Title: Defining Division

Brief Overview:

Students will explore the use of fractions as a powerful strategy for constructing their own division strategies. They will develop their understanding of fractions as parts of unit wholes, as parts of a collection, and as divisions of whole numbers to solve multi-step problems. Throughout the problem-solving process students will develop mathematical arguments and proofs for their solutions as well as analyze and evaluate the mathematical thinking and strategies of each other. Finally, students will construct their own understandings for how fractions are connected to the function of division.

NCTM Content Standard/National Science Education Standard:

Number and Operations
Reasoning
Communication
Connections

Grades/Levels: 3-5

Duration/Length:

3-4 days for 50-60 minutes daily; 1 day will be used for assessment

Student Outcomes:
Students will:

- Identify fraction concepts in relationship to division concepts by comparing part/part whole relationships.
- Identify quotients and remainders mathematically in the context of a problem.
- Support or refute mathematical statements or solutions.

Materials and Resources:

Lesson 1

- Paper bags (1 per each pair)
- Hershey bars (fun size, regular, and movie theater size)
- Brown crayon (1 per student)
- Calculators (1 per student)
- Hershey bar sheet (Student Resource 1) (1 per student)
- Overhead transparency of Hershey Bar Sheet (Student Resource 1).
- Paper Hershey bars for lesson 1 (Student Resource 2)
- Share Fairly! Record Sheet (Student Resource 3) (1 for each pair)
- Overhead transparency of Share Fairly! Record Sheet (Student Resource 3)

- Base ten blocks (10 rods, 15 units for each pair)
- Overhead base ten blocks (1 set)
- Observation Checklist (Teacher Resource 2)
- Overhead transparency of Hershey Bar Survey (Teacher Resource 1)

Lesson 2

- Paper bags (1 per each pair)
- Calculators (1 per student)
- Paper Hershey bars for lesson 2 (Student Resource 4)
- Base ten blocks (10 rods, 15 units for each pair)
- Overhead base ten blocks
- Share Fairly! Record Sheet (1 for each pair) (Student Resource 5)
- Overhead transparency of Share Fairly! Record Sheet (Student Resource 5)
- A Remainder of One, by Elinor J. Pinczes.
- Internet access

Lesson 3

- Base ten blocks (4 flats, 10 rods, 15 units for each pair)
- Overhead base ten blocks (1 set)
- Calculators (1 per student)
- Chart paper
- Marker

Advanced Preparation for the Share Fairly Game:

Since this game will be played during two of the lessons you will create two versions. Lesson one's version of the game strictly deals with numbers that are divisible by 2. To prepare this version copy and cut enough sets of Student Resource 2 so that each partnership will have one set. Put each set into a paper bag. Lesson two's version of the game involves numbers that are not divisible by 2. Again, to prepare this version copy and cut enough sets of Student Resource 4 so that each partnership will have one set. Put each set into a paper bag.

Development/Procedures:

Lesson 1

Preassessment –

- Write the following journal prompt on the chalkboard or overhead projector:

  Is a half always equal to a half? Use numbers, words, and/or drawings to explain your thinking.

- Instruct students to write their answers in a journal. Allow students about 7 minutes to answer the question on their own. Carefully observe students as they

complete the task. Look for students who display confidence in solving the problem as well as students who struggle. Reassure students that you do not expect them to know the answer and that they will have an opportunity to change their answers at another time. After seven minutes, ask for volunteers to share their thinking. On the chalkboard, or on chart paper, record three students' thinking. (Save this to refer to at the beginning of the lesson 2.)

Launch –

- Display 3 different sizes of Hershey chocolate bars. (fun, regular and movie theater size bars) Tell students to point to the chocolate bar that they would want to share with a friend. Instruct students to discuss why they chose a particular chocolate bar with partner. Survey the students by asking them to raise their hands if they chose the fun size bar. Tally the results on the Hershey Bar Survey Form (Teacher Resource 1). Continue to survey students until you have tallied all of the students' choices. The expected result of the data is that most, if not all, students chose the movie theater-sized Hershey bar. Ask students to explain why they think most students would choose this bar. Students should be able to explain that the half of the largest candy bar is more than halves of the other two candy bars (fun size and regular).

Teacher Facilitation –

- After discussing with students the idea that the size of the candy bars can help us determine which half is the biggest, ask "How much is each half of each of the bars? How could we find out?" Allow students time to think and rehearse their thinking with a partner. Then, ask students to share their thinking.

- Pass out one Hershey Bar Sheet (Student Resource 1) to each of the students and instruct them to take out a brown crayon.

- Display a transparency of Hershey Bar Sheet (Student Resource 1). Tell students that this drawing represents the movie theater candy bar without the wrapper.

- Ask, "How much is ½ of this candy bar?" Instruct students to shade the area on the bar and to hold up their papers once they have completed the task.

- Ask, "What helped you to decide how much to color of the candy bar?" Students need to explain that the number of parts of the whole and how many people are sharing will determine the value of ½ for the candy bar.

- Write the following equations on the transparency.

  16 ÷ 2 = 8 **16/2=8** **½ of 16 is 8** **½ x 16 = 8**

- Ask students to compare the equations and determine if one or more of them describe how we determine what half of the candy bar was.

- Engage students in a discussion of what they noticed regarding similarities and differences of the equations and lead them to discover that all of the equations have the same meaning. Develop the idea that a fraction can be thought of as division of the numerator by the denominator. (a/b = a ÷b)

- Repeat the above activity with students for the regular-sized Hershey bar. Guide students to develop four equations for determining the value of 1/2 of this bar.

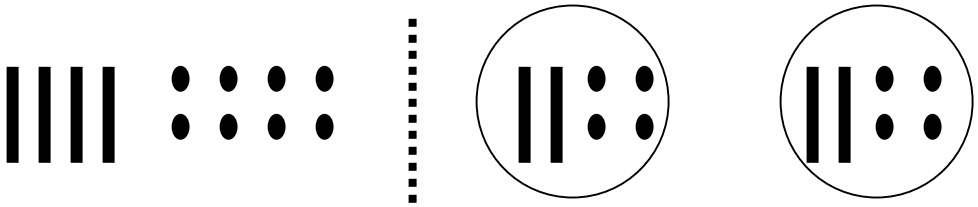
- Through questioning, construct with students the meaning of how using the fraction of half helps students to understand that the divisions of a whole are fractional parts.

  Question Stems
  - How do you know …
  - Tell me more about …
  - How could you show me …

Student Application –

- Model how to play the "Share Fairly Game" for students. Begin by removing a paper Hershey bar from a paper bag. Tell students that the number on each of the bars tells them the number of parts for each bar. For example, if the number 48 is written on the bar there are 48 pieces or parts that make up the whole candy bar.

- Using transparent base ten blocks, construct 48 (4 rods, 8 units).



- Ask, "How can we share this candy bar fairly with a partner?" Allow students to construct 48 with their base ten blocks and work with a partner to equally share the number of pieces of this bar.

- Give each pair of 2 students the Share Fairly! Record Sheet (Student Resource 3)

- Model for students how to complete the table on the Share Fairly! Record Sheet (Student Resource 3) and to play the game.

Trial	Value of Hershey bar	Value of each share	Equation
1	48	24	48/2 = 24 or 48 ÷ 2 = 24 ½ or 48 is 24 ½ x 48 = 24

Embedded Assessment –

- While students are playing the game observe and record behaviors on the observation checklist (Teacher Resource 2). Share with students that you will assess their performance as they play the game. Discuss with students the observable behaviors you will be looking for and establish criteria with the students on what these behaviors may look and sound like.

Reteaching–

- If a student doesn't understand the concept, play a few rounds of the game with them in a small group while other students work with a partner or independently. Continuously question students as they construct their own meanings for fractions and division.

Extension -

- Students who understand the lesson should create a model, or diagram, to explain the idea that fractions are divisions of a whole number.

Lesson 2

Preassessment –

- Rewrite the journal prompt from lesson 1 on the chalkboard or overhead projector:

  Is a half always equal to a half? Use numbers, words, and/or drawings to explain your thinking.

- Instruct students to reflect on their previous response and evaluate their original thinking. Students need to decide if they think they need to revise their original answer and to explain why they made these revisions. Allow students about 7 minutes to write their reflections. Once again, observe students as they complete the task. After seven minutes, ask for volunteers to share how their thinking changed and why. Through discussion, help students draw the conclusion that new learning helps us to revise our thinking.

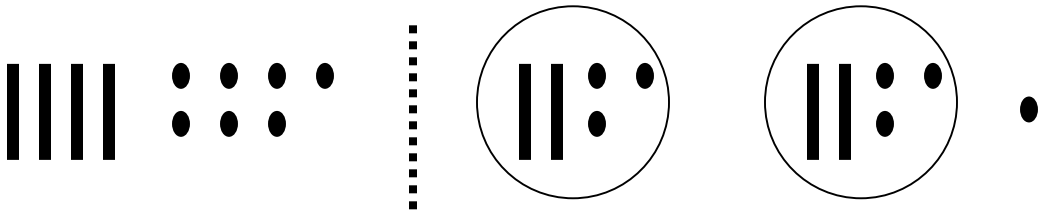
Launch –

- Read the story, A Remainder of One, by Elinor J. Pinczes. Discuss examples/circumstances when it is hard to share amounts equally. Before reading the story, have students predict how quotients would be different if all of the dividends were odd numbers. Ask, "What if candy bars were made with an odd number of sections?" The students will predict how the value of their portion is going to change from yesterday. **Essential Question:** If two people share a candy bar, is each person going to receive an equal amount of candy?

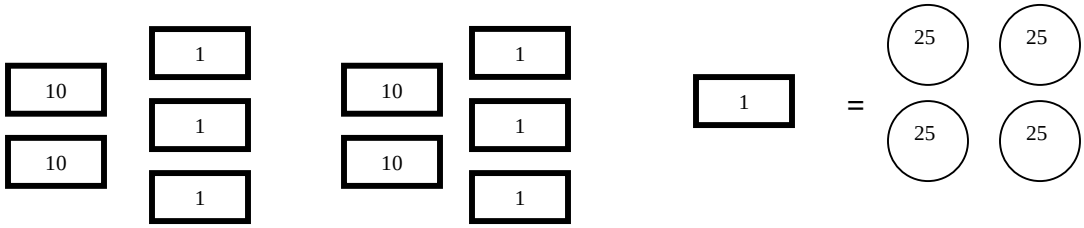
Teacher Facilitation –

- Model how to play version two of the "Share Fairly Game" for students. Begin by removing a paper Hershey bar (Student Resource 4) from a paper bag. Remind students that the number on each of the bars tells them the number of parts for each bar. For example, if the number 47 is written on the bar there are 47 pieces or parts that make up the whole candy bar.

- Ask, "How can you share this candy bar fairly with your partner so that each of you gets an equal share?" Instruct students to construct 47 with their base ten blocks and work with a partner to fairly divide the pieces of the candy bar.



- Ask students to share their solutions. Help students to explain the idea of a remainder. Have students decide what should be done with the remainder. Direct students to recognize that the one remainder is a whole. If students continue to have difficulty understanding the concept, relate it to money. For example, ask students to think of all the different ways that 47 could be written as money. ($47, $47.00, etc) Then draw a picture of 47 dollars, or use overhead bills and coins. With students decide how much is ½ of $47.00.



- When students understand that the one-dollar can be exchanged for 4 quarters and that ½ of $47 is $23.50, ask "What fraction of a dollar is 50 cents?"

- Brainstorm possible equations that mathematically explain how they solved the problem.

  ½ of 47 is 23 ½ **47 ÷ 2 = 23 r 1** **47/2 = 23 ½** **47 ÷ 2 = 23.5**

Student Application –

- Students play the revised version of the Share Fairly! Game with a partner.

- Give each pair a bag of Hershey bars that have odd numbers on them and Share Fairly! Record Sheet (Student Resource 5).

- If necessary, model for students how to complete the table on the Share Fairly! Record Sheet (Student Resource 5).

Embedded Assessment –

- While students are playing the game observe and record behaviors on the observation checklist (Teacher Resource 2). Again watch at least one-third of the students in your classroom. Review the established criteria for behaviors you expect to hear and see while students work together. Revise as needed.

Reteaching/Extension -

- Work with struggling students in a small group to play a few rounds of the game until their understanding is fully developed.

- Early finishers may visit the web site: Interactive Mathematics: http://matti.usu.edu/nlvm/nav/frames_asid_193_g_2_t_1.html

Lesson 3

Preassessment –

- Write the following problem on the chalkboard or overhead projector.

  Michael made 342 cookies. He gave one-third of the cookies to the school to sell in the PTA Bake Sale. He gave three-fourths of what was left to his family. How many cookies did Michael save for himself?

DO NOT SHARE WITH STUDENTS!
Solution for the teacher – Michael saves 57 cookies for himself.

1/3 of 342 is 114 (342 ÷ 3 = 114)

114	114	114

228 cookies are left over

114 + 114 = 228 or 114 x 2 = 228

114	**228**

¾ of 228 is 171

114	**57**	**57**	**57**	57

57 + 57 + 57 = 171 or 57 x 3 = 171

Michael saves 57 cookies for himself.

114	57	57	57	**57**

- Put students into groups of 2 to solve the following problem.

- Encourage students to use strategies to solve the problem by providing a variety of manipulatives (base ten blocks, calculators, paper, etc.)

- Observe students and ask questions while they are working. Look for students who understand that one-third of 342 is 342/3 or 342 ÷ 3. Students may have difficulty understanding what is meant by "He gave three-fourths of what was left to his family." Encourage them to use base ten blocks to create models representing the cookies or draw pictures. It is possible that the majority of the students will not be able to solve the problem at this time. This is good!

Launch –

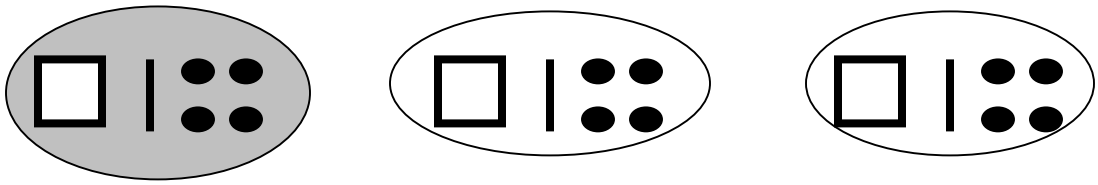
- After students have had an opportunity to develop solutions, or partial solutions, ask for volunteers to share. Record students' solutions on the overhead projector, or chart paper, as an anchor for future discussion.

Teacher Facilitation –

- Focus discussion on understanding the problem. Ask, "What does the phrase *"one-third of the cookies"* mean mathematically?" Write student responses.

Possible responses are 1/3 of 342, 1/3 x 342, 342 ÷ 3, 342/3. If students cannot explain this idea construct a model with base ten blocks. Ask, "How many cookies did Michael bake?" With students, construct 342. (3 flats, 4 rods, 2 units)

- Then, ask "What does one-third mean?" (One-third means to divide 342 into 3 equal sets.) With students, divide 342 into 3 sets.



- Ask: "How many of the 342 cookies is one-third? How do you know?" Allow students to share their thinking with a partner or group. Listen for students who justify their thinking with mathematical reasoning and proof.

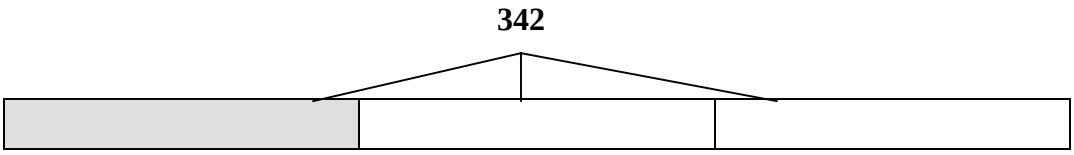
- Record responses and ask students to evaluate each others thinking.

- Ask: "How are fractions and division related?" and "How can fractions help us to solve division problems?" (To understand what it means to divide is to understand that the quotient is the result of dividing equally the numerator (dividend) by the denominator (divisor).

- Continue to develop this understanding by designing a diagram to show another representation of the problem. Draw a long rectangle and divide it into 3 equal parts. Instruct students to do the same and to color one-third.

- Ask, "If this entire rectangle's area is 342, then what is the value of one-third?

Possible diagram:



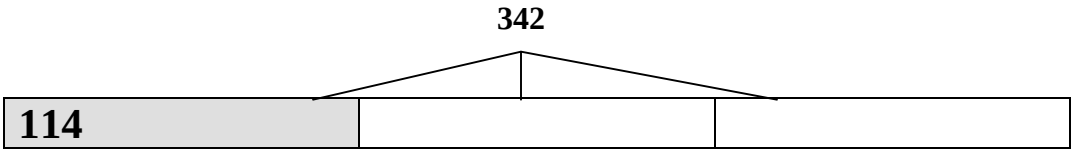
- Use a Math Explorer overhead calculator to solve this part of the problem. Give students calculators to perform the task with you.

- Enter 1/3 x 342 while saying "one-third of 342."

- Enter = and the display will read 342/3. Ask students to share their thinking and discuss the connections they have made. When they have determined that 342/3 is also 342 ÷ 3, perform the operation on the calculator. (342 ÷ 3 = 114)

- Instruct students to write the quotient in the part they shaded on the rectangle.

Possible diagram:



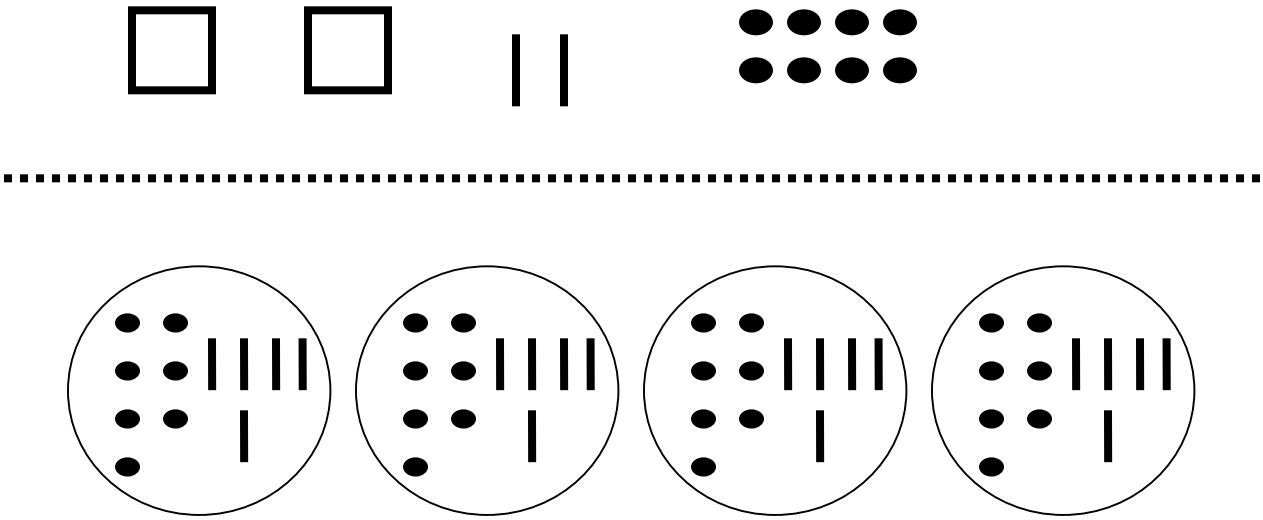
- Continue to focus discussion on understanding the problem. Ask, "What is the mathematical meaning of the sentence, *"He gave three-fourths of what was left to his family."*

- Guide students to determine the value of the number of cookies that were left. Allow students to solve with a partner. When students have determined that there were 228 cookies left, ask volunteers to either show or tell how they solved. (Students need to explain that the other two parts of the above diagram are both equal to 114 and 114 doubled is 228.)

Student Application –

- Instruct students to solve the problem with a partner and to construct models, diagrams and/or equations to explain their thinking.

Possible model:



Possible diagrams with equations:

1/3 of 342 is 114 (342 ÷ 3 = 114)

114	114	114

228 cookies are left over 114 + 114 = 228 or 114 x 2 = 228

114	**228**

¾ of 228 is 171

114	**57**	**57**	**57**	57

57 + 57 + 57 = 171 or 57 x 3 = 171

- Ask: "Do we know how many cookies Michael gave his family?" "How do we know?" Listen for solid reasoning by the students. They need to be able to fully explain their answer. Engage students in discussions in which they evaluate the solutions and strategies of their classmates.

- Ask: "Do we know how many cookies Michael saved for himself?" "How do we know?" Again, listen for solid reasoning by the students. They need to be able to fully explain their answer.

Embedded Assessment -

- While students are working to solve the problem, observe and complete the teacher checklist for the remaining third of your students. In addition, question students to determine their levels of understanding.

Reteaching -

- Pull students to a small group. Instruct students to count 15 counters.

- Ask, "If 15 counters are a whole set, how many counters make one-fifth?" "How could we find out?" Write 1/5 of 15 on a white board or chalkboard. Identify if students understand that the denominator determines the number of new sets. Once the students have determined that one-fifth of 15 is 3 perform the task on the calculator. Repeat this activity as often as need with different amounts of counters.

Extension –

- Challenge students to determine the fraction that reflects how many cookies Michael saved for himself.

- Ask, "What fraction of the cookies does Michael save for himself?"

57	57	57	57	57	57

- The answer is 1/6.

Summative Assessment –

Students will demonstrate an understanding of fractions as parts of unit wholes, as parts of a collection, and as divisions of whole numbers by completing two brief constructed responses and a diagram (Student Resource 6). They will need to use appropriate mathematical reasoning to justify their thinking. Answer key is on Teacher Resource 3.

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Hershey Bar Survey

Fun Size	Regular Size	Movie Theater Size

Student Resource 1

Movie Theater-Sized Bar

HERSHEY’S	HERSHEY’S	HERSHEY’S	HERSHEY’S
HERSHEY’S	HERSHEY’S	HERSHEY’S	HERSHEY’S
HERSHEY’S	HERSHEY’S	HERSHEY’S	HERSHEY’S
HERSHEY’S	HERSHEY’S	HERSHEY’S	HERSHEY’S

Regular-Sized Bar

HERSHEY’S	HERSHEY’S	HERSHEY’S	HERSHEY’S
HERSHEY’S	HERSHEY’S	HERSHEY’S	HERSHEY’S
HERSHEY’S	HERSHEY’S	HERSHEY’S	HERSHEY’S

Hershey's 52	Hershey's 74	Hershey's 86
Hershey's 34	Hershey's 88	Hershey's 90
Hershey's 76	Hershey's 36	Hershey's 92

Share Fairly! Game

Version I

Object of the game: To complete as many trials as possible in 8 minutes.

Materials:

- ✓ 1 paper bag with Hershey bar cut outs
- ✓ Set of base ten blocks
- ✓ Pencil
- ✓ Paper

Directions:

1) Pull a Hershey bar from the paper bag.
2) Construct a model for the value of the Hershey bar using your base ten blocks
3) Record your data in the table below.

Trial	Value of Hershey bar	Value of each share	Equation
1			
2			

Think and Reflect:

How did you find equal shares?

Hershey's 57	Hershey's 75	Hershey's 81
Hershey's 33	Hershey's 87	Hershey's 91
Hershey's 73	Hershey's 39	Hershey's 97

Share Fairly! Game

Version II

Object of the game: To complete as many trials as possible in 8 minutes.

Materials:

- ✓ 1 paper bag with Hershey bar cut outs
- ✓ Set of base ten blocks
- ✓ Pencil
- ✓ Paper

Directions:

4) Pull a Hershey bar from the paper bag.
5) Construct a model for the value of the Hershey bar using your base ten blocks
6) Record your data in the table below.

Trial	Value of Hershey bar	Value of each share	Equation
1			
2			

Think and Reflect:

How did you find equal shares?

Did you need to rename or regroup your base ten blocks? Why?

If you had extra units, or remainders, what did you do?

Summative Assessment

Name_____ Date _____

Assessment

1. Jeremy has 18 feet of rope. He cuts off one-fifth of the rope to make a jump rope for his sister. How much rope did Jeremy use to make the jump rope?

   A. Write at least 2 equations to show how to solve the problem.

   _____

   B. Using what you know about fractions and division, explain why your answer is correct. Use words, numbers, and/or diagrams to support your response.

_____

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2. Pat picked 15 apples off of the tree. She put three-fifths of the apples into one basket. How many of the 15 apples are in the basket?

   A. Solve. Show your work. _____ apples are in the basket

   B. Use what you know about fractions to explain how you know that your answer is correct. Fully explain your thinking using words, numbers, and/or symbols.

   In addition determine how many apples are not in the basket and explain how you know.

_____

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Summative Assessment

Assessment Key

1. Jeremy has 18 feet of rope. He cuts off one-fifth of the rope to make a jump rope for his sister. How much rope did Jeremy use to make the jump rope?

A. Write at least 2 equations to show how to solve the problem.

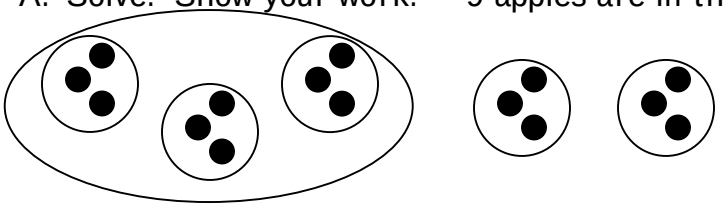
1/5 x 18 = 3.6, 18/5, 3 3/5
18 ÷ 5 = 3.6, 18/5, 3 3/5
18/5 = 3.6, 18/5, 3 3/5
1/5 of 18 = 3.6, 18/5, 3 3/5

B. Using what you know about fractions and division, explain why your answer is correct. Use words, numbers, and/or diagrams to support your response.

Score	Criteria
2	✓ Effective use of a strategy to correctly solve the problem ✓ Applied mathematical thinking and reasoning correctly ✓ Used appropriate vocabulary to fully explain the solution ✓ Used numbers, words, and/or diagrams
1	✓ Used a strategy to solve the problem ✓ Attempted to apply mathematical thinking and reasoning ✓ Used appropriate vocabulary to fully explain the solution ✓ Used numbers, words, and/or diagrams
0	✓ Ineffective use of a strategy to solve the problem correctly ✓ No attempt to apply mathematical thinking and reasoning ✓ Left the answer blank

2. Pat picked 15 apples off of the tree. She put three-fifths of the apples into one basket. How many of the 15 apples are in the basket?

   A. Solve. Show your work. 9 apples are in the basket



   B. Use what you know about fractions to explain how you know that your answer is correct. Fully explain your thinking using words, numbers, and/or symbols.

   In addition determine how many apples are not in the basket and explain how you know. (6 apples were not in the basket)

Score	Criteria
3	✓ Very effective use of a strategy to correctly solve the problem ✓ Applied mathematical thinking and reasoning correctly ✓ Used appropriate vocabulary to fully explain the solution ✓ Used numbers, words, and/or diagrams effectively
2	✓ Effective use of a strategy to correctly solve the problem ✓ Applied mathematical thinking and reasoning correctly ✓ Used appropriate vocabulary to somewhat explain the solution ✓ Used numbers, words, and/or diagrams
1	✓ Used a strategy to solve the problem ✓ Attempted to apply mathematical thinking and reasoning ✓ Used appropriate vocabulary to fully explain the solution ✓ Used numbers, words, and/or diagrams
0	✓ Ineffective use of a strategy to solve the problem correctly ✓ No attempt to apply mathematical thinking and reasoning ✓ Left the answer blank