Title: Geometric Giants

Brief Overview:

The unit introduces the ideas of ratios and similarity using body size. The students will measure people, calculate ratios, and make predictions. The graphing calculator will be used to extrapolate data. Geometric relationships involving areas and volumes of similar figures will help answer the question: Can there be giants?

Links to NCTM Standards:

- **Mathematics as Problem Solving**
  The student will find ratios of the human body and make a mathematical model to predict body size.

- **Mathematics as Communication**
  The student will express, orally and in writing, ratios of the human body.

- **Mathematics as Reasoning**
  The student will formulate common ratios of the human body and will then logically conclude whether a giant could exist based on their findings.

- **Functions**
  The student will find a line of best fit and use it to explore relationships of the human body using actual forensic data.

- **Geometry from a Synthetic Perspective**
  The student will classify figures as similar and apply the properties of similarity to predict body size.

- **Geometry from an Algebraic Perspective**
  The student will use dilations to investigate the feasibility of the existence of a giant.

- **Number Relationships**
  The student will find ratios in the human body.

- **Statistics**
  The student will find averages of the ratios in the human body and will analyze data from the human body to explore relationships.

Links to Maryland High School Mathematics Core Learning Goals:

- **1.1.1**
  The student will recognize, describe, and extend patterns and functional relationships that are expressed numerically, algebraically, and geometrically.

- **2.1.3**
  The student will use transformations to move figures, create designs, and demonstrate geometric properties.

- **2.2.1**
  The student will identify and verify congruent and similar figures and apply equality or proportionality of their corresponding parts.

- **3.2.2**
  The student will make predictions by finding and using a line of best fit and by using a given curve of best fit.

Grade/Level:

Grades 8 - 12; Geometry, Algebra

Duration/Length:

Two to three 45-minute periods.

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Measuring
- Finding ratios and solving proportions
- Calculating an average of a set of numbers
- Graphing ordered pairs

Objectives:

Students will be able to:

- use proportions to predict the value of an unknown.
- recognize similar figures.
- discover the Fundamental Theorem of Similarity.
- explain why there can't be giants.

Materials/Resources/Printed Materials:

- Student Activity Sheets 1-3
- Measuring tapes, pencils
- TI-82/TI-83 overhead calculator
- Calculator and Graphing calculators (optional)
- Video: "Honey, I Blew Up the Kid"
- Transparencies 1-3
- Photograph of a young child with head circumference and height information for that child
- Information sheet on world's tallest man (see Teacher's Notes)

Development/Procedures:

First 45-minute period

1. The teacher will begin with **Transparency 1**, a story about the existence of Big Foot. The class should discuss how we could predict Big Foot's height from his foot size, leading to the idea of proportions.

2. Students work in groups of 3-4 measuring each other and completing the first chart on **Student Activity Sheet 1**. The teacher should move among groups making sure that students are being consistent in their measurement techniques. A class discussion will help students to decide how to fill in the food and drink intake chart (perhaps using 1/4 lb. hamburgers and 8 oz. glasses of water as a guide).

3. One student will measure the teacher and record the dimensions on **Transparency 2**. The class will work together to calculate ratios of body dimensions of the teacher. They will then write the measurements for each group into the chart on the transparency. A discussion of whether using one person's measurements to predict the measurements of the whole human population should lead to the next activity.

4. Working in groups, they will decide if their own measurements are in the same ratio as the teachers. Ratios will be filled in on **Student Activity Sheet 2** and on the transparency. A discussion of how this information could be made more useful should lead to the idea of taking an average of the ratios. Averages will be calculated by groups and filled in on the activity sheet and transparency.

5. Each group will measure the head circumference of one person in its group who was not previously measured. Using this measurement, the students will use the average ratio to predict the height of this person. They will then measure his height to compare.

6. Using the head circumference of a small child, provided by the teacher, the students will again use this ratio to estimate the height of the child. The teacher will have the exact height of the child (perhaps with a photograph) to show that ratios in children are different from those of adults. The concept of "similar" figures will be discussed and defined.

7. A homework assignment will be given to measure the height and foot length of 1-2 other people (measured in cm). The teacher should encourage them to measure people of different ages.

Second 45-minute period:

8. During the next class, a scatter plot of foot length vs height will be created on the **TI-82/83 graphing calculator**. Before beginning, the teacher should help students understand why this is appropriate rather than using their average ratios. (A scatter plot is good for showing relationships between 2 variables and avoids the need to calculate and average the ratios each time. The class will use height and foot length because we only know Big Foot's foot length.) From the scatter plot, a line of regression can be found. Using this model, the class will find the height of Big Foot.

9. A 10 minute portion of the movie, "Honey, I Blew Up the Kid," will be viewed (begin 51min.45 sec. from the beginning of the movie). Since the height of the "blown-up" baby is given as 50 feet, the students should use proportions to change feet to centimeters. Students will work in groups to answer the questions on **Student Activity Sheet 3** concerning the feasibility of the existence of Big Foot and other giants.

10. Continuing on **Student Activity Sheet 3**, students will work together to find the areas and volumes of cubes and calculate the ratios of their area to side and volume to side. The class will go over the answers and the significance of the results.

11. Students will work in groups to discuss the questions on **Transparency 3** and then share their answers with the entire class. The teacher will tell them facts about the tallest man, as recorded in the Guiness Book of Records, to help students to understand some of the difficulties that giants would encounter. This will lead them to conclude that giants could not exist if their body parts were proportional to humans.

Extension/Follow Up:

1. Discuss the feasibility of "shrinking" humans. Is it possible? The teacher could reference movies such as "Honey, I Shrunk the Kids" or "The Incredible Shrinking Man."

2. Make scale drawings or models of buildings using ratio and proportion.

Teacher's Notes

1. The teacher may want to use a drill on ratio and proportion before beginning the unit.
2. When calculating ratios, the teacher may need to discuss writing them as decimals and may want to write them in the form "decimal:1".
3. A spreadsheet could be used to do the averages if students are familiar with them.
4. If students are familiar with scatter plots on graphing calculators, they could do that exercise on their own or in groups. Otherwise the teacher can do this on the overhead graphing calculator.
5. When solving for the height of the baby given the head circumference, the teacher may need to review solving a proportion.

World's Tallest Man

According to the Guinness Book of World Records, the world's tallest known human being was Robert Wadlow, the "Alton giant," who was 8'11". He lived from 1918-1940. His foot length was 47 cm. He had to wear ankle braces to support his weight and died of a septic ulcer caused by the brace.
(reference: galileo.enc.org/online/ENC1665/1665b45.html)

Directions for Scatter Plots for TI-82 or TI-83:

1. **Clearing the Lists:**
Under the STAT menu, choose EDIT. Clear all lists by putting the cursor at the top of a column, say on L_1, and press CLEAR and then ENTER.

2. **Entering Data:**
Under L_1 put in the lengths of the people's feet (all measurements should be in cm).
Under L_2 put in the people's height.

3. **Making the Scatter Plot:**
Under the STAT PLOT menu, select the first Plot. (Make sure all other plots are off).
Turn PLOT1 On.
For Type, choose the scatter plot (the first picture).
For XList, choose L_1
For YList, choose L_2
For Mark, choose the box (the first picture). You may use other marks such as the "plus" or the "dot", but the "box" is easiest to see.

Go to the Zoom menu and select ZoomStat. You will then see the plots of each data point. You may then discuss that the correlation between height and foot size is linear.

4. Finding **the Regression Equation:**
Go to the STAT menu, tab over to CALC. Choose LinREG(ax+b). Then, enter.

5. **Graphing the Regression Equation:**
Go to Y=, and clear all equations. Then, place the cursor after Y_1=. Under the VARS menu (or Y-VARS on the TI-82), choose Statistics. Tab over to the EQ menu and choose RegEQ. This will put the Linear Regression Equation you've just calculated into Y_1. Then, press GRAPH.

6. **Using the Regression Equation to predict data:**
You can use the TRACE or TABLE feature to predict data.
Using TRACE:
There will a blinking cursor on the line of regression. The x-values and y-values are shown at the bottom of the screen. Trace to the x-value of the foot size. The y-value will be the predicted height.
Using TABLE:
Go to the TblSet menu (2nd WINDOW) and put in the foot size you want to use to find someone's height . Go to the TABLE to view the y-value.

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Student Activity Sheet 1

Name _____
Date _____

Geometric Giants

Measure one member of your group and record the following information:

Height	cm
Weight(estimated)	lbs.
Arm Length	cm
Foot Length	cm
Foot Width	cm
Floor to knee	cm
Ankle circumference	cm
Neck circumference	cm
Head circumference	cm

Estimated food intake per day	lbs.
Estimated fluid intake per day	oz.

Student Activity Sheet 2

Name _____
Date _____

Ratios (in decimal form to 2 decimal places):

	Teacher	Group1	Group2	Group 3	Group 4	Group 5	Group 6
Weight:Height							
Arm Length:Height							
Foot Length:Height							
Foot Width:Height							
Floor to knee:Height							
Ankle circ.:Height							
Neck circ.:Height							
Head circ.:Height							

Average Ratios:

	Average Ratio
Weight:Height	
Arm Length:Height	
Foot Length:Height	
Foot Width:Height	
Floor to knee:Height	
Ankle circ.:Height	
Neck circ.:Height	
Head circ.:Height	

Height prediction:
If a person's head circumference is _____, approximately how tall is the person?

If a person's head circumference is _____, approximately how tall is the person?

Student Activity Sheet 3

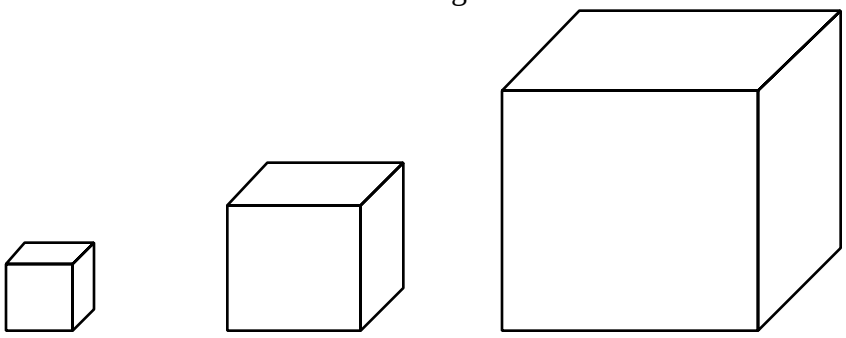
Name _____
Date _____

1. The movie, "Honey, I Blew Up the Kid," told us that the baby was "blown up" to a height of 50 feet. Based on the fact that a average 1 year old's height is 28 inches, weight is 21 lb., and head circumference is 13 inches (according to *Your Growing, Child,* Time-Life), find the "blown up" baby's weight and head circumference.

2. How much do you think he would eat? Why?

3. Could giants exist based on the physical capacity of the "Blown Up" child?

4. Find the areas and volumes of the following cubes.



2 cm 4 cm 6 cm

	Length of side	Area of bottom	Volume
Small	2 cm		
Medium	4 cm		
Large	6 cm		

	Ratio of sides	Ratio of areas	Ratio of volumes
Medium:Small			
Large:Small			

Is the small cube similar to the medium cube? _____

Is the small cube similar to the large cube?_____

Conjectures:_____

_____

Transparency 1

Last week I went camping at Deep Creek Lake. During the night, the earth shook and I thought I heard footsteps. I was too scared to move. When I woke up in the morning, I saw footprints outside my tent. I measured the footprints and found them to be 75 cm long. I packed up as fast as I could and went home.

When I told my friends what had happened, one friend told me that Big Foot had been sighted near Deep Creek Lake. I exclaimed, “Wow! I was almost face to face with Big Foot. I wonder how tall he is.” But my other friend said, “There’s no such thing as Big Foot.”

What do you think?

Transparency 2

Measurements:

	Teacher	Group1	Group 2	Group 3	Group 4	Group 5	Group 6
Height							
Weight(estimated)							
Arm Length							
Foot Length							
Foot Width							
Floor to knee							
Ankle circumference							
Neck circumference							
Head circumference							

Ratios (in decimal form to 2 decimal places):

	Teacher	Group1	Group2	Group 3	Group 4	Group 5	Group 6
Weight:Height							
Arm Length:Height							
Foot Length:Height							
Foot Width:Height							
Floor to knee:Height							
Ankle circ.:Height							
Neck circ.:Height							
Head circ.:Height							

Average Ratios:

	Average Ratio
Weight:Height	
Arm Length:Height	
Foot Length:Height	
Foot Width:Height	
Floor to knee:Height	
Ankle circ.:Height	
Neck circ.:Height	
Head circ.:Height	

Transparency 3

Discussion Questions:

1. What does the calculation of the cubes' areas and volumes have to do with what we were discussing before?

2. Suppose the small cube weighed 5 lb. What would be the weights of the medium and large cubes? Why?

3. What is the ratio of the "blown-up" baby's foot length to the teacher's foot length?

4. Is it possible to find the ratio of the areas of the baby's foot to the teacher's foot without calculating their areas? If not, how would you figure out the areas? If so, find the ratio of the areas of the baby's foot to the teacher's foot?

5. What is the ratio of the baby's volume to the teacher's volume?

6. What is the ratio of the baby's weight to the teacher's weight?

7. Go back and reconsider the questions at the top of Activity Sheet 3. Do you want to change your answers? Why or why not?

Performance Assessment

Teacher's Guide

Introduction

The purpose of the assessment activity is to provide feedback to you so that appropriate instructional decisions can be made and modifications can be made to the lessons.

Objectives Covered

This task assumes that your students have received instruction and have been assessed on the following objectives:

Use proportions to predict the value of an unknown
Recognize similar figures
Discover the Fundamental Theorem of Similarity
Explain why there can't be giants

Tools/Materials Needed for Assessment

Pencil, calculator
Measuring tape (for checking feasibility of answers)
Copies of assessment

Administering the Assessment

This assessment should take approximately 30 minutes. Distribute the papers to the students (and calculators, if necessary).

Name _____
Date _____

Performance Task

Solve each proportion.

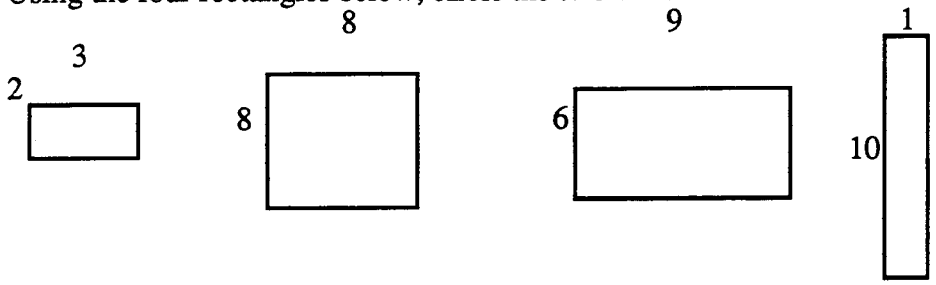
1. $\frac{x}{3} = \frac{4}{7}$

2. $\frac{4}{5} = \frac{16}{x}$

3. Three pencils cost $.85. Solve each problem showing your work.
a.) How many pencils can be bought with $1.58?

b) How much would 25 pencils cost?

4. Using the four rectangles below, circle the two which are similar to each other.



5. In the chart below, the ratios of two similar figures are given. Find the missing ratios.

Ratio of Sides	4:5		
Ratio of Areas		9:25	
Ratio of Volumes			1:27

6. Is foot size a good indication of a person's height? Why or why not?

_____

_____

_____

_____

_____

7. Suppose Mattel decided to make a motion picture starring a real-life Barbie. The director decided that Barbie should be 5' 10". You are the casting director for the movie. You have already asked your assistant to find the measurements of a Barbie doll, and they are listed below. You need to find the measurements (to two decimal places) of the actress who will play Barbie.

Barbie's Measurements	Actress's Measurements
Height = 11.65 in	Height = 5' 10" = _____ in
Foot Width = .24 in	Foot Width = _____ in
Foot Length = .91 in	Foot Length = _____in
Head Circumference = 4.09 in	Head Circumference = _____ in
Bust = 5.91 in	Bust = _____in
Neck Circumference = 1.77 in	Neck Circumference = _____in
Weight = .28 lb.	Weight = _____lb.

As casting director, you will need to advertise for a person who fits this description. Give a physical description of the actress. Think back to the discussion about the existence of giants; is it possible for you to find an actress to fit this description? Why or why not? Support your answer.

_____

_____

_____

_____

_____

_____

_____

Name ***KEY***
Date _____

Performance Task

Solve each proportion.

1. $\frac{x}{3} = \frac{4}{7}$

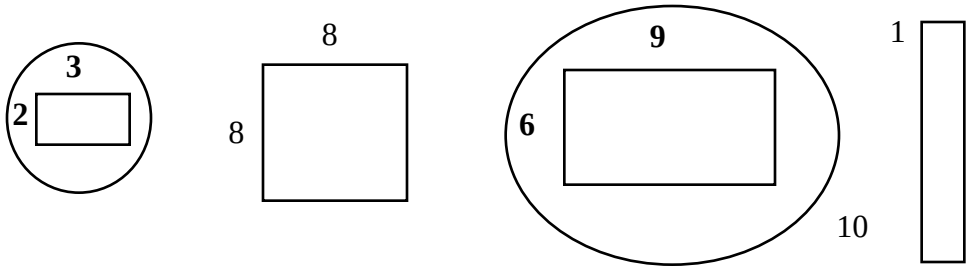
x = (12/7) or 1.71

2. $\frac{4}{5} = \frac{16}{x}$

x = 20

3. Three pencils cost $.85. Solve each problem showing all steps.
   a. How many pencils can be bought with $1.58?
   $\frac{3}{.85} = \frac{x}{1.58}$ *x = 5.576 which should be rounded to **5***
   b. How much would 25 pencils cost.
   $\frac{3}{.85} = \frac{25}{x}$ *x = 7.08*

4. Using the four rectangles below, circle the two which are similar to each other.



$\frac{3}{2} = \frac{9}{6}$

5. In the chart below, the ratios of two similar figures are given. Find the missing ratios.

Ratios of Sides	4:5	3:5	1:3
Ratio of Areas	16:25	9:25	1:9
Ratio of Volumes	64:125	27:125	1:27

6. Is foot length a good prediction of a person's height? Why or why not?

see Scoring Guide

7. Suppose Mattel decided to make a motion picture starring a real-life Barbie. The director decided that Barbie should be 5' 10". You are the casting director for the movie. You have already asked your assistant to find the measurements of a Barbie doll, and they are listed below. You need to find the measurements (to two decimal places) of the actress who will play Barbie.

Barbie's Measurements	Actress's Measurements
Height = 11.65 in	Height = 5' 10" = 70 in
Foot Width = .24 in	Foot Width = 1.44 in
Foot Length = .91 in	Foot Length = 5.47 in
Head Circumference = 4.09 in	Head Circumference = 24.58 in
Bust = 5.91 in	Bust = 35.51 in
Neck Circumference = 1.77 in	Neck Circumference = 10.64 in
Weight = .28 lb.	Weight = 60.74 lb.

As casting director, you will need to advertise for a person who fits this description. Give a physical description of the actress. Think back to the discussion about the existence of giants; is it possible for you to find an actress to fit this description? Why or why not? Support your answer .

see Scoring Guide

Scoring Guide

Question 6

Both a "Yes" and a "No" answer will be scored as acceptable if supported correctly.

3: If the student uses correct terminology to support his answer (such as "linear correlation" or "in proportion").

2: If the student uses non-mathematical terms to support his answer (such as "as one gets bigger, the other one gets bigger").

1: If the student states that a relationship exists, but does not specifically state what the relationship is.

0: A blank response or no support following "Yes" or "No".

Extended Response for Question 7

Only a "No" response is correct.

3: If the student mentions a specific inequitable proportion of her body (such as the length of her foot will not support her height or weight).

2: If the student mentions the proportions would be inequitable but does not mention a specific example.

1: If the student gives a negative response with no explanation.

0: A blank or positive response.