

Title: Polygon-Tessellation Exploration

Link to Outcomes:

- **Problem Solving** Students will use critical thinking skills to determine methods for creating tessellations using *The Geometer's Sketchpad*TM.
- **Cooperative Learning** Working in a cooperative setting students will demonstrate their ability to tessellate polygons.
- **Technology** Students will use *The Geometer's Sketchpad*TM to discover the interior and exterior angle theorems and to create a tessellation.
- **Patterns and Relationships** Students will use data collected in tables to discover several polygonal relationships.
- **Connections** Students will integrate their knowledge of *The Geometer's Sketchpad*TM and regular polygons to create a tessellation.
- **Reasoning** Students will utilize polygonal relationships to logically determine which regular polygons tessellate.
- **Communication** Students will explain in writing the relationships and theorems discovered in their polygon and tessellation investigations.
- **Geometry** Students will demonstrate their ability to construct and measure segments, angles, and polygons using *The Geometer's Sketchpad*TM.

Brief Overview:

In this unit students will use computers and hands-on activities to: 1) investigate and explain relationships involving angles of polygons and 2) develop a basic understanding of tessellations. Students will then apply this knowledge to construct a tessellation.

Grade/Level:

Grades 9-12, Geometry

Duration:

3-4 days

Prerequisite Knowledge:

- Classification of polygons and related definitions
- Familiarity with theorem: The sum of the angles of a triangle is 180° .
- Basic knowledge of *The Geometer's Sketchpad*TM

Objectives:

- To discover and verify the formula for the sum of the angles of a convex polygon with n sides
- To discover the sum of the exterior angles of a convex polygon with n sides
- To find the measure of an interior angle of a regular polygon with n sides
- To define, name, and classify basic tessellations
- To determine which regular polygons tessellate
- To construct a tessellation using *The Geometer's Sketchpad*TM

Materials/Resources/Printed Materials:

- Computer
- *The Geometer's Sketchpad*TM Software
- Student worksheets
- Calculator
- Scissors

Development/Procedures:

Throughout this unit students will work cooperatively in groups. Students will complete two worksheets and a project.

Worksheet I requires investigations of interior and exterior angle sums of polygons. These investigations involve organizing and analyzing information in a table, and constructing polygons and measuring angles using *The Geometer's Sketchpad*TM.

In *Worksheet II* students will investigate relationships of regular polygons. This worksheet involves several activities. First, students will explore relationships between the number of sides and exterior and interior angle measures of regular polygons. Second, students will then cut out shapes from the *Regular Polygon Activity Sheet* and manipulate these shapes to determine which regular polygons tessellate. Third, students will become familiar with semiregular and demiregular tessellations.

Before students begin work on their final project, the teacher will demonstrate the process of creating a tessellation using **rotate**, and **translate** on *The Geometer's Sketchpad*TM. Students will complete a final project utilizing information from the worksheets. This project involves the construction of a tessellation using *The Geometer's Sketchpad*TM.

Evaluation:

The teacher will circulate around the classroom to make sure all students are on task. Student work done using *The Geometer's Sketchpad*TM will be printed and collected with other worksheets. Assessment of student understanding will be based on the tessellation project produced.

Extension/Follow Up:

- The teacher may discuss and explore with students Escher-like tessellations.
- Have students create Escher-like tessellations using *The Geometer's Sketchpad*[™].

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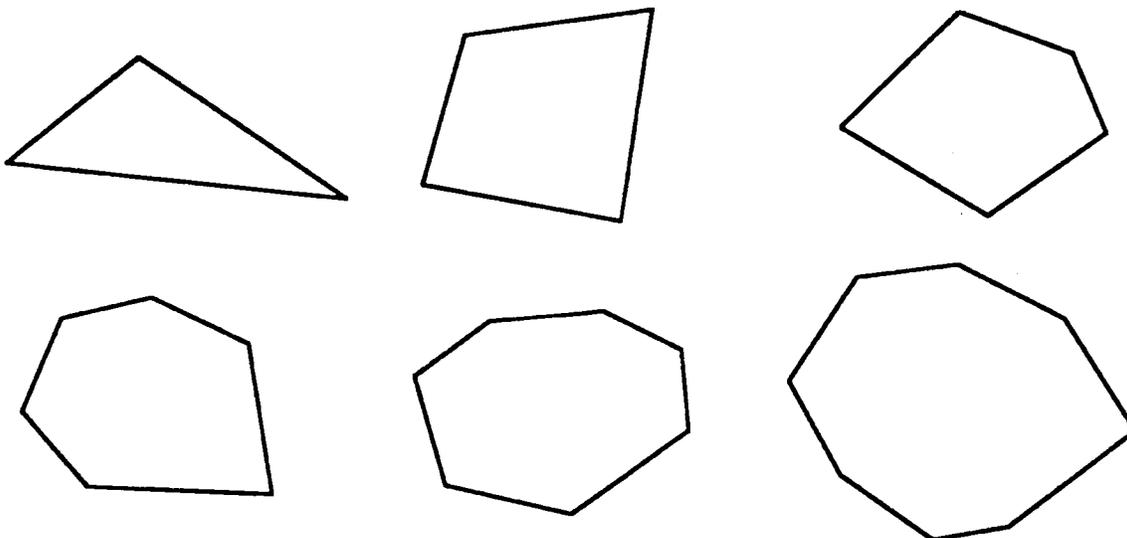
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Worksheet I - Interior & Exterior Angle Sums of Polygons

NAME _____ DATE _____ PERIOD _____

Part A: Deriving a formula for the interior angle sum of a convex polygon

Draw all the diagonals from just **one vertex** for each of the following convex polygons, and then complete the chart



Number of Sides	3	4	5	6	7	8
Number of Triangles	1					
Interior Angle Sum	180 deg					

Complete.

1. Using the information above, predict the following for a decagon:
 - a. the number of triangles **formed** _____
 - b. the interior angle sum _____
2. What is the relationship between the number of sides of a convex polygon and the number of triangles sketched within that polygon? _____

3. Derive a formula for the interior angle sum of a convex polygon with n sides. _____

Worksheet I - Interior & Exterior Angle Sums of Polygons
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Part B: Verifying the formula for the interior angle sum of a convex polygon

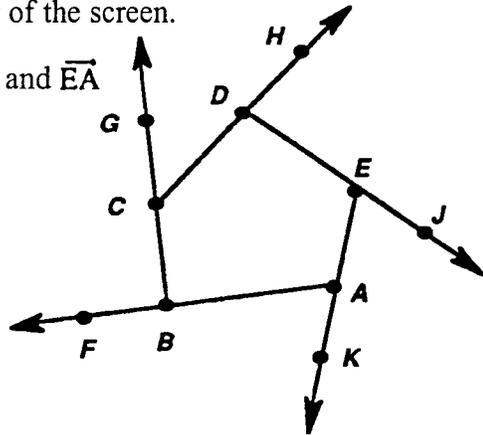
Use *The Geometer's Sketchpad*TM to complete the following.

1. Type your name(s) in the upper right-hand corner of the screen.
2. Use points to **construct** a convex polygon with at least four sides.
3. **Measure** each angle of your polygon and **calculate** the sum of these angles. Use **tabulate** to display your measurements in a table. Change the shape of your polygon several times, retabulating your measurements after each change. Use this information to verify the formula derived in **Part A**.
4. **Print** your sketch and measurements.

Part C: Investigating the sum of the exterior angles of a convex polygon¹

Use *The Geometer's Sketchpad*TM to complete the following.

1. Type your name(s) in the upper right-hand corner of the screen.
2. Use the **ray** tool to create rays \overrightarrow{AB} , \overrightarrow{BC} , \overrightarrow{CD} , \overrightarrow{DE} , and \overrightarrow{EA} to form a convex pentagon.
3. **Construct** the following points on each ray:
 - a. point F on \overrightarrow{AB}
 - b. point G on \overrightarrow{BC}
 - c. point H on \overrightarrow{CD}
 - d. point J on \overrightarrow{DE}
 - e. point K on \overrightarrow{EA}



4. **Measure** one exterior angle at each vertex, and then **calculate** the sum of these angles.
5. **Print** your sketch and measurements.
6. Using a procedure similar to the one above create a new polygon with a different number of sides. Find the sum of the exterior angles. **Print your** sketch and measurements.
7. Compare the exterior angle sum of both constructions. Describe your findings. _____

¹ Adapted from
Bennett, Dan, *Exploring Geometry with The Geometer's Sketchpad*TM, Key Curriculum Press, 1993

Worksheet II - Regular Polygon Investigation

NAME _____ DATE _____ PERIOD _____

Part A: Investigating the interior and exterior angle measures of a regular polygon

1. Define a *regular polygon*. _____

2. State the relationship between the exterior and interior angles of a polygon at a given vertex.

3. Complete the chart for *regular polygons* with the given number of sides.

Number of Sides	3	4	5	6	7	8
Each Exterior Angle						
Each Interior Angle						
Interior Angle Sum						

4. Determine the measure of one interior angle of a regular 45-gon. _____
5. Determine the number of sides of the regular polygon with exterior angle 4° . _____
6. Determine the number of sides of the regular polygon with interior angle 162° . _____

Part B: Investigating which regular polygons tessellate

1. Define a *tessellation*. _____

2. Give examples of tessellations found in the real world. _____

3. Cut out the figures on the **Regular Polygon Activity Sheet**.
4. Combine your cut-outs with the cut-outs of members in your group. Sort the polygons by shape. **Tessellations formed by using only one shape are called *pure tessellations***. Use these cut-outs to determine which regular polygons form a *pure tessellation*, and list them below.

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5. Explain why you think these regular polygons tessellate. (Consider and incorporate the findings from the investigation in **Part A** when giving your answer.)
-
-

Part C: Tessellations of two or more regular polygons

1. Tessellations formed by using two or more shapes are called ***semipure tessellations***. Tessellations formed by two or more ***regular*** polygons are either ***semiregular*** or ***demiregular***.

A ***semiregular tessellation*** consists of two or more regular polygons with edges of equal length that has an identical combination of polygons at every vertex point. Here are three examples:

Figure 1

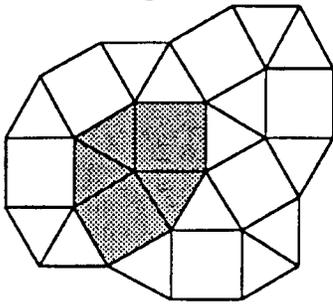


Figure 2

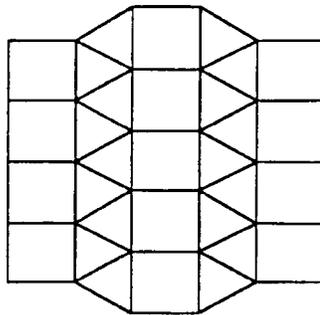
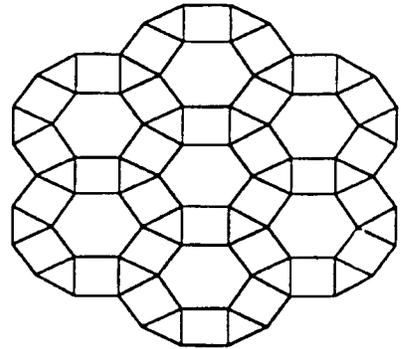


Figure 3



Name: 3.3.4.3.4

In ***demiregular tessellations***, more than one possible combination occurs. Here are two examples:

Figure 4

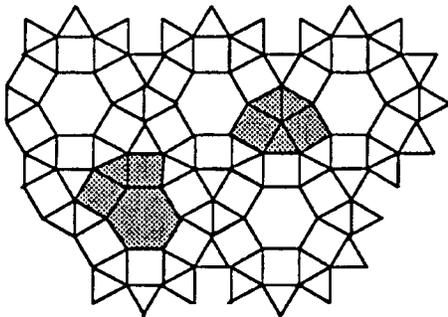
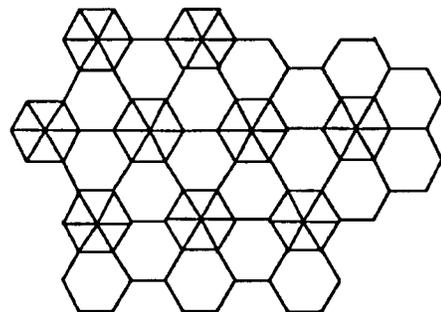


Figure 5



3.4.6.4, 3.3.4.3.4

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Page 3

Both *semiregular* and *demiregular* tessellations are identified by listing the number of sides of the polygon about the vertices.

STEPS to name both types of tessellations:

Step 1: Choose a vertex.

Step 2: Start with the polygon with the least number of sides.

Step 3: List the number of sides of each polygon as you move (clockwise or counter-clockwise) about the vertex. The direction is determined by moving towards the adjacent polygon with the least number of sides.

- a. Notice the given name of Figure 1. About any vertex there are two adjacent triangles, a square, a triangle, and a square. Hence the name 3.3.4.3.4.

Give the numerical name for Figures 2 and 3.

Name of Figure 2: _____ Name of Figure 3: _____

- b. The *demiregular* tessellation in Figure 4 has combinations 3.4.6.4 and 3.3.4.3.4 as labeled.

Find two combinations in Figure 5 and give their numerical names.

Name of Figure 5: Combination 1 _____ Combination 2 _____

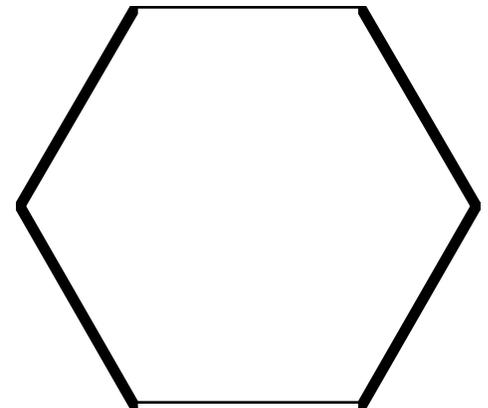
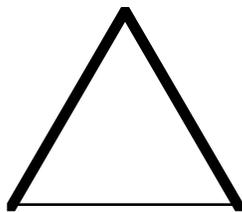
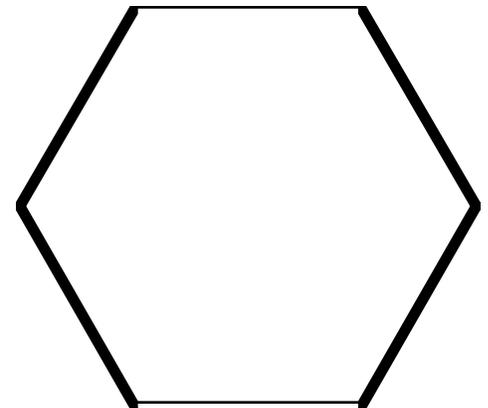
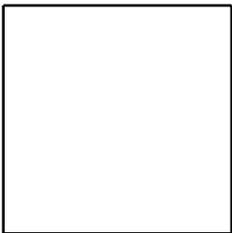
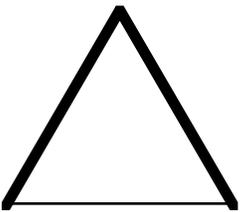
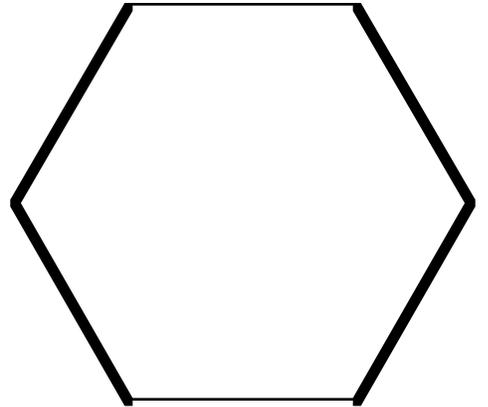
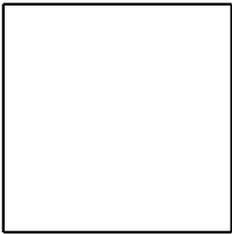
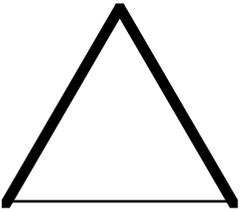
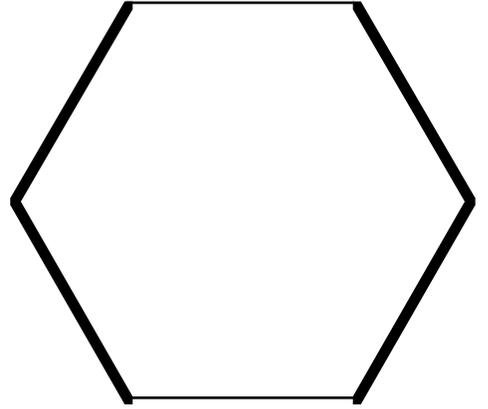
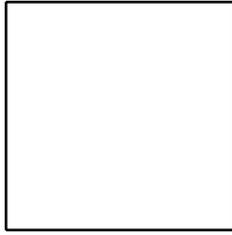
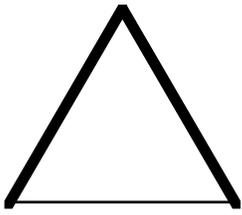
2. Combine two or more different shapes of the cut-outs to form a *semipure tessellation* unlike the ones given above.

- a. Make a rough sketch of your tessellation.

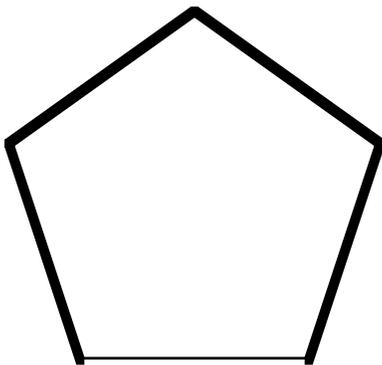
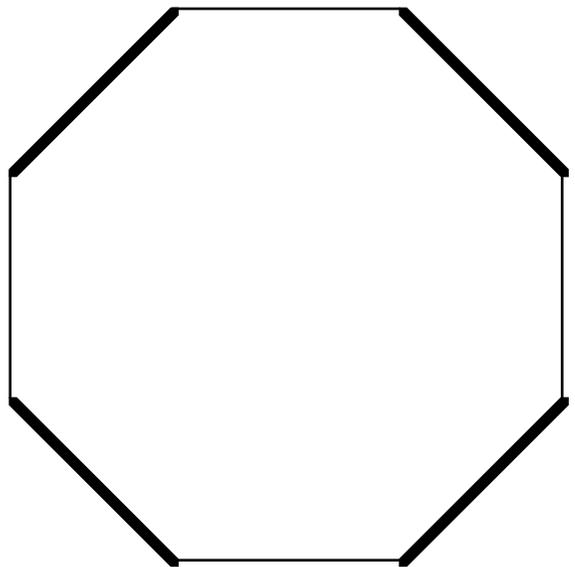
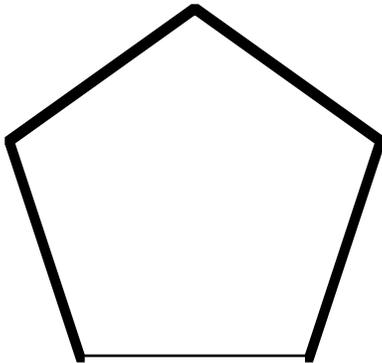
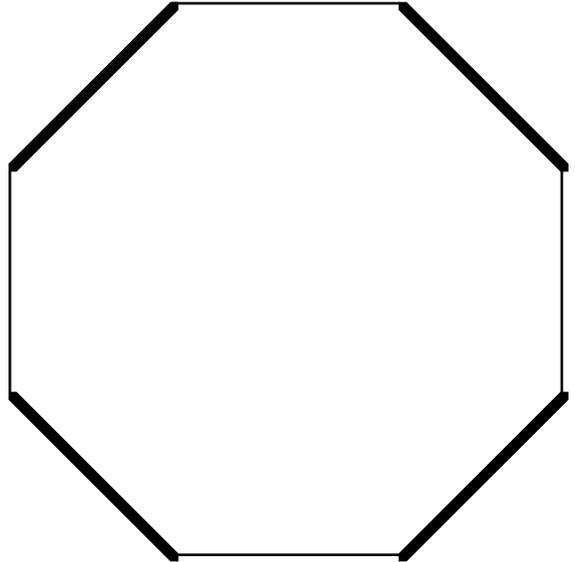
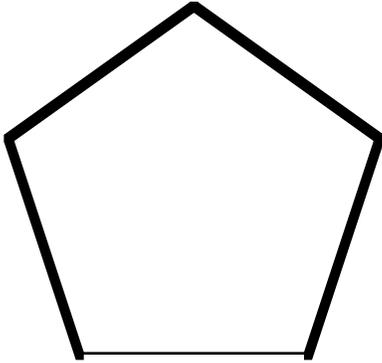
- b. Give the numerical name of your tessellation.

- c. Identify your tessellation as *semiregular* or *demiregular*. Be sure to include all numerical combinations if it is demiregular. _____

Regular Polygon Activity Sheet



Regular Polygon Activity Sheet
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Tessellation Project

Project Objective: Constructing a semiregular or demiregular tessellation using *The Geometer's Sketchpad*TM

Directions

1. Type your name(s) in the upper right-hand corner of the screen.
2. Construct a tessellation with at least two different regular polygons. All of the sides of each of these polygons should be the same length.
3. When constructing a polygon it will be necessary to rotate each side about its interior angle measure as follows:
 - a. **Construct** segment.
 - b. **Select** segment and endpoints.
 - c. **Mark** one endpoint as center. This is the point of rotation. (Direction of rotation is counter-clockwise.)
 - d. **Rotate** segment.
 - e. Repeat these steps for all segments until the polygon is formed.
4. After sketching your polygons, use **rotate** and **translate** to complete your tessellation.
5. You may choose to **shade** various polygons and/or **hide** all vertices to make your tessellation look more attractive. If a color printer is available you may choose to use colors as well.
6. When you have completed your creation print it and turn it in.

Evaluation

Your project will be graded on the following components:

- Completion
- Neatness/Appearance
- Creativity

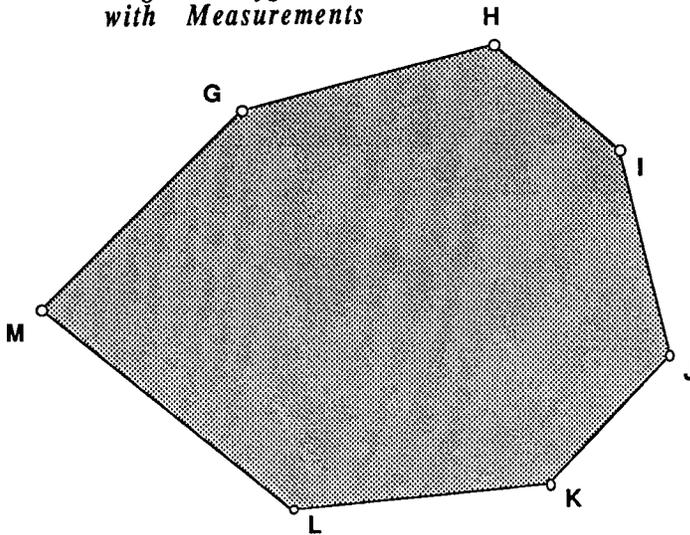
Sample Construction from Worksheet I - Part B

Student's Name _____

Date _____

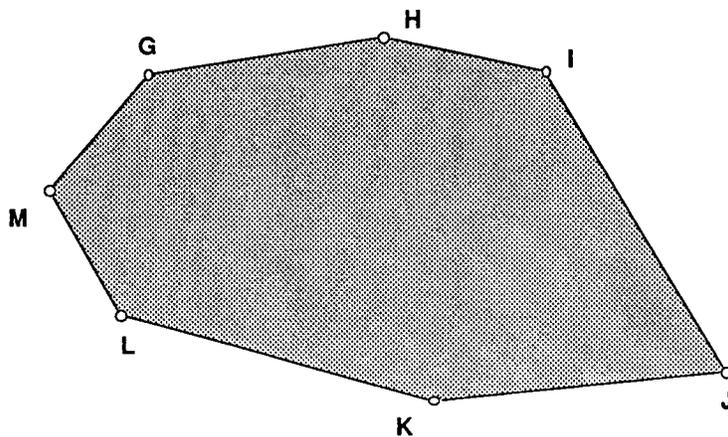
Period _____

*Original Polygon
with Measurements*



Angle(LMG)	83.72
Angle(MGH)	149.14
Angle(KLM)	135.76
Angle(JKL)	138.95
Angle(IJK)	122.67
Angle(HIJ)	143.44
Angle(GHI)	126.32
SUM	900.00

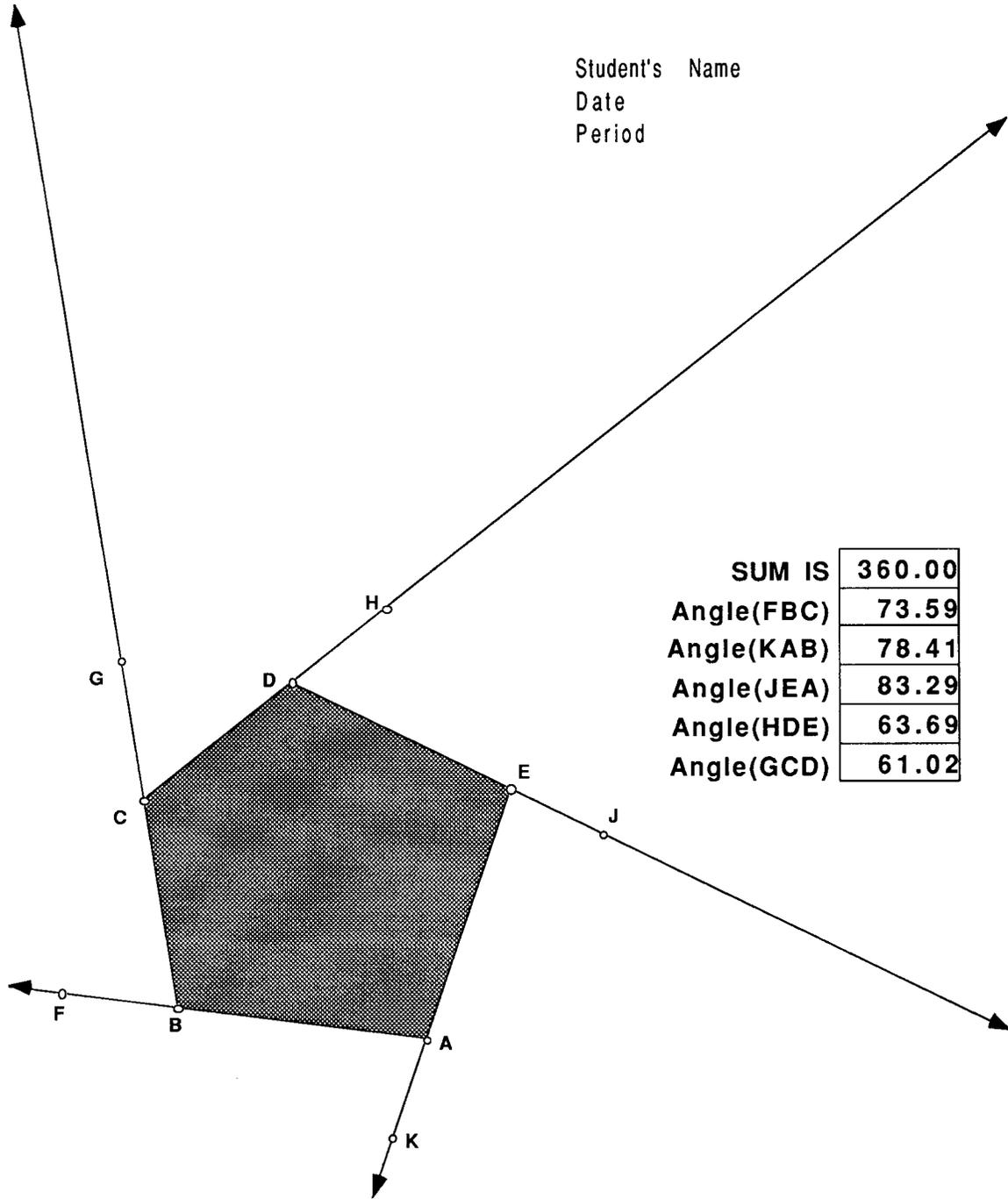
*Polygon and Measurements
After One Transformation*



Angle(LMG)	83.72	110.33
Angle(MGH)	149.14	138.95
Angle(KLM)	135.76	134.11
Angle(JKL)	138.95	159.73
Angle(IJK)	122.67	63.90
Angle(HIJ)	143.44	132.77
Angle(GHI)	126.32	160.21
SUM	900.00	900.00

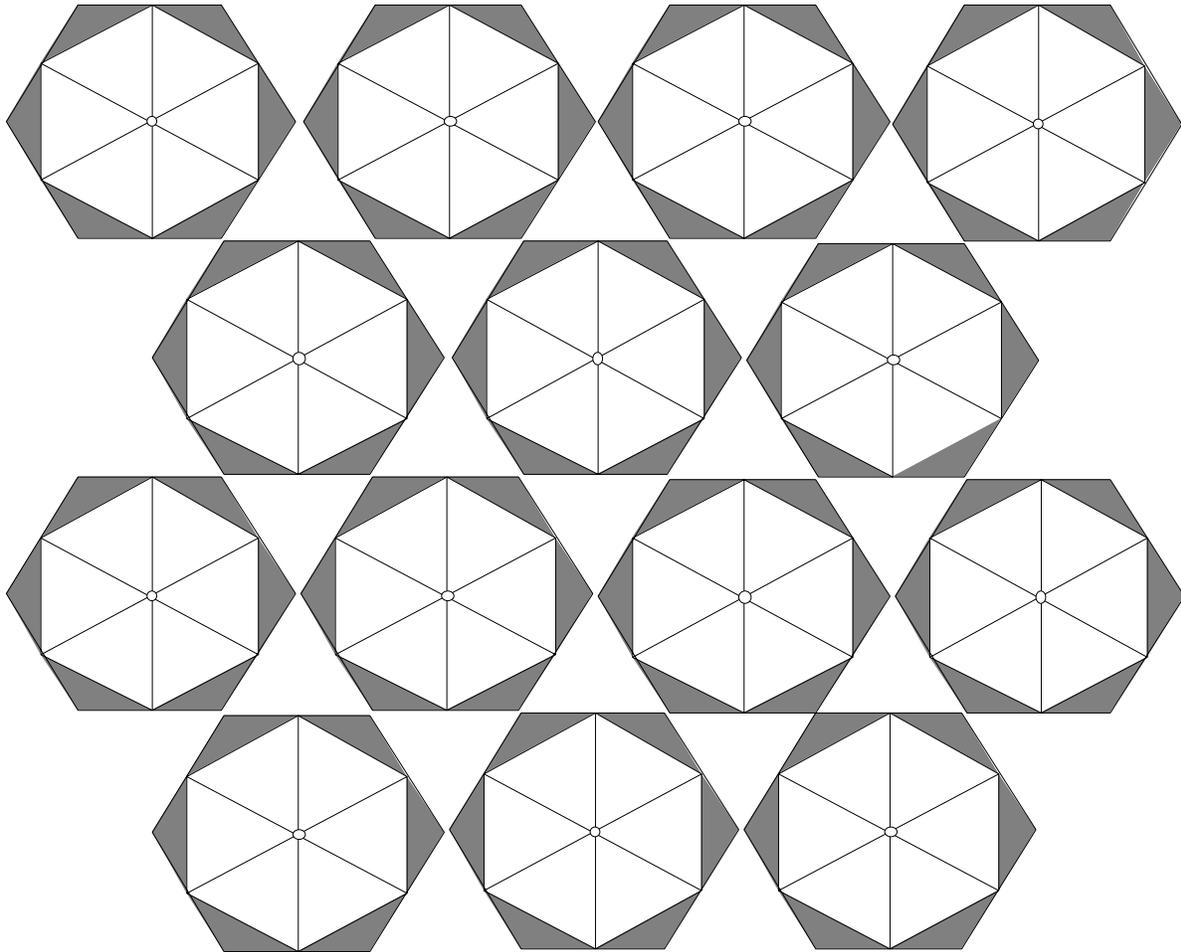
Sample Construction from Worksheet I - Part C

Student's Name
Date
Period



SUM IS	360.00
Angle(FBC)	73.59
Angle(KAB)	78.41
Angle(JEA)	83.29
Angle(HDE)	63.69
Angle(GCD)	61.02

Tessellation Project Sample



¹ Adapted from Bennett, Dan, *Exploring Geometry with The Geometer's Sketchpad™*, Key Curriculum Press, 1993.