Title: Banneker Park

Brief Overview:

Students will construct and analyze circles, discovering and investigate the formulas for arc length and area of the sector of a circle. The unit assessment will reflect applications of arcs related to a traffic round-about. The students will use Geometer's Sketchpad, TI-92 plus calculators (with Geometer's Sketchpad) to create the constructions and solve problems.

NCTM 2000 Principles for School Mathematics:

- **Equity:** *Excellence in mathematics education requires equity - high expectations and strong support for all students.*
- **Curriculum:** *A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.*
- **Teaching:** *Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.*
- **Learning:** *Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.*
- **Assessment:** *Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.*
- **Technology:** *Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.*

Links to NCTM 2000 Standards:

- **Content Standards**

<u>Number and Operations</u>
Students will calculate the area, circumference, and arc length of a circle. They will use the appropriate approximate for Pi (3.14) to find solutions to a real-life situation.

<u>Algebra</u>
Students will use area and perimeter formulas to calculate parts of the round-about.

<u>Geometry</u>
Students will analyze the characteristic and properties of circles. They will develop a geometric model to solve a word problem.

Measurement

Students will measure arcs using degrees, arc length using feet, and area using square feet. They will use appropriate measurements and formulas for the proposed problem.

- **Process Standards**

Mathematics as Problem Solving, Reasoning and Proof, Communication, Connections, and Representation

These five process standards are threads that integrate throughout the unit, although they may not be specifically addressed in the unit. They emphasize the need to help students develop the processes that are the major means for doing mathematics, thinking about mathematics, understanding mathematics, and communicating mathematics.

The students have a hypothetical situation for computing the measurement and designing a city intersection that is in the form of a circle. The students' preparation should be to use the correct formula and calculations, as well as explain the situation in writing. They should be able to express verbally and in a written form the problem using mathematical terminology specifically related to the problem. They also should be able to relate this lesson and identify the components used in another class not related to mathematics. There should be group efforts displayed in the classroom environment. Students should be able to model and construct information that is pertinent to the class assignment. There should be a display of understanding through modeling that generates feedback within the classroom.

Links to Maryland High School Mathematics Core Learning Units:

Geometry, Measurement, and Reasoning

- **2.1.1**
  Students will describe the characteristics of geometric figures and will construct geometric figures using technology.

- **2.1.4**
  Students will validate properties of geometric figures using appropriate and technology.

- **2.2.2**
  Students will solve problems using two-dimensional figures**.**

- **2.3.2**
  Students will use techniques of measurement and will estimate, calculate, and compare perimeter, circumference, and area of two dimensional figures and their parts. . The results will be expressed with appropriate precision.

Grade/Level:

Grades 9 – 12, Geometry

Duration/Length:

Three class periods, including assessment, 45 minutes each

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Calculating the circumference of a circle
- Comparing values to determine whether they form a constant ratio
- Calculating the area of a circle
- Using Geometer's Sketchpad to construct, calculate, and tabulate.
- Calculating special right triangles (30°-60°-90°; 45°-45°-90°)

Student Outcomes:

Students will:

- construct a traffic roundabout/circle using Geometer's Sketchpad.
- interpreting and comparing arc length with circumference and arc angle with degrees of a circle.
- calculate the arc length and sector area of circle using the developed formulas.

Materials/Resources/Printed Materials:

- Worksheet 1
- Worksheet 2
- Worksheet 3
- BANNEKER PARK Assessment
- Geometer's Sketchpad

Development/Procedures:

- Teacher will review any necessary computer/calculator skills procedures. (Worksheets and Assessment where developed using Geometer's Sketchpad 3.0 and teachers need to adjust instructions for TI-92 plus calculators.)
- The students will complete Worksheet 1 using **Geometer's Sketchpad**.
- The teacher should lead a class discussion and present the formulas for arc length and sector area. $\text{Arc length} = \frac{n}{360}(2\pi\ \text{r})$ and $\text{Sector area} = \frac{n}{360}(\pi\ \text{r}^2)$
- Giving an example as practice #1: a circle with an arc of 60 and a radius of 10 inches.
- Assign Worksheet 2 for homework.
- The students will complete Worksheet 3 on the second day.
- The students will complete the BANNEKER PARK assessment to conclude the unit.

Assessment:

Teacher's Guide

Introduction

The assessment will determine whether the students can apply arc formulas related to arc length and areas of sectors. The students will use Geometer's Sketchpad or TI- 92 Plus calculators (with Geometer's Sketchpad) to create the constructions and solve a problem related to a traffic round-about/circle.

Objectives Covered

Students will:

- construct a traffic roundabout/circle using Geometer's Sketchpad.
- calculate the arc length and sector area of circle using the developed formulas.

Tools/Materials Needed for Assessment

- BANNEKER PARK Assessment
- Geometer's Sketchpad
- Pencil and Paper, Calculator

Administering the Assessment

The solutions for the assessment problems are included at the end of this guide.
A sample solution for the construction also is included.

Extension/Follow Up:

Students may need more practice and exposure to finding the area between the circle and various polygons. Other application topics such as geometry probability are developed by the instructor's edition of your geometry text and ancillary materials.

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Date _____ Pd _____

Worksheet 1 – Discovering Arc Length and Sector Areas in Circles

Part I: Constructing Arcs in a Circle

1. Open Geometer's Sketchpad. Choose the circle tool. Click and drag to construct a circle with center A and a point on the circle being B.

2. Choose the point tool. Click on the circle in various places to construct point C, D, E, and F on circle A.

3. Choose the selection tool. Click on any three of the four points in order (DO NOT CHOOSE Point B) to construct an arc (be sure to use the shift key). Go to the Construct Menu and choose arc by three points.

4. Select the arc. Go to the Measure Menu and choose arc angle.

5. Select the arc again. Go to the Measure Menu and choose arc length.

Part II: Comparing the Arc Length and Circumference

6. Select the circle. Go to the Measure Menu and choose Circumference.

7. Select the arc length and the circumference measurements. Go to the Measure Menu and choose Calculate.

8. Click on Values and drag to arc length. Click on divide. Click on Values and drag to circumference. Finally click on OK.

Part III: Comparing the Arc Angle and Degrees in a Circle (360°)

9. Select the arc angle measurement. Go to the Measure Menu and choose Calculate.

10. Click on Values and drag to arc angle. Click on divide. Type in 360. Go to Units and drag to degrees. Finally click on OK.

Part IV: Creating a Table to Compare Ratios

11. Select the two ratios created in part II and part III; go to the Measure Menu and choose Tabulate.

12. Choose one of the endpoints of the arc and move it on the circle; double click on the table.

13. Repeat step 12.

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Part IV: Writing Conclusions

14. What conjecture can you make about $\frac{\text{arc angle}}{360^0}$ and $\frac{\text{arc length}}{\text{circumfere nce}}$?

_____

_____

_____

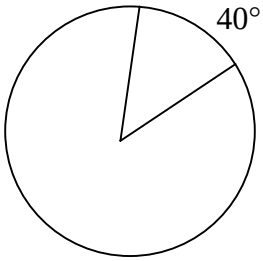
Use this space to write the formulas the class discovered and practice problems.

Name _____
Date_____ Pd _____

Worksheet 2 – Homework Assignment

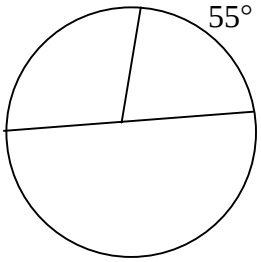
Part I: Find the circumference and then arc length in the following circles:

1.



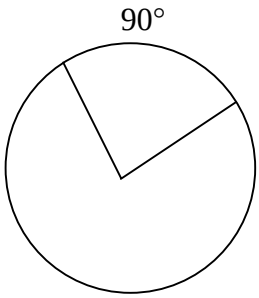
Radius = 16cm

2.



Diameter = 10 ft

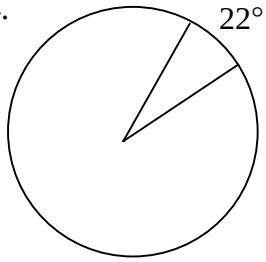
3.



Radius = 23 inches

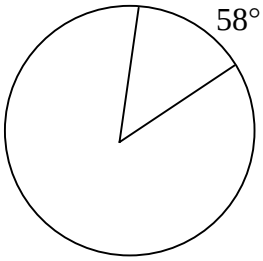
Part II: Find the area and then area of the sector cut by the radii in the following circles:

4.



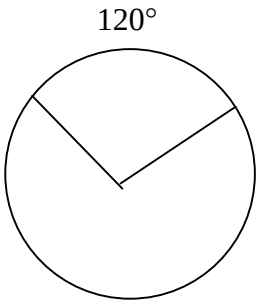
Radius = 7 in

5.



Diameter = 30 yards

6.

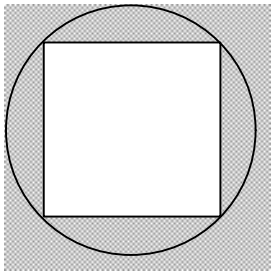


Radius = 35 cm

Name _____
Date _____ Pd _____

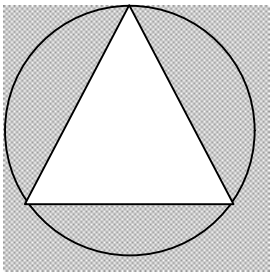
Worksheet 3 – Finding shaded areas between circles and polygons

1.



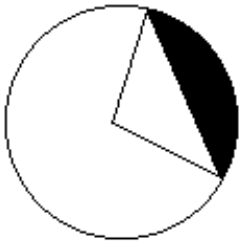
Find the shaded area inside the circle and outside the square when the circle has a radius of 10 cm.

2.



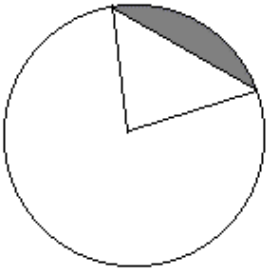
Find the shaded area inside the circle and outside the equilateral triangle when the circle has a diameter of 50 ft.

3.



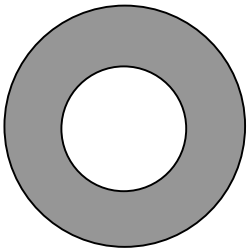
Find the shaded area of the circle that has a radius of 9 cm and the arc is 120°.

4.



Find the shaded area of the circle that has a radius of 24 inches and the arc is 90°.

5.



Find the shaded area when the larger circle has a diameter of 6 ft and the smaller circle has a diameter of 4 ft.

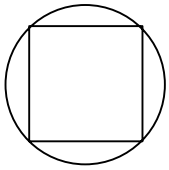
6.

Find the shaded area when the 5-12-13 yard triangle is inscribed in the semicircle.

BANNEKER PARK ASSESSMENT

Part I: Choose A, B, C or D for each correct answer.

1. In the figure how much fabric is wasted if the largest possible square is cut from a circle of radius r.



A. $\pi r^2 - 2\pi r^2$

B. $r^2 - 2\pi r^2$

C. $\pi r^2 - r^2$

D. $2\pi r^2 - 2$

2. The arc length of a circle with a radius of 6 feet and arc angle of 30° is

A. 37.68 feet B. 3.14 feet C. 6.28 feet D. 12.56 feet

Part II: Solve and grid your response.

3. Find the shaded area created by an 8-15-17cm right triangle inscribed in the semicircle.

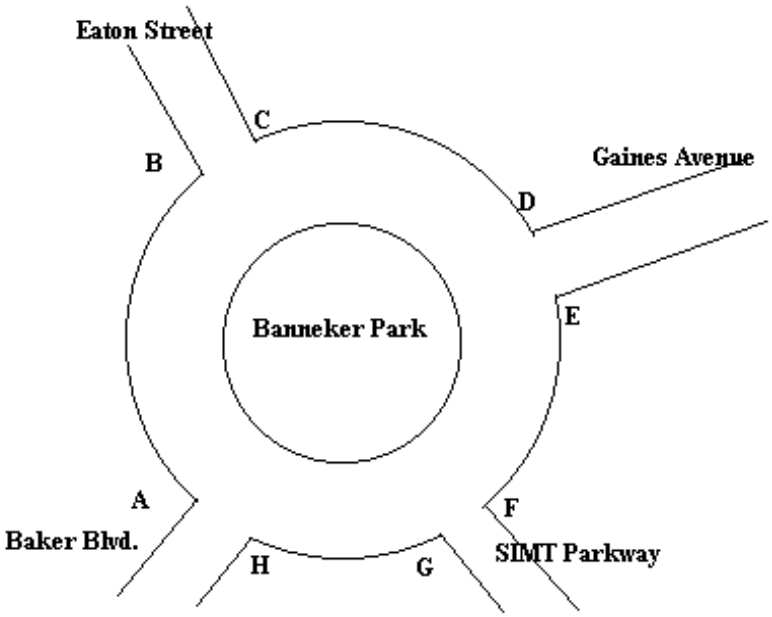
Answer: _____

4. Find the area of the sector of the circle whose diameter is 10 centimeters and arc has a measure of 76°.

Answer: _____

5. In Mathland, a traffic roundabout/circle is in the middle of downtown Euclidville. The center of the roundabout is BANNEKER PARK, which is 100 feet in diameter. The entire traffic circle is 216 feet in diameter. The four roads enter the intersection at different angles so that arc AB = 134°, arc CD = 63°, arc EF = 57°, and arc HG = 66°.

   - Use a construction on Geometer's Sketchpad to reproduce the drawing below.
   - Find the total feet of curbing needed around the Park and among the arcs of the traffic roundabout.
   - Find the area of the asphalt road between the traffic roundabout/circle and the Park.
   - Justify your answer using mathematics.



BANNEKER PARK Unit -- SOLUTIONS

Worksheet 1

14. Students should mention that both ratios are the same value, even when the arc changes measure.

Practice #1: arc length = 10.47 inches sector area = 52.33 square inches

Worksheet 2

1. C = 100.48 cm; arc length = 11.16 cm
2. C= 31.4 ft; arc length = 4.80 ft
3. C = 144.44 inches; arc length = 36.11 inches
4. A= 153.6 sq. in ; area of sector = 9.40 sq. in
5. A = 706.5 sq. yd.; area of sector = 113.83 sq. yd.
6. A= 3846.5 sq. cm; area of sector = 1282.17 sq. cm

Worksheet 3

1. A = 214 sq. cm
2. A= 1150.6 sq. ft
3. A = 84.78 sq. cm
4. A= 164.16 sq. in
5. A = 15.7 sq. ft
6. A= 36.33 sq. yd

Assessment

1. C
2. B
3. 53.43 (grid)
4. 16.57 (grid)
5. Curbing = 916.88 feet; Asphalt = 28775 sq. feet
   Construction – see Sketchpad sample

   Justification should include:
   For construction, how the student constructed circles and arcs, hide circle and points, using same center for both circles.

   For solution of curbing, student must find circumference and arc lengths of the four arcs.

   For solution of asphalt, student must find the area of the larger circle and subtract the area of the smaller circle.

Rubric for Problem 5

4	**Student demonstrated a knowledgeable understanding of arc length and area between two-dimensional figures.** Student construction is labeled and accurate. Student answers are correct and contain correct units. Student correctly explains the process of constructing the figure and finding curbing and asphalt.
3	**Student demonstrated an understanding of arc length and the area between two-dimensional figures.** Student construction is labeled and accurate. Some student answers are correct and contain units. Student explains the process of constructing the figure and finding curbing and asphalt.
2	**Student demonstrated some understanding of arc length and area between two-dimensional figures.** Student construction is labeled. Some student answers are correct. Student gives some explanations of the process of constructing the figure and finding curbing and asphalt.
1	**Student demonstrated little understanding of arc length and area between two-dimensional figures.** Student construction is incomplete. Student answers are incorrect. Student gives no explanation of the process of constructing the figure and finding curbing and asphalt.
0	**Student makes no attempt to solve the problem.**

Sample Construction Solution