**Title: Distance, Velocity, Acceleration**

**Brief Overview:**

Students will use the ultrasonic motion detector to generate a time versus distance graph of an object dropping. The data will be used to determine the acceleration due to gravity. The connection between distance, velocity, and acceleration will be analyzed using calculus techniques in conjunction with the generated data and the TI-82 calculator.

**Link to Standards:**

- **Problem Solving** Students will demonstrate the ability to apply derivatives to real-world data involving velocity and acceleration.
- **Connections** Students will make connections between calculus and physics.
- **Functions** Students will be able to analyze data from a physical experiment by graphing and determining the equation of certain functions.
- **Calculus** Students will be able to determine the relationship between a distance function, its velocity, and its acceleration in relationship to the first and second derivatives of the function.

**Grade/Level:**

Grades 11 - 12; Calculus

**Duration/Length:**

This lesson will take 2 periods (45 min.).

**Prerequisite Knowledge:**

Students should have working knowledge of the following skills:

- Using the TI- 82 Stat mode to determine regression equations
- Determining the 1st and 2nd derivative of a function
- Applying the relationships between 1st and 2nd derivatives, velocity, and acceleration, and their graphs
- To have some familiarity with the CBL system and the ultrasonic motion detector

**Objectives:**

Students will:

- use the CBL system to generate a distance versus time graph of a free-falling object.
- determine the "best fit" equation of this graph.
- graph the velocity and acceleration functions in conjunction with the distance function using the nDeriv on the TI-82.
- graph the first derivative (velocity) and second derivative (acceleration).
- analyze the velocity and acceleration in relationship with the position function.
- determine the value of acceleration due to gravity.

**Materials/Resources/Printed Materials:**

- TI-82 graphics calculator
- CBL System
- Vernier Ultrasonic Motion Detector
- 1 ball per motion detector (such as a baseball)
- Student worksheet
- Programs for TI-82 (See Appendix)
  - RAMP
  - ALLSELECT
  - SORTLIST
  - AVEL
  - ACCEL

**Development/Procedures:**

- Preload the RAMP, ALLSELCT, SORTLIST, AVEL, and ACCEL programs onto the TI-82 calculators.
- Place the ultrasonic motion detector at a height of at least 7 feet. The detector should be facing directly toward the floor.
- Arrange students individually or in groups of two.
- Distribute the student worksheets. The worksheets are divided into 4 parts.
  - PART I - DATA COLLECTION
    - procedures using the CBL, RAMP and ALLSELECT programs to generate the free-fall distance vs. time data.
  - PART II - INTERPRETATION
    - expressing the distance vs. time data as an algebraic equation
  - PART III - ANALYSIS OF VELOCITY
    - comparing the first derivative to the physical quantity of velocity
  - PART IV - ANALYSIS OF ACCELERATION
    - comparing the second derivative to the physical quantity of acceleration
- Stop after each section of the worksheet and discuss the results with students in order to check for understanding.

**Evaluation:**

- Student evaluation will be based upon the completion of the student worksheet and task.

**Extension/Follow Up:**

- Students will be able to translate the information about velocity and acceleration in relationship to the distance function to different types of functions.

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# DISTANCE, VELOCITY, ACCELERATION STUDENT WORKSHEET

The TI-82 should already have the programs RAMP, ALLSELCT, SORTLIST, AVEL, AND ACCEL loaded onto it.

PART I - Data Collection

A. Proceed to the workstation and link the CBL to the TI-82 with the link cable, making sure the connectors are pushed in firmly.

B. Press [PRGM], move the cursor to the program RAMP, and press [ENTER].

C. Follow the instructions given in the program with the following modifications:

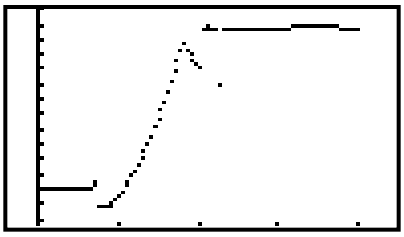
1. OPTIONS -- Choose 1. Collect Data

2. SEE DIRECTIONS -- Choose 2. No

3. When the screen prompts you to "Enter the estimated collection time in seconds", enter 2 seconds.
4. When the screen prompts you to "Move the cart halfway up the ramp, hold it in place, and hit [ENTER] to zero the probe", hold a ball right up to the motion detector, being careful not to touch the detector.
5. When the screen prompts you to "Return the cart . . .", hold the ball about 1.5 feet below the motion detector.
6. When the screen prompts you to "Activate the CBL and set the cart in motion at the same time", simply press [ENTER] and drop the ball to collect your data. You should drop the ball right after you begin to hear the clicking noise of the motion detector.

D. Wait for the CBL system to say DONE.

E. You should have a graph that looks like this:



What is the horizontal axis representing?

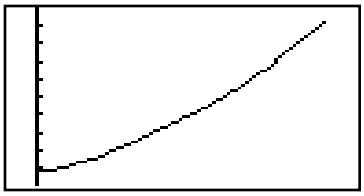
What is the vertical axis representing?

Think back to the experiment. Which part of the graph best represents the data collected from this experiment?

F. In order to extract this portion of the graph, follow these steps:

1. Press [PRGM] on the TI-82, move the cursor to the program ALLSELCT, and press [ENTER].
2. The values of X are located in L1 and the values of Y are located in L2.
3. Trace the graph and choose a point where you believe is the best representation of where the data begins as your lower bound, and press enter.
4. Trace the graph and choose a point where you believe is the best representation of where the data begins as your upper bound, and press enter.
5. The new values of X are now located in L3, and the new values of Y are now located in L4.

G. You should have a graph that looks like this:



Press [2nd] [Y=] and press [ENTER]. Highlight the following items under Plot 1: On; the left-most plot type; L3 for the X list; L4 for the Y list; the box for the mark type. Then press [GRAPH].

PART II - EXPRESSING THE DISTANCE - TIME DATA AS AN ALGEBRAIC EQUATION

A. Observe the graph. Which of the following functions could you reasonably eliminate as not being able to produce a graph similar to the distance-time graph you obtained?

$c/(x^2)$, $c/x$, $c$, $cx$, $cx^2$ where c is a constant

B. To determine the best-fit equation for the distance-time data, do the following:

1. Press [STAT], highlight CALC, and observe the different equations from 5 through B on the calculator.
2. Highlight a function that you listed as a possible function for this data and press [ENTER]. At the cursor, press [L3][,][L4] and press [ENTER].

The information for the regression equation should appear on the screen.

C. To store this equation as a Y variable, do the following:

1. Press [Y=] and scroll to the first available Y.
2. Press [VARS], press [5](Statistics), highlight EQ at the top of the menu, and press [7](RegEq).

D. To graph the data points and the regression equation simultaneously, do the following:

1. Make sure all Y equations are off except for the one that you wish to graph.
2. Press [GRAPH].

E. Observe how well the function fills the data points and record these observations below.

F. Turn Y1 off by placing the cursor on top of the = and press [ENTER].

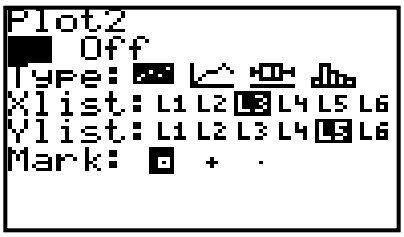
PART III - ANALYSIS OF VELOCITY

A. Run the program AVEL which will determine the velocity of the object at each point in time.

B. Press [2nd][STATPLOT] and press [4] (PlotsOff).

C. Press [2nd][STATPLOT] and press [2] (Plot2...).

D. Adjust the settings to look like this.




E. Press [ZOOM] [9] (ZoomStat).

F. What type of function is this? Why?

G. To determine the best-fit equation for the velocity-time data, do the following:

1. Press [STAT],highlight CALC, highlight the function you chose in part F, and then press [ENTER].
2. At the cursor, press [L3][,][L5], and press [ENTER].
3. Store this equation in Y2 by doing the following.
   a. Press [Y=] and scroll to Y2.
   b. Press [VARS], [5] (Statistics), highlight EQ at the top of the menu, and press [7] (RegEq).

H. Take the derivative of Y1 . You may round all the coefficients in the equation to the nearest hundredths before taking the derivative. Store this equation in Y3. Press [GRAPH].

Compare the graphs of velocity equation (Y2) and the derivative of the function (Y3). Record your observations.

Compare the equations in Y2 and Y3. Record your observations about these two equations.

Draw a conclusion about relationship between velocity equation and the distance equation.

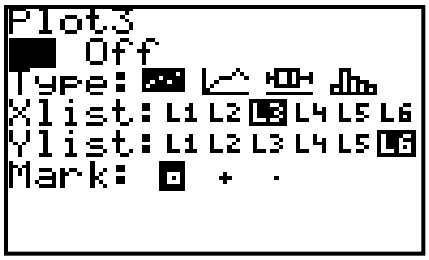
PART IV - ANALYSIS OF ACCELERATION

A. Run the program ACCEL which will determine the acceleration of the object at each point in time.

B. Press [2nd][STATPLOT] and press [4] (PlotsOff).

C. Press [2nd][STATPLOT] and press [3] (Plot3...).

D. Adjust the settings to look like this:




E. Press [WINDOW]. Let YMIN = 25, YMAX = 40, YSCL = 1. Press [GRAPH].

F. Trace the function. What do you observe about the y-values? What type of a function is this? Determine the equation of this function and store it in Y3. (The function previously stored in Y3 may be cleared.)

G. Take the derivative of Y2 (the velocity function). Compare this equation to the equation you stored into Y3. Record your observations about these two equations.

Draw a conclusion about the relationship between velocity equation and the acceleration equation.

Draw a conclusion about the relationship between distance equation and the acceleration equation.

H. A force causes a mass to accelerate. Did you exert any downward force on the ball at its highest point?

What is the ball's initial velocity?

If you exerted no force on the ball initially, what force is acting on the ball?

What is the numerical value of the acceleration that the force caused?

# APPENDIX A

PROGRAMS FOR THE DISTANCE, VELOCITY, ACCELERATION LEARNING UNIT

## ALLSELECT (From 1995 SIMT)

```
:ClrHome
:Disp "YOU WILL NEED"
:Disp "TO KNOW THE LIST"
:Disp "UNDER WHICH THE"
:Disp "X-LIST AND"
:Disp "Y-LIST ARE"
:Disp "STORED TO"
:Disp "CONTINUE"
:Pause
:prgmSORTLIST
:ClrHome
:ClrDraw
:FnOff
:PlotsOff
:AxesOn
:Plot1(xyLine,L1,L2,·)
:ZoomStat
:Text(2,2,"LOWER BOUND?")
:Trace
:X→A
:Vertical A
:Text(2,2,"UPPER BOUND?")
:Trace
:X→B
:Vertical B
:dim L1→N
:1→C
:Text(2,2,"ANALYZING...")
:For(I,1,N,1)
:If L1(I)≥A and L1(I)≤B
:Then
:L1(I)→L3(C)
:L2(I)→L4(C)
```

```
:C+1→C
:End
:End
:ClrHome
:Disp "X-LIST: L3"
:Disp "Y-LIST: L4"
:Disp ""
:Disp "HIT ENTER TO SEE"
:Disp "SELECTED PLOT."
:Pause
:Plot1(xyLine,L3,L4,·)
:ZoomStat
:Stop
```

## SORTLIST (From the 1995 SIMT)

```
:ClrHome
:Menu("WHERE IS X-LIST?","L1",1,"L2",2,"L3",3,"L4",4,"L5",5,"L6",6)
:Lbl 1:1→Q:Goto 0
:Lbl 2:2→Q:Goto 0
:Lbl 3:3→Q:Goto 0
:Lbl 4:4→Q:Goto 0
:Lbl 5:5→Q:Goto 0
:Lbl 6:6→Q:Goto 0
:Lbl 0
:ClrHome
:Menu("WHERE IS Y-LIST?","L1",A,"L2",B,"L3",C,"L4",D,"L5",E,"L6",F)
:Lbl A:1→R:Goto G
:Lbl B:2→R:Goto G
:Lbl C:3→R:Goto G
:Lbl D:4→R:Goto G
:Lbl E:5→R:Goto G
:Lbl F:6→R:Goto G
:Lbl G
:If R=1 and Q≠3
:Then:L1→L3:Goto H:End
:If R=1 and Q=3
:Then:L1→L2:Goto H:End
:Lbl H
```

```
:If Q=2
:Then:L₂→L₁:Goto J:End
:If Q=3
:Then:L₃→L₁:Goto J:End
:If Q=4
:Then:L₄→L₁:Goto J:End
:If Q=5
:Then:L₅→L₁:Goto J:End
:If Q=6
:Then:L₆→L₁:Goto J:End
:Lbl J
:If R=1 and Q≠3
:Then:L₃→L₂:Goto K:End
:If R=3
:Then:L₃→L₂:Goto K:End
:If R=4
:Then:L₄→L₂:Goto K:End
:If R=5
:Then:L₅→L₂:Goto K:End
:If R=6
:Then:L₆→L₂:Goto K:End
:Lbl K
:ClrList L₃,L₄,L₅,L₆
:Return
:Stop
```

## ACCEL

```
:dim L₃→D
:1→A
:Lbl 1
:nDeriv(Y₂,X,L₃(A))→L₆(A)
:A+1→A
:If A≤D
:Goto 1
```

## AVEL

```
:dim L3→D
:1→A
:Lbl 1
:nDeriv(Y1,X,L3(A))→L5(A)
:A+1→A
:If A≤D
:Goto 1
```

## RAMP

This program is available through Texas Instruments' Real-World Math with the CBL System by Brueningsen, Bower, Antinone, and Brueningsen, c. 1994.