

Step 2:

Write the systems of inequalities from the descriptions in the problem.

<table>
<thead>
<tr>
<th></th>
<th>75</th>
<th>30</th>
<th>(&lt; 40*60=2400)</th>
</tr>
</thead>
<tbody>
<tr>
<td>time</td>
<td>75</td>
<td>30</td>
<td>(&lt; 2400)</td>
</tr>
<tr>
<td>pie for week</td>
<td></td>
<td>(p)</td>
<td>(&lt; 60)</td>
</tr>
</tbody>
</table>

\[ 75c + 30p < 2400 \quad p < 60 \]
Step 3:

Graph the system of inequalities.

Step 4:

Locate the vertices of the feasible region.

The polygon formed is a quadrilateral with vertices at (0, 60), (8, 60), (32, 0), and (0, 0).
Step 5:

Determine the function used to maximize or minimize the values on the feasible region.

\[ f(c, p) = 34c + 16p \]

Step 6:

Evaluate the function at each vertex.

\[
\begin{array}{|c|c|c|}
\hline
(c, p) & \text{Plug Vertices into Formula} & f(c, p) \\
\hline
(0, 60) & 34(0) + 16(60) & $960.00 \\
(8, 60) & 34(8) + 16(60) & $1232.00 \\
(32, 0) & 34(32) + 16(0) & $1088.00 \\
(0,0) & 34(0) + 16(0) & $0.00 \\
\hline
\end{array}
\]

Step 7:

Determine the vertex that provides the optimal value.

If Ingrid sells 8 cakes and 60 pies, she will make $1232.00 per week.
1. Identify the system of inequalities that represents the shaded area in the graph.

A. \[ \begin{align*} x - y & \geq -2 \\ y & \leq 4 \end{align*} \]
B. \[ \begin{align*} y - x & \geq -2 \\ y & \leq 4 \end{align*} \]
C. \[ \begin{align*} y - x & \geq -2 \\ y & \leq 4 \end{align*} \]
D. \[ \begin{align*} y - x & \geq -2 \\ y & \leq 4 \end{align*} \]

2. In honor of Thanksgiving, the M&M company produced bags of brown and green M&Ms. Each bag contains approximately 75 M&Ms. The number of brown M&Ms is twenty-four less than twice the number of green M&Ms. Let \( b \) represent the number of brown M&Ms and \( g \) represent the number of green M&Ms. Which linear system below represents this situation?

A. \[ \begin{align*} b + g & = 75 \\ g & = 2b - 24 \end{align*} \]
B. \[ \begin{align*} b + g & = 75 \\ b & = 2g - 24 \end{align*} \]
C. \[ \begin{align*} b + g & = 75 \\ g & = 2b + 24 \end{align*} \]
D. \[ \begin{align*} b + g & = 75 \\ b & = 2g + 24 \end{align*} \]

3. Choose the true statement if \( P = 4x - 2y \).
   A. The maximum profit is 22 at the vertex (4, -3).
   B. The maximum profit is –12 at the vertex (-2, 2)
   C. The minimum profit is 8 at the vertex (0.5, -3).
   D. The minimum profit is 12 at the vertex (4, 2).
4. A snack bar sells burritos and nachos during basketball games. To make a profit, the snack bar must sell at least 10 burritos but cannot prepare more than 40. It must also sell at least 30 nachos but cannot prepare more than 70. The snack bar cannot prepare more than a total of 90 burritos and nachos altogether. The profit is $0.95 on a burrito and $0.45 on a nacho. How many of each kind of snack should be sold to make the maximum profit?

a. Identify the variables

b. Write the system of inequalities from the description of the problem

c. Graph the system.

d. Find the maximum profit.
5. Identify the system of inequalities that represents the shaded area in the graph.

A. \[ \begin{cases} x - y \geq 2 \\ y \leq 4 \end{cases} \]
B. \[ \begin{cases} y - x \geq 2 \\ y \leq 4 \end{cases} \]
C. \[ \begin{cases} y - x \geq 2 \\ x \leq 4 \end{cases} \]
D. \[ \begin{cases} y - x \geq 2 \\ x \leq 4 \end{cases} \]

6. In honor of Thanksgiving, the M&M company produced bags of brown and green M&Ms. Each bag contains approximately 75 M&Ms. The number of brown M&Ms is twenty-four less than twice the number of green M&Ms. Let \( b \) represent the number of brown M&Ms and \( g \) represent the number of green M&Ms. Which linear system below represents this situation?

A. \[ \begin{cases} b + g = 75 \\ b = 2g - 24 \end{cases} \]
B. \[ \begin{cases} b + g = 75 \\ b = 2g - 24 \end{cases} \]
C. \[ \begin{cases} b + g = 75 \\ g = 2b + 24 \end{cases} \]
D. \[ \begin{cases} b + g = 75 \\ g = 2b + 24 \end{cases} \]

7. Choose the true statement if \( P = 4x - 2y \).
   
   A. The maximum profit is 22 at the vertex (4, -3).
   B. The maximum profit is –12 at the vertex (-2, 2)
   C. The minimum profit is 8 at the vertex (0.5, -3).
   D. The minimum profit is 12 at the vertex (4, 2).
A snack bar sells burritos and nachos during basketball games. To make a profit, the snack bar must sell at least 10 burritos but cannot prepare more than 40. It must also sell at least 30 nachos but cannot prepare more than 70. The snack bar cannot prepare more than a total of 90 burritos and nachos altogether. The profit is $0.95 on a burrito and $0.45 on a nacho. How many of each kind of snack should be sold to make the maximum profit?

c. Identify the variables

\[ b = \text{burritos} \quad n = \text{nachos} \]

d. Write the system of inequalities from the description of the problem

\[
\begin{align*}
    b &\geq 10 & b &\leq 40 \\
    n &\geq 30 & n &\leq 70 \\
    b + n &\leq 90
\end{align*}
\]

e. Graph the system.

f. Find the maximum profit.

\[ f(b,n) = 0.95b + 0.45n \]

The maximum profit is earned by selling 40 burritos and 50 nachos.