

Title: Introduction to Research in Mathematics

Link to Outcomes:

- **Problem Solving** Students will study problems whose solutions are unknown. They will use problem solving skills to determine the best way to come to a solution quickly.
- **Communication** Students will be presented with several research problems and will be asked to select one. This learning unit covers one such research problem.
- **Reasoning** Students will investigate and gather information on a mathematical research problem.
- **Statistics** Students will collect, organize, and describe data. They will create tables and use these tables to make hypotheses.
- **Arithmetic Operations** Students will use arithmetic operations to generate sequences.
- **Patterns/ Relationships** Students will look for patterns in various sequences. Using these patterns, they will be able to make generalizations about given sequences.

Brief Overview:

This lesson provides a first look at research in mathematics by generating and examining sequences.

Grade/Level:

Grades 11-12

Duration/Length:

Actual class time is 2-3 days. Students will write a computer program outside of class to generate counting sequences.

Prerequisite Knowledge:

Students should be able to write computer programs and have a basic understanding of statistics. Students should also be able to work fairly independently and have an interest in experimentation. An article on counting sequences can be found in June-July 1994 *American Mathematical Monthly*.

Objectives:

The students will gain experience in the field of research mathematics.

Materials/Resources/Printed Materials:

- Access to a computer
- Worksheets

Development/Procedures:

- Ask students to fill in the blank: 2, 12, 1112, _____, 211213.
- Provide the answer, 3112, and explain that the sequence was generated in the following way:
 - A) Start with any number N (which does not contain 0 as a digit).
 - B) Count the number of times that each digit appears in N .
 - C) Beginning with the digit 1 and ending with 9, if a digit appears at least once, write down the number of times that the digit appeared and then the digit itself.
- Use this technique to generate the $(I+1)$ st element of the sequence from the i th element.
- Ask students to practice generating these sequences using the worksheets.
- Discuss interesting features of sequence A (the sequence eventually repeats with period one).
- Discuss interesting features of sequence B (the sequence eventually repeats with period two).
- Discuss interesting features of sequence C (the sequence eventually repeats with period three).
- Ask students to write a computer program which will generate sequences. Print out the number of iterations until the sequence is periodic and print out the period.
- Use the results of the program to fill out the table worksheet.

Evaluation:

When the project has been completed, students will compare tables and discuss other observations concerning properties of the sequences.

Extension/Follow Up:

Ask the students if any sequences were found that were not eventually periodic. What other periods besides 1,2, and 3 were found?

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Generating Counting Sequences (Solutions)

<u>Sequence A</u>	<u>Sequence B</u>	<u>Sequence C</u>
1	46	57
11	1416	1517
21	211416	211517
1112	31121416	31121517
3112	4112131416	4112131517
211213	5112132416	511213141517
312213	412213141516	611213142517
212223	512213241516	51221314151617
114213	413213142516	61221314251617
31121314	412223241516	51321314152617
41122314	314213241516	51222314251617
31221324	412223241516	41421314251617
21322314	314213241516	51221334151617
21322314	412223241516	51222314251617
21322314	314213241516	41421314251617
21322314	412223241516	51221324151617

Period and Iteration Data

Directions:

Use the program that generates sequences to complete the following table.

number of digits	average period	shortest period	longest period	average number of iterations	fewest number of iterations	greatest number of iterations
1						
2						
3						
4						

Period and Iteration Data (Solutions)

number of digits	average period	shortest period	longest period	average number of iterations	fewest number of iterations	greatest number of iterations
1	1	1	1	11.3	9	13
2	1.4	1	3	10.1	2	15
3	1.8	1	3	10.9	6	15
4	2.03	1	3	11.1	6	19