Title: Tweaking a Trigonometric Function -- An Exploratory Lesson

Link to Outcomes:

- **Problem Solving** The students demonstrate the ability to predict changes in trigonometric graphs due to changes in equations.
- **Communication** The students will be able to communicate to other students the results given by the calculator.
- **Reasoning** The students will be able to predict future phenomena from previous examples.
- **Technology** The students will use graphing calculators to investigate changing the parameters in trigonometric equations.
- **Cooperation** The students will work in groups of three with one calculator per group. Each member of the group will have a designated task: operator, report writer, and group manager. Students will use a calculator as a tool to investigate the changing trig equations.

Brief Overview:

This lesson will demonstrate the various translations of $y=A\sin B(x+C)+D$ and $y=A\cos B(x+C)+D$ based upon changes in A, B, C, and D. The graphing calculator's overlay graphs make it easy for students to detect, understand, and predict translations in periodic curves.

Grade/Level:

Grades 11th and 12th; Trigonometry/Trig-Analysis/Functions/Precalculus

Prerequisite Knowledge:

Students must have a basic knowledge of how to use a graphics calculator; i.e., how to input a function and how to generate the graph in an arbitrary viewing window. These are two of the four prerequisite skills required for the AP Calculus Examination.

Objectives:

- The student will be able to predict the resulting graph from any single change of parameter in a trigonometric function.
- Students in each group must confer and agree on the prediction of the above changes.
- Student groups must use graphing calculators to examine parameter changes.
- Each student will be required to perform a specific duty in compiling the discoveries of the group.

Materials/Resources/Printed Materials:

- TI-82 overhead projector (1)

Per group:

- TI-82 graphics calculator (1)
- Group organization sheet (1)
- Group exploration sheets (5)
- Group report sheet (1)

Development/Procedures:

For each of four translations of the graph of the trigonometry function:

$$y = A \sin B(x + C) + D$$

the teacher will demonstrate, using the list capability of the TI-82 calculator, how a change of parameter will affect the graph of the function. After the demonstration, student groups will explore and discover patterns of change in graphs by varying the target parameters of the worksheet. For each parameter, there is included a teacher demo sheet and a master group exploration sheet.

Students will be arranged into groups of three. Each group must work cooperatively to reach conclusions. By methodically varying the parameters of a trigonometry function, the students will generate and describe the patterns formed.

The learning unit can be used over a 3 to 5 day period, depending on the size and ability of the class. The slowest of pacing involves one demo and exploration per day, for four days, and group reports on day five. A more advanced class may do the first two demos/explorations on day one, the third on day two, and the fourth and the group report on day three.

Evaluation:

Student achievement will be based on the student report sheets for each group, plus knowledge gained by the teacher while moving about the room among the groups serving as a resource for individual students.

Extension/Follow Up:

In future lessons explore periodic real-world phenomena using the sine and cosine functions–sound waves, radio frequency, tides and hours of daylight.

Authors:

Kevin Caughlan	Patricia Myers	John Forrest
DeMatha Catholic H.S.	DeMatha Catholic H.S.	Suitland High School
Prince George's County	Prince George's County	Prince George's County

Group Name: _____
Group Manager: _____
Calculator Operator: _____
Data Recorder: _____
Calculator Number: #_____

GROUP ORGANIZATION SHEET
DUTIES:

GROUP MANAGER

Keeps the group focused on the task. Assists calculator operator to interpret calculator results. Assists recorder in final wording of report.

CALCULATOR OPERATOR

Inputs information into calculator. Helps interpret output of calculator.

RECORDER

Completes group exploration sheets and assists group in interpreting results for group report sheet.

GROUP NAME:_____

GROUP EXPLORATION SHEET

AMPLITUDE

$y = A \sin x$

Sketch the graph of:

$y = \sin x$

Sketch the curve which results from graphing each function:

$y = 3 \sin x$ $\quad\quad$ $y = -3 \sin x$
$y = 4 \sin x$ $\quad\quad$ $y = -4 \sin x$

Sketch the graph of:

$y = \cos x$

Sketch the curve which results from graphing each function:

$y = 5 \cos x$ $\quad\quad$ $y = -3 \cos x$
$y = 4 \cos x$ $\quad\quad$ $y = -4 \cos x$

Summarize how a change in the AMPLITUDE changes the graph of a trigonometric function.

GROUP NAME:_____

GROUP EXPLORATION SHEET

FREQUENCY

$y = A \sin Bx$

Sketch the curve which results from graphing each function:

$y = \cos 2x$
$y = \cos 3x$
$y = \cos 4x$

$y = \sin 2x$
$y = \sin 3x$
$y = \sin 4x$

Summarize how a change of FREQUENCY changes the graph of a trigonometric function.

Another property of a trigonometric function is its PERIOD. The period of a sine or cosine function is calculated by dividing 2π by the frequency of the trigonometric function. The period of a trigonometric function represents the width of one cycle of the curve. Period is crucial to know when you are graphing with paper and pencil. It is less crucial to calculator-assisted graphs.

Give the period of each of the following functions:

$y = 3 \sin 2x$
$y = -6 \cos 4x$
$y = 4 \sin 1/2x$

GROUP NAME:_____

GROUP EXPLORATION SHEET

PHASE SHIFT

$y = A \sin B(x + C)$

Sketch the curve which results from graphing each function:

$y = \sin (x + \pi/4)$
$y = \sin 3(x + \pi)$
$y = \sin 3(x + \pi/4)$

$y = \sin 3(x - \pi)$
$y = \sin 3(x - \pi/4)$

$y = \cos (x + \pi/4)$
$y = \cos 2(x + \pi/3)$
$y = \cos 2(x + \pi/4)$

$y = \cos 2(x - \pi/3)$
$y = \cos 2(x - \pi/4)$

Summarize how a change in the parameter added to/subtracted from x affects the graph of the function. Can you determine how that parameter affects the DISTANCE that the graph is shifted?

EXTRA

Can you describe the relationship between $y = \sin x$ and $y = \cos x$ which is exhibited by the following functions?

Graph each pair of functions and compare the resulting curves:

$y = 2 \sin x$
$y = 2 \cos (x - \pi/2)$

$y = 3 \cos x$
$y = 3 \sin (x - \pi/2)$

GROUP NAME:_____

GROUP EXPLORATION SHEET

VERTICAL SHIFT

$y = A \sin Bx + D$

Sketch the curve which results from graphing each function:

$y = \sin x + 1$
$y = \sin x + 1$
$y = \sin x + 2$
$y = \sin x + 3$

$y = \sin x - 1$
$y = \sin x - 2$
$y = \sin x - 3$

$y = \cos x + 1$
$y = \cos x + 1$
$y = \cos x + 2$
$y = \cos x + 3$

$y = \cos x - 1$
$y = \cos x - 2$
$y = \cos x - 3$

Summarize how a change in the constant affects the graph produced.

GROUP NAME:_____

GROUP REPORT SHEET

Given the following trigonometric function:

$$y = 3 \sin 4(x + \pi/4) + 1$$

write out, in your own words, what would happen to the graph of this function when:

the AMPLITUDE is changed from 3 to 4

the AMPLITUDE is changed from 3 to -2

the FREQUENCY is changed from 4 to 2

the FREQUENCY is changed from 4 to 8

$(x + \pi/4)$ is changed to $(x + \pi/3)$

$(x + \pi/4)$ is changed to $(x - \pi/3)$

the CONSTANT is changed from 1 to 3

the CONSTANT is changed form 1 to -2

Sketch the graph of the original function: $y = 3 \sin 4(x + \pi/4) + 1$

_____

GRADING

NEATNESS:_____
ACCURACY:_____
OVERALL UNDERSTANDING:_____

TEACHER GUIDE

AMPLITUDE

Using a TI-82 overhead projector or having students follow your directions, input into the function screen:

$$Y1 = \{1,2,3\} \sin X$$

Graph Y1, using zoom trig and radian, connected, and sequential mode. The ENTER key can be used to pause the graphing at any point. Discuss.

Input into the function screen:

$$Y2 = \{2, -2\} \sin X$$

Graph Y2 and discuss the results.

TEACHER GUIDE

FREQUENCY

Input into the function screen:

$$Y1 = \sin \{1,2,3\} X$$

Graph the function and discuss the results.

If you wish to discuss the concept of PERIOD, then it is interesting to use the ROOT function, found on the CALC screen, to demonstrate the period of a sine curve.

TEACHER GUIDE

PHASE SHIFT

Input into the function screen:

$$Y1 = \sin\ (X - \pi/4)$$

$$Y2 = \sin\ (X + \pi/3)$$

Graph each function in turn, discussing the direction of the shift briefly.

TEACHER GUIDE

VERTICAL SHIFT

Input the following functions into the function screen:

$$Y1 = \sin X + \{1,2,3\}$$

$$Y2 = \sin X - \{1,2,3\}$$

Graph the functions and briefly discuss the vertical shifting.

TRIGONOMETRY EXTENSION

Frequency—sound waves

Have a student bring in one of their music instruments. An oscilloscope would then be hooked up to a microphone and different pitches would be tried. For each pitch, one group of students will come up and carefully figure out the period of the curve. The frequency may now be found by using the equation $f = 2\pi/p$ where f is the frequency, and p is the period of the graph.

In place of an oscilloscope, the TI Calculator Based Laboratory (CBL) may be used.

Alternate Assessment

Trig Test-Part I
Chapter 2
50 Points
Using Calculators

Names_____
_____
_____
_____

I. Given the following trigonometric function:

ORIGINAL EQUATION: **y=sin (x+45)** SKETCH:

answer the following questions. (the group must agree on each answer)

1. What window do you need to see this graph?

2. Where does one cycle begin? _____ Where does it end? _____

3. Change the amplitude to 3. Do you need to change your window? _____. If so, what is your new window? _____ Max= _____ Min=_____ Describe in your own words what happens to the original graph when you make this change.

4. Change the amplitude to -2. Now what is the Max= _____ Min= _____ Where does this graph cross the y-axis? _____ Describe in your own words what happens to the graph from #3 when you make this change.

5. Go back to ORIGINAL EQUATION and change the frequency to 2. Where does the first cycle begin? _____ Where does the first cycle end? _____ Where does the graph cross the y-axis? _____

6. Go back to ORIGINAL EQUATION and change (x+45) to (x-60). Now where does the cycle begin? _____ Where does it end? _____ Describe what effect this change has on the original graph.

7. Go back to ORIGINAL EQUATION and add the constant +3 to the end of the equation. Sketch the new graph. Max= _____ Min= _____ Where does it cross the y-axis? Describe what effect this change has on the original graph.

Sketch:

II.

8. On your calculator, graph these two equations at the same time.

y = sin x	y = 2sin (x-30)

On the positive side of the x-axis where do these graphs intersect for the first time?

value of x _____ (nearest degree)

value of y _____ (nearest tenth)

Max of equation #2 _____

Min of equation #2 _____

Where equation #2 crosses

9. On your calculator, graph these two equations at the same time.

y = sin x + 2	y = 2sin (x-30)

What do you notice that is different about these two graphs compared to the graphs in problem 8.

Bonus

Enter the equation y = cot x into your calculator and graph it. Sine and cosine both have a period of 360° if the frequency is 1. What is the length of the period for this trigonometric function? _____ Max = _____ Min = _____ Sketch this graph.