Title: How High are the Clouds?

Link to Outcomes:

Mathematics Outcomes

- **Problem-Solving** Students will demonstrate their ability to solve mathematical problems with open-ended answers, problems which are solved in a cooperative atmosphere, and problems which are solved with the use of technology.
- **Connections** Students will demonstrate their ability to connect mathematics topics within the discipline and with other disciplines.
- **Technology** Students will demonstrate their ability to solve problems using arithmetic operations with technology where appropriate.
- **Statistics** Students will demonstrate their ability to collect, organize, and display data and will interpret information obtained from displays.
- **Patterns & Relationships** Students will demonstrate their ability to recognize numeric relationships and will generalize a relationship from data.
- **Algebra** Students will demonstrate their ability to perform algebraic operations and will be able to model algebraic concepts using concrete materials.

Science Outcomes

- **Content** Students will demonstrate their acquisition and integration of major concepts and unifying themes from earth science.
- **Nature of Science** Students will demonstrate the ability to interpret and explain information generated by their exploration of scientific phenomena.
- **Habits of Mind** Students will demonstrate ways of thinking and acting inherent in the practice of science.
- **Attitudes** Students will demonstrate a positive attitude toward science and its relevance to the individual, society, and the environment and will demonstrate confidence in their ability to practice science.
- **Science Process** Students will demonstrate the ability to employ the language, instruments, methods, and materials of science for collecting, organizing, interpreting, and communicating information.

Brief Overview:

The students will gather data in an experiment in order to compute the dewpoint, the altitude of cloud formation, the altitude of ice crystal formation, and relative humidity. They will use the data to draw linear graphs and solve simultaneous equations to accomplish these goals. Even though the lesson recommends the use of the TI-81/82 graphics calculators, graphs can be represented on graph paper if calculators are not available.

Grade/Level:

Grades 7 - 12; Pre-algebra through Algebra II

Duration/Length:

This lesson is expected to take two or three 45-minute class periods.

Prerequisite Knowledge:

The students should be able to read a thermometer (or use CBL system with temperature probe), graph a linear function, and solve a system of two linear equations. The use of the TI-81/82 graphics calculators is an option.

Objectives:

- Identify rate of change as slope.
- Apply the concept of slope to graph a line.
- Solve a system of two linear equations by the graphing method.
- Interpret the solution in the context of the problem.

Materials/Resources/Printed Materials:

- Thermometer (or CBL system with temperature probe)
- Coffee cans (small colored cans, 12 oz., are the best to use)
- Water
- Ice
- Graph paper, pencil, and ruler
- TI-81/82 graphics calculator

Development/Procedures:

Teacher Preparation:

The class will be divided into groups of 2 or **3** students. Have one coffee can and one thermometer per group. Fill the coffee cans half full of water and place outside. They will need to be there long enough so that the water reaches air temperature before the students can perform the experiment. Hang a thermometer outside for the students to record the air temperature. Have available an ice chest filled with ice. The graphing calculators, graph paper, pencils, and rulers should be available in the classroom.

Activity 1: Collecting Data to Find the Dewpoint[1]

The teacher should explain to the class that dewpoint is the temperature at which condensation occurs. Examples of dewpoint are dew on the grass in the morning, and the condensation on a glass of cold soda on a humid day. The students will perform an experiment to find the dewpoint.

---

1 The measurements in this learning unit are taken from the National Oceanic and Atmospheric Administration.

Assign the following roles to group members: one student will be the recorder and reader of the thermometer, the second student will stir the water in the **can** and will add the ice, and the third will look for condensation.

Upon arriving outside, the recorder will record the air temperature. To begin the procedure for finding the dewpoint, a thermometer should be placed in the water and the temperature noted. The water temperature should be at or near the air temperature. One ice cube should be added to the water. The water should be stirred constantly so that the cooler water from the ice is evenly distributed. If the ice cube melts and no condensation has appeared on the outside of the can, another ice cube should be added and the procedure repeated until condensation appears. [Note: The students should be careful not to splash any water on the outside of the can when they put in the ice.]

When condensation occurs the thermometer should be immediately read and the temperature recorded. This temperature in the dewpoint [Note: Condensation occurs rapidly and the students need to be very observant in order to collect the most accurate data. A suggestion would be to have students run their finger over the outside of the can below the water level to test for condensation. When it occurs, a "trail" through the condensation wil1be visible.]

When the students are finished this activity, they will have two temperatures, the air temperature and the dewpoint.

Activity 2: Finding the Equations of the Linear *Functions*

Explain to the students that the air temperature drops 5.4 degrees F per 1000 feet altitude, and the dewpoint drops 1 degree F per 1000 feet. Theses rates of changes can be expressed as the slopes of linear functions.

The slope of the air temperature function is -5.411000 (or -.0054) and the slope of the dewpoint temperature function is -1/1000 (or -.001). The slope is negative because the temperature decreases as altitude increases.

The air temperature and the dewpoint are the y-intercepts of the two linear functions, respectively. They are the y-intercepts because the students found the air temperature and dewpoint at ground level (0 feet altitude).

The equations of the linear functions in slope-intercept form are:

or $y = -.0054x + a$ (a = air temperature)

or $y = -.001x + d$ (d = dewpoint)

Activity 3: Finding the Altitude of Cloud Formation

The solution to the system from Activity 2 will be an ordered pair with the x-coordinate representing the altitude of cloud formation and the y-coordinate representing the temperature at that altitude. [Note: At this altitude the air is completely saturated, thus the formation of clouds.] This system can be solved by the graphing method (on paper or on the graphing calculator) or any other simultaneous equation method depending on the level of the students. [Note: See sample graph and explanation.]

Enter the two equations into the graphing calculator (or graph them on the graph paper). For the TI-81, use the trace and zoom functions to find the intersection of the two lines. For the TI-82, use the intersect function under the Calc menu to find the intersection of the two lines. The value of the x-coordinate represents the altitude of the lowest cloud formation. The value of the y-coordinate represents the temperature at that altitude.

Activity 4: Finding the Altitude of Ice Crystal Formution

Explain to the students that once the air is saturated the drop in temperature as a function of altitude changes. The air temperature now drops 3.5 degrees F for every 1000 feet of altitude. A new linear equation must be written using the slope of -3.5/1000, and the coordinates found in Activity 3.

or

[where (x_1, y_1) are the coordinates found in Activity 3]

This equation should be entered and graphed on the graphing calculator. If using the TI-81, use the trace and zoom functions to find point whose x-coordinate is 32. If using the TI-82, graph the function y = 32 and use the intersect function to find the y-coordinate of the point of intersection. This is the altitude at which ice crystals form.

Activity 5: Finding the Relative Humidity

A Relative Humidity Data Sheet is provided that shows the amount of water vapor (in g I m^3) in the air at a given temperature in degrees F. Using the data sheet, find the amount of water vapor present at the air temperature and the dewpoint temperature from Activity 1. The ratio of the dewpoint temperature water vapor to the air temperature water vapor is the relative humidity, which is usually expressed as a percent.

Activity 6: Find the Equation of the Water Vapor Function (for advanced classes)

Use finite differences to find the degree of the polynomial function representing the water vapor in the air at a given temperature. [Note: See explanation of finite differences.] Using the values on the Relative Humidity Data Sheet, enter the data into the stat lists of the TI-82 graphics calculator and graph. Use the stat function and the fact that it is a polynomial equation of degree 4 to find the equation and graph.

Evaluation:

Given the dewpoint of 50 degrees F and the air temperature of 80 degrees F, the students will find the highest altitude a hot air balloon can reach before it enters a cloud. or

Extension/Follow Up:

Using the data from the Relative Humidity Data Sheet, use matrices to find the coefficients of the function in order to write its equation.

Investigate how the altitude of cloud formation affects air traffic control at an airport.

Name of Developer, Location, Phone Number:

Lisa Ceccarelli
Notre Dame Preparatory School
(410) 825-6202
Baltimore County

Pat Hemler
Randallstown H.S.
(410) 887-0748
Baltimore County

John Maienshein
Randallstown **H.S.**
(410) 887-0748
Baltimore County

Explanation of Sample Data

Sample for Activity ***2:***

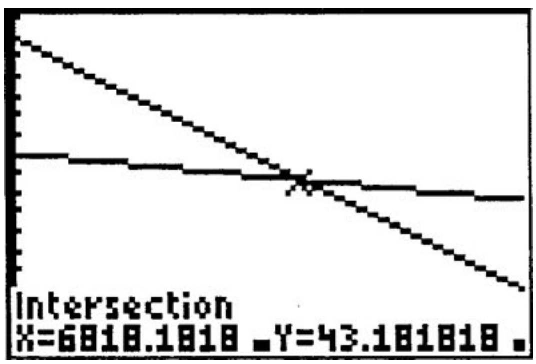
For the sample data, the air temperature is 80 degrees F and the dewpoint is 50 degrees F. The equations for this data are:

$$y = \frac{-5.4}{1000}x + 80 \quad \text{or} \quad y = .0054x + 80$$

$$y = \frac{-1}{1000}x + 50 \quad or \quad y = -.001x + 50$$

Sample for Activity 3:

The TI-82 graph appears below and a graph is included on the page: Altitude of Cloud Formation.



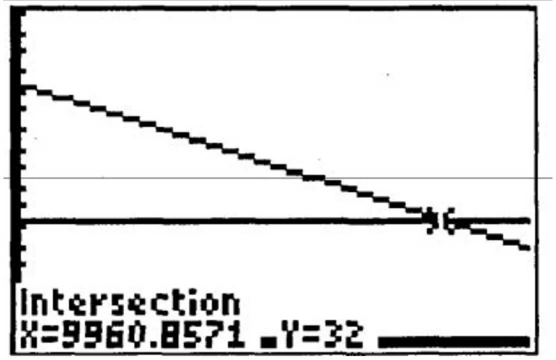
The solution to the simultaneous equations is (6818,43). Therefore, the altitude of cloud formation is 6818 feet and the temperature at that altitude is 43 degrees F.

Sample for Activity 4:

To find the altitude of ice crystal formation, the equation is:

$$y = \frac{-3.5}{1000}(x - 6818) + 43 \quad or \quad y = -.0035(x - 6818) + 43$$

The TI-82 graph appears below and a graph is included on the page: Ice Crystal Formation.



Ice crystals form at 9%0 feet altitude at a temperature of **32** degrees F (which is the temperature at which water freezes.)

Sample for Activity 5:

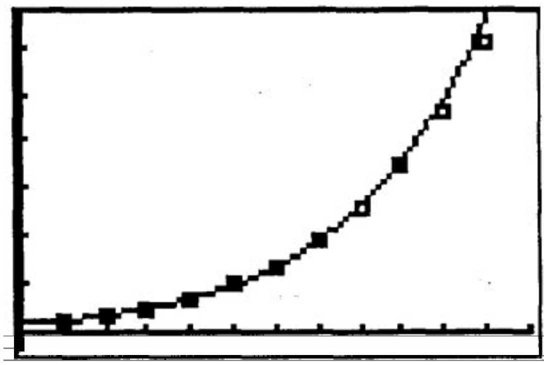
Using the Relativity Humidity Data Sheet, the ratio of the water vapor for 50 degree dewpoint to 80 degree air is 9.40 to 25.49 which equals **.37** or **37%** relative humidity.

Sample for Activity 6:

See Finite Differences explanation to find the degree of the function. It is a 4th degree polynomial. Then the stat function of the graphing calculators **can** be used to **find** the equation. The equation is:

$$y = .0000004\,x^4 - .00002\,x^3 + .002x^2 + .04\,x + 1.4$$

The TI-82 graph showing the data points and the graph of the function appears below.



Relative Humidity Data Sheet

Degrees F	Water Vapor
10	2.03
20	3.06
30	4.53
40	6.57
50	9.40
60	13.33
70	18.58
80	25.49
90	34.49
100	46.05
110	60.73

Note: Water vapor is given in g/m^3 dry air

Finite Differences

Given points that are solutions to a polynomial, the degree of the polynomial may be found using finite differences. Select the independent variables (x-coordinates) to have a constant difference. Then by taking the differences of the dependent variables (y-coordinates) the degree of the polynomial is found. If the differences between the dependent variables are constant the first time (call these differences d_1),the degree is one. If these differences are not constant, then find the differences of the values of d_1. If these differences (call these differences d_2) are constant, then the degree of the polynomial is two. Keep finding the differences until they are constant. The degree is equal to the level of differences.

Example of a Quadratic Equation: $y = x^2$

x values:	1		**2**		**3**		**4**		**5**		**6**	
y values:	**1**		4		9		16		25		**36**	
d1		**3**		*5*		7		**9**		11		
d2			2		2		2		2			

Since the differences were constant at the second level, this confirms that the polynomial has degree two.

Example: Find the degree of the polynomial with the following values.

x values:	**2**		4		6		**8**		10
y values:	9		65		217		**513**		**1001**
d_1		56		**152**		**296**		488	
d_2			96		**144**		**192**		
d_3				**48**		*48*			

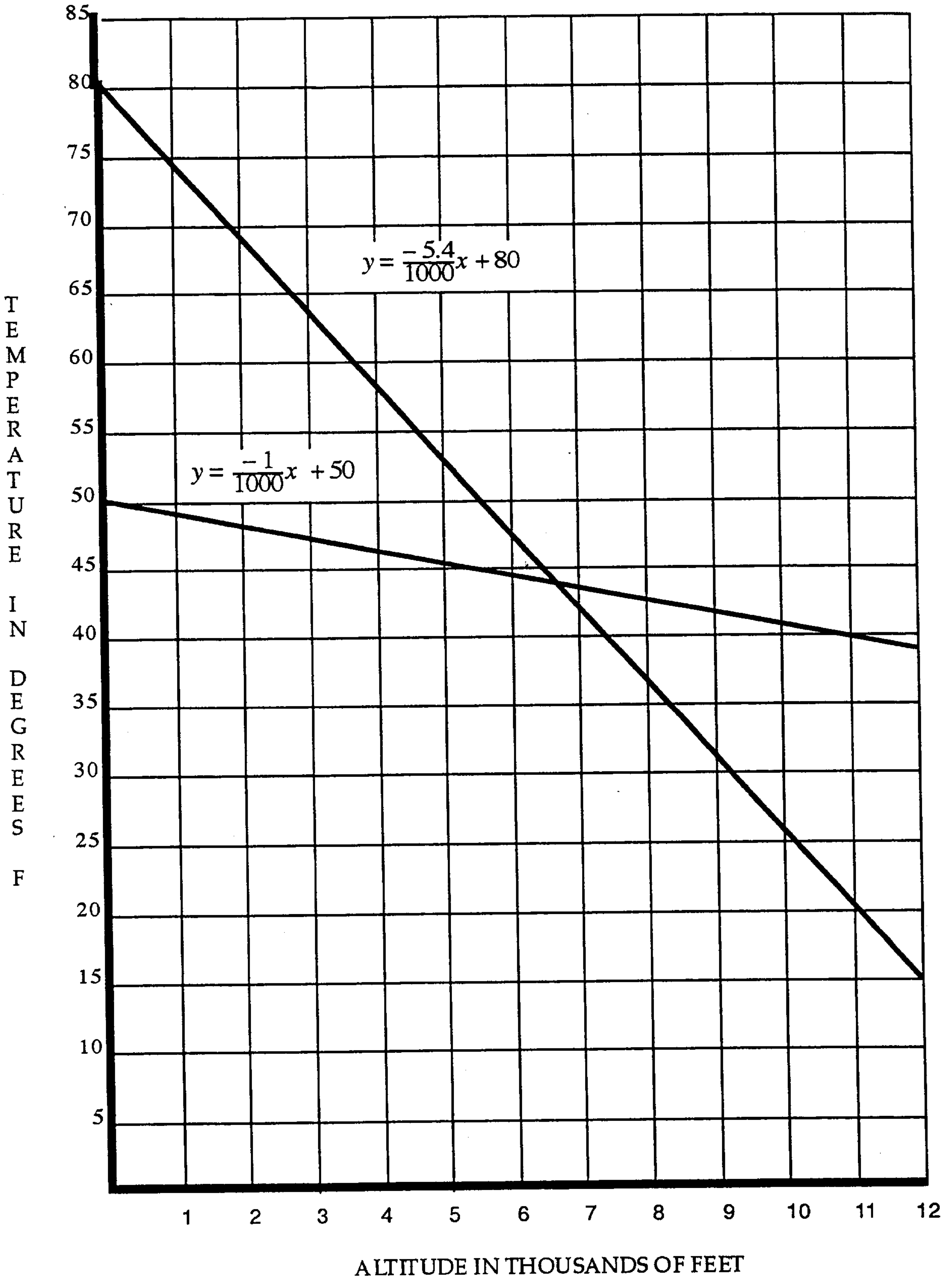
The degree is **3** since the differences were constant at the third level.

Now, use finite differences to find the degree of the function represented by the data on the Relative Humidity Data Sheet.

x values:	**10**		20		30		40		50		60
y values:	**2.03**		3.06		4.53		6.57		9.40		13.33
d_1		1.03		1.47		2.04		2.83		3.93	
d_2			.44		.57		.79		1.1		
d_3				.13		.22		.31			
d_4					.09		.09				

Therefore, the degree is 4.

Altitude of Cloud Formation



The Altitude of Ice Crystal Formation