Title: Cooling It with Calculus

Link to Outcomes:

- **Problem Solving** Students will use the TI Calculator Based Laboratory (CBL) to demonstrate their ability to use a mathematical model to solve an authentic problem.

- **Communication** Students will demonstrate their ability to communicate the results of an experiment using the language of mathematics. They will use the methods of science to organize and interpret data.

- **Reasoning** Students will demonstrate their ability to make and test conjectures and examine possible extensions of the problem situation.

- **Connections** Students will demonstrate their ability to make connections between integration in calculus and formulas in science.

- **Technology** Students will demonstrate their ability to use the Calculator Based Laboratory in conjunction with other technology where appropriate.

- **Cooperation** Students will demonstrate their ability to solve a problem in a cooperative atmosphere.

- **Real-World Applications** Students will demonstrate their ability to solve mathematics problems using technology as a tool.

- **Algebra** Students will demonstrate their ability to apply algebraic concepts to solve problems using Newton's Law of Cooling.

- **Functions** Students will demonstrate their ability to model real-world phenomena using functions.

- **Statistics** Students will demonstrate their ability to apply statistical concepts to problem solving by drawing inferences and making predictions from data.

Brief Overview:

Students will use a temperature probe along with the Calculator Based Laboratory and temperature programs in the TI-82 graphics calculator to determine an equation for the rate of cooling water. According to Newton's Law of Cooling, if the temperature surrounding an object remains constant, then under ideal conditions, temperature of the object will satisfy the equation

$$\frac{d\mathrm{T}}{dt} = k(\mathrm{T} - \mathrm{A})$$

where T is the temperatureof the object and A is the temperaturesurrounding the object

The students will work in groups to collect data points. Each group will plot their data points and graph a curve of best fit.

Each group will report to the class and compare their results. As a class, the students will discuss the conclusions reached and relate the experiment which they have just completed to selected problems in several calculus texts.

Grade/Level:

9 - 12 Calculus Students

Duration/Length:

The unit will require two or three 45 minute classes.

Prerequisite Knowledge:

Students should

- know safety procedures for a laboratory experiment.
- know how to use the calculator to graph data points.
- know how to determine a curve of best fit.
- understand linear, logarithmic, exponential, and power regression equations.

Objective:

- Students will be able to use technology to derive the formula for Newton's Law of Cooling.
- Students will be able to communicate the results of their experiment using the language of mathematics.

Materials/Resources/Printed Materials:

- Beaker of hot water
- CBL with temperature probe
- TI-82 graphics calculator with appropriate programs
- Student Worksheet
- Graph Paper or TI-Graph Link

Development/Procedures:

The class should be divided into groups of four or five students. Each group will be given a beaker containing hot water and a CBL unit with a temperature probe. Students will collect data points of the cooling water in degrees Celsius at intervals determined by the teacher. The data will be stored in L_2 and L_4 on the TI-82 calculator. Students will copy their data to the worksheet provided. The calculator will be used to make a scatter plot of the data points and a graph of the curve of best fit. Students will transfer these to graph paper or use the TI-Graph Link to make a hard copy. The students will write a statement of how the decision was made for the selection of the curve of best fit.

When each group has completed the lab experiment, the class will discuss what they know about cooling models. Some of the students may be familiar with Newton's Law of Cooling.

The teacher will present the differential equation for Newton's Law of Cooling and ask students to use their data to write an equation. At this point, the teacher will discuss separation of variables and how this type of integration can be used in other problems.

The students will be assigned exercises which require the use of Newton's Law of Cooling and use techniques of algebra and calculus to solve them.

Evaluation:

The teacher will circulate around the room to be sure students are on task and understand the requirements of the experiment. Evaluation will be based on performance in data collection, calculations, and oral report. Students will also be assigned application problems using the formula they have derived.

Extension/Follow Up:

Assign homework exercises which require the use of Newton's Law of Cooling.
Discuss other opportunities to use the CBL.

Authors:

Leil Waggoner	Linda Kreitlow
Walkersville High School	Hammond High School
Frederick County	Howard County

Temperature Chart

Room Temperature:________

Time (L_2)	Temperature(L_4)

Use graph paper or TI-Graph Link to copy the scatter plot and the curve of best fit.

- Why did you select your regression equation?
- Predict when the water temperature will be 30˚ C.
- Will the water temperature continue to decrease? To what temperature? Explain your answer.
- What factors affect the rate of cooling?

Student Worksheet

Procedure:

- Connect the CBL unit to the TI-82 Graphics Calculator with the unit-to-unit cable using the I/O port located on the bottom edge of each unit. (Be sure that the cable is pushed in firmly.)
- Connect the temperature probe to the Channel 1 (CH1) input on the top of the CBL unit.
- Determine the room temperature by reading a thermometer or by the CBL. Record this value on the worksheet.
- Turn on CBL and the calculator. Select the CBL and TEMP programs or another appropriate program. Press {HALT} key on the CBL unit.
- Place the probe in the beaker of hot water and wait until the temperature stops increasing. At this point, press {ENTER} to begin collecting data.
- Record data on worksheet.
- Select a curve of best fit using the regression equations on the calculator.
- Copy the scatter plot and curve of best fit onto worksheet or use the TI-Graph Link.

Notes to Teacher

- A variety of container sizes or volumes of water can be used to produce different exponential equations.
- Students have an exponential equation which they have copied from their calculators.
- The differential equation for Newton's Law of Cooling is $\frac{dT}{dt} = k(T - A)$, which is a linear equation.
- Discuss with students how they could manipulate their data to get a linear equation of the form y = mx + b. Allow students to make suggestions. Students at the North Carolina School of Science and Mathematics used (midpoint of T, dT/dt). If you wish to emphasize data analysis procedures, you might ask students to do a residual plot to see the "goodness" of fit of their linear model.
- When students are convinced that they have manipulated their data to produce an acceptable linear model of the form y = mx + b, write y as $\frac{dT}{dt}$ so $\frac{dT}{dt} = mT + b$.
- At this point, use the technique of separation of variables to solve the differential equation $\int \frac{1}{mT + b} dT = \int dt$
- Continue this discussion to find the function whose differential equation is given above.
- Assign problems from your calculus text. Students find the exercise in *Calculus*, Deborah Hughes-Hallett, Andrew M. Gleason, et al., 1996 pg 506 an interesting application of Newton's Law of Cooling.