Optimization for Breakfast!

Algebraic and Technological Investigations of Cereal Boxes and Their Contents

Brief Overview:

Students will use graphs, algebra, and graphing calculators to investigate and optimize box dimensions, meet dietary requirements, and find good consumer value using cereal boxes and their contents. Students will employ algebraic geometry, linear programming and curve fitting. Optional extensions include field work to measure sizes and evaluate unit prices versus purchase prices.

NCTM Content Standard/National Science Education Standard:

Algebra:

- Understand patterns, relations, and functions;
- Represent and analyze mathematical situations and structures using algebraic symbols;
- Use mathematical models to represent and understand quantitative relationships;
- Analyze change in various contexts;

Geometry:

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships;

Data Analysis & Probability:

- Develop and evaluate inferences and predictions that are based on data

Problem Solving:

- Apply and adapt a variety of appropriate strategies to solve problems.

Grade/Level:

Lesson 1: Algebra 1 (Grade 9) Oh My! What to Buy?
Lesson 2: Algebra 1 (Grade 7-Accelerated Algebra) Building the Best Cereal Box
Lesson 3: Algebra 2 (Grade 11) Milk for Breakfast?

Duration/Length:

Oh My! What to Buy? – One 90-minute class.
Building the Best Cereal Box – Two 60 minute classes.
Milk for Breakfast? – One 50-minute class.

Student Outcomes:

Students will:

- Use systems of inequalities and linear programming to optimize the cost of a nutritional breakfast.
- Create a scatter plot and line of best fit using technology.
- Analyze slope and y-intercept in the context of the problem.
- Determine the best consumer buy through mathematics.
- Calculate perimeter, area, and volume.
- Translate area and volume relationships into functional algebraic forms.
- Graph functions on a calculator.
- Find minimum and maximum values using trace and solve.
- Complete and replicate proofs of maximum area and maximum volume. (Advanced)

Materials and Resources:

Student Worksheets
Overhead Calculator / Student Calculators (TI-83+ or TI-84+)
Graph Paper
Cardboard Shoe Box

Development/Procedures:

Lesson 1
Lesson 2
Lesson 3

Summative Assessment:

Summative assessments are attached to the end of each lesson.

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Development/Procedures:

Lesson 1

Preassessment – Students should have the ability to do the following: graph ordered pairs and lines, create a scatter plot, write a linear equation given two points, identify the independent and dependent variables from a set of data, recognize the general idea of a line of best fit, identify the slope and *y*-intercept of a linear equation. Prior to using this lesson, students should all have TI-Interactive (TI-83+) or TI-Transformation Graphing (TI-84+) installed on their calculators.

Launch – You may want to group students prior to starting the lesson. Read introduction at the top of the worksheet aloud to the students. Ask students why they might want to do comparison shopping. Ask students if they can tell what the relationship is from such a large set of data, especially data that is not organized in any particular fashion. Bring two different size cereal boxes to class. Which cereal would you pick? Why? What kind of data sets do we normally use in class or see on county assessments? Is that the type of data we are likely to see in the real world?

Teacher Facilitation – Have the students read the directions at the bottom of the first worksheet. Explain the symbols. These symbols let them know what types of activities they should expect. Each time they get to analysis questions they should answer them and then wait for teacher direction.

Lead them into the first analysis question. Does weight depend on price or does price depend on weight? Ask the students to explain their choice. Direct them to complete the first action item (entering data and creating scatter plot). Once they get to choosing window, you will have to lead a discussion about how to go about choosing an appropriate window. Give them a few minutes to examine their graph and then facilitate discussion of correlation. Have the students discuss in their groups circumstances where there might be a negative correlation instead.

Direct the students to complete the next action item; writing the linear equation given two points. Students should work with their group to complete this item. You may want to have a representative from each group write their equation on the board. This will lead into the interpretation of what slope and *y*-intercept mean with respect to the data. Direct them to graph their line with their data and facilitate discussion of line of best fit and drawbacks to this method.

Initiate TI-Interactive/Transformation Graphing exercise. The purpose of this activity is to let the students visually examine changes in slope and *y*-intercept with respect to the data and the axes. Let the groups have a fairly lengthy amount of time to manipulate their equation. Circle the room assisting them with step increases, settings, etc. Give them a time frame for them to choose their final interactive equation. Lead a discussion of drawbacks to this method.

Direct the students to find the linear regression equation. Have students examine all three equations and discuss among themselves which is the best one. Lead class discussion about which one is the best and why.

Student Application – Students will analyze the data and models throughout the lesson, paying particular attention to the representation of slope and y-intercept in the context of the problem.

Embedded Assessment – Class discussion of analysis items and circulating during action items should give the teacher a good indication of student progress. Additionally, the evening activity is a good review for students. The teacher should be available for help/questions about the evening activity.

Reteaching/Extension – The extension activity could be assigned as part of the homework or used as a warm-up the next day. This activity is almost all analysis and has a legitimate link to real world application. There is also a summative assessment that could be used as a short quiz the next day.

NOTE: Data was originally collected at Giant grocery store in Silver Spring, Maryland. Numbers were then adjusted to fit the activity.

Oh My! What to Buy?!

Welcome! Today we are going to investigate the weight of several cereals versus their price per box. We would like to discover if there is a correlation between the two. If so, we would like to model the

Cereal Brand	Price ($)	Weight (oz)
Cheerios (Big)	4.79	20
Cheerios	4.19	15
Multi-Grain Cheerios	3.99	16
Lucky Charms	4.09	14
Trix	3.79	12
Wheaties	4.39	18
Apple Jacks	4.59	19.1
Cocoa Krispies	3.79	17.5
Cracklin' Oat Bran	4.49	17
Crispix	3.59	12
Froot Loops	4.59	19.7
Corn Flakes	3.49	13.1
Mini-Wheats	3.79	19
Special K	3.69	12
Cap'n Crunch	3.99	16
Life	4.29	21
Oatmeal Squares	3.79	16
Go Lean	3.19	14.1
Go Lean Crunch	3.99	15
Good Friends	3.29	14
Raisin Bran	4.99	25.5
Grape-Nuts	4.49	24
Lilo & Stitch Cereal	2.99	11.8
All Bran	3.29	11.4
Honeycombs	3.49	14.5
Fruit & Nature	3.13	12.2
Granola	3.39	16
Shredded Wheat	3.59	19
Honey Crunch	3.09	14.5
Crispers	2.99	12
Oat O's	3.19	15

relationship using a variety of methods.

Represents an analysis item. Consider these questions carefully and write an answer. Class discussion will follow.

Represents an action item. You should perform whatever steps follow this marker.

What is the independent variable with respect to this data? What is the dependent variable? Explain.

Enter the data into your lists and create a scatter plot using an appropriate window.

ENTERING DATA:

Press STAT, ENTER
Type your independent data into L1.
Type your dependent data into L2.
Press 2ND MODE to Quit.

CREATING A SCATTER PLOT:

Press 2ND Y=
Press ENTER to choose Plot 1:
Turn Plot 1 ON
Make your Xlist: L1 and Ylist: L2

CHOOSING A WINDOW:

Press WINDOW

Considering your data, choose an appropriate starting and stopping value for the independent (X) and dependent (Y) data.

Xmin:	Ymin:
Xmax:	Ymax:
Xscl:	Yscl:
Xres: 1	

Press GRAPH

What type of correlation exists between the weight of the cereal and price of the cereal? Explain.

Can you think of a situation where the relationship would have the opposite correlation? Give an example.

Choose two points from the data set and write a linear equation.
Round to two decimal places!!!

Points: ___________ ____________

Slope: ___________

Y-intercept: __________

Linear Equation: _________________________________

What does the *y*-intercept represent in the context of the problem?

What factors would cause the *y*-intercept to be something other than zero?

What does the slope represent in the context of the problem?

Graph your linear equation with the data:

Press Y=
Type your equation into Y1
Press GRAPH

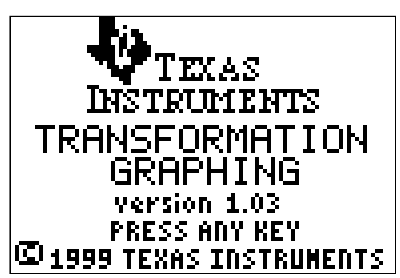
Do you think your line is **<u>the</u>** line of best fit for the data? How could you make it fit better?

What are the drawbacks of using this method?

TI-Interactive/Transformation Graphing allows us to make small adjustments without having to retype/edit the equation each time. Let us examine how!

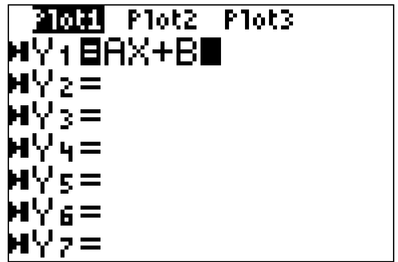
TI-Interactive/Transformation Graphing

TURN ON PROGRAM
Press APPS
Choose Interact (TI-83+) or Transfrm (TI-84+)

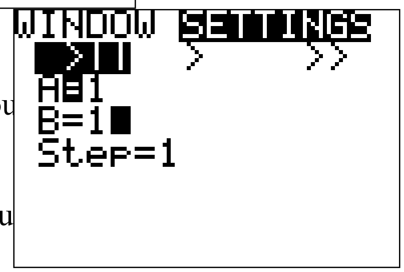


Press Any Key

Press Y=
Type Y1 = AX + B



Press WINDOW
Arrow UP to get to SETTINGS
Select Play/Pause-First Option



Choose values for A and B
(You should have some idea where to start based on you

Choose your step increase.
(Should you count by whole numbers, halves, tenths, hu

Press Graph
Highlight the = after A.
Use your right and left arrows to change A.
Adjust B the same way.

Write your final interactive/transformation equation:___________________________________

Are there any drawbacks to this method?

<u>CALCULATING A LINEAR REGRESSION EQUATION</u>:

Press STAT
Right arrow over to CALC
Choose 4:LinReg (ax+b)
Press ENTER

Write your final linear regression equation:____

Compare your derived linear equation, your interactive/transformation graphing equation, and the linear regression equation. Are they similar? Which one do you think is the best?

Cereal	Sugar (g)	Price ($)
Wheaties	4	4.39
Crispix	3	3.89
Corn Flakes	7	3.49
Special K	4	3.69
Life	6	4.29
Oatmeal Squares	10	3.79
Good Friends	9	3.29
Grape-Nuts	5	4.49
Lilo & Stitch Cereal	15	2.99
Honeycombs	12	3.49
Fruit & Nature	10	3.13
Granola	12	3.39
Shredded Wheat	11	3.59

Evening Activity:

- Enter the data into your calculator.
- Create a scatter plot of the data.
- What type of correlation does this data have, if any? Explain.

Find a line of best fit for the data. Choose any of the three methods we used in class.

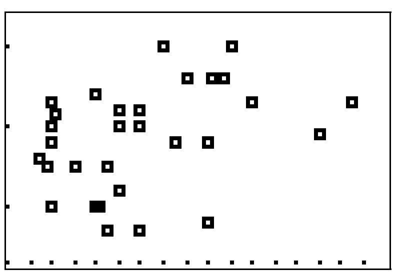
Equation: ____

What does the y-intercept mean in the context of the problem?

What does the slope mean in the context of the problem?

Extension:

Here is a picture of the **REAL** data collected from our local grocery store.



Does there appear to be a relationship between weight of the cereal and the price? Why or why not?

How can we decide which cereal to buy?

Let's say we have a box of Cheerios for \$4.99 and Lucky Charms for \$4.39. Which one would you buy if this is the only information you have?

What if I tell you the Cheerios is a 20 oz. box and the Lucky Charms is a 14 oz. box. Does that affect your decision?

How can I mathematically determine which one is the better buy?

Cereal Brand	Price ($)	Weight (oz)
Cheerios (Big)	4.79	20
Cheerios	4.19	15
Multi-Grain Cheerios	3.99	16
Lucky Charms	4.09	14
Trix	3.79	12
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Cracklin' Oat Bran	4.49	17
Crispix	3.59	12
Froot Loops	4.59	19.7
Corn Flakes	3.49	13.1
Mini-Wheats	3.79	19
Special K	3.69	12
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All Bran	3.29	11.4
Honeycombs	3.49	14.5
Fruit & Nature	3.13	12.2
Granola	3.39	16
Shredded Wheat	3.59	19
Honey Crunch	3.09	14.5
Crispers	2.99	12
Oat O's	3.19	15

Grocery stores are required to provide this information to the consumers. Watch out!! They frequently give these numbers in different units making it very difficult to make an accurate comparison. General rule of thumb….store brand is typically cheaper than name brand and the ingredients are almost always the same!!! Check it out for yourself!

Oh My! What to Buy?! (KEY)

Welcome! Today we are going to investigate the weight of several cereals versus their price per box. We would like to discover if there is a correlation between the two. If so, we would like to model the relationship using a variety of methods.

Represents an analysis item. Consider these questions carefully and write an answer. Class discussion will follow.

Represents an action item. You should perform whatever steps follow this marker.

What is the independent variable with respect to this data? What is the dependent variable? Explain.

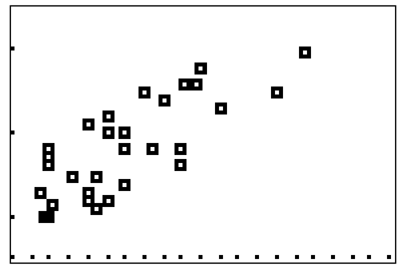
The independent variable is the weight of the cereal. The dependent variable is price since the price of the cereal depends on how much (weight) you receive.

Enter the data into your lists and create a scatter plot using an appropriate window.

ENTERING DATA:

Press STAT, ENTER
Type your independent data into L1.
Type your dependent data into L2.
Press 2ND MODE to Quit.

CREATING A SCATTER PLOT:



Press 2ND Y=
Press ENTER to choose Plot 1:
Turn Plot 1 ON
Make your Xlist: L1 and Ylist: L2

CHOOSING A WINDOW:

Press WINDOW

Considering your data, choose an appropriate starting and stopping value for the independent (X) and dependent (Y) data. **Suggested Window**

Xmin: **10**	Ymin: **2.5**
Xmax: **30**	Ymax: **5.5**
Xscl: **1**	Yscl: **1**
Xres: 1	

Press GRAPH

Is there a relationship between the weight of the cereal and price of the cereal? Explain.

There is a positive correlation. As weight of the cereal increases, price increases.

Can you think of a situation where the relationship would have the other correlation? Give an example.

Answer may vary. When you buy in bulk, the price per unit can sometimes decrease. The more you buy, the better the deal, i.e. bags of mulch are more expensive per square yard than truckloads of mulch per square yard.

Choose two points from the data set and write a linear equation.
Round to two decimal places!!! **Answers will vary. Example answer.**

Points: **(16 , 3.99) (13.1 , 3.49)**

Slope: **0.17**

3.99 = 0.17 (16) + b
3.99 = 2.72 + b

3.99 – 2.72 = b

Y-intercept: <u>**1.27**</u>

Linear Equation: $\underline{y = 0.17x + 1.27}$

What does the *y*-intercept represent in the context of the problem?

The y-intercept represents the price of a cereal that weighs zero ounces.

What factors would cause the *y*-intercept to be something other than zero?

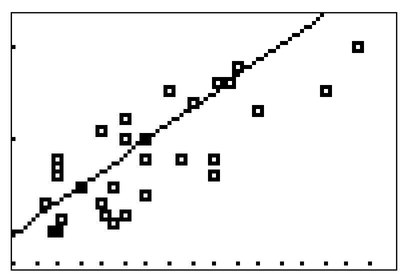
Answers may vary. The cost of production, warehouse, packaging, marketing, paying employees, distribution, etc.

What does the slope represent in the context of the problem?

Slope represents the increase in cost per ounce of cereal.

Graph your linear equation with the data:

Press Y=
Type your equation into Y1
Press GRAPH



Do you think your line is <u>**the**</u> line of best fit for the data? How could you make it fit better?

Answers may vary. Increasing/Decreasing slope, increasing/decreasing *y*-intercept.

What are the drawbacks of using this method?

Answers may vary. Using two points with a data set this size will not necessarily produce the most accurate line of best fit.

TI-Interactive/Transformation Graphing

<u>TURN ON PROGRAM</u>

Press APPS
Choose Interact (TI-83+) or Transfrm (TI-84+)

Press Any Key

Cereal	**Sugar (g)**	**Price ($)**
Wheaties	4	4.39
Crispix	3	3.89
Corn Flakes	7	3.49
Special K	4	3.69
Life	6	4.29
Oatmeal Squares	10	3.79
Good Friends	9	3.29
Grape-Nuts	5	4.49
Lilo & Stitch Cereal	15	2.99
Honeycombs	12	3.49
Fruit & Nature	10	3.13
Granola	12	3.39
Shredded Wheat	11	3.59

Press Y=
Type Y1 = AX + B

Press WINDOW
Arrow UP to get to SETTINGS
Select Play/Pause-First Option

Choose values for A and B
(You should have some idea where to start based on your linear equation.)

Choose your step increase.
(Should you count by whole numbers, halves, tenths, hundredths, etc.?)

Press Graph
Highlight the = after A.
Use your right and left arrows to change A.
Adjust B the same way.

Write your final interactive/transformation equation: **Answers will vary.**

Are there any drawbacks to this method?

Answers may vary. These equations still carry an element of human error since we are adjusting our equations based on our perception of where the line of best fit should be with respect to the data.

CALCULATING A LINEAR REGRESSION EQUATION:

Press STAT

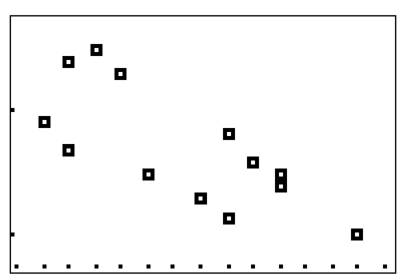
Right arrow over to CALC
Choose 4:LinReg (ax+b)
Press ENTER

Write your final linear regression equation: $\underline{y = 0.12x + 1.90}$

Compare your derived linear equation, your interactive/transformation graphing equation, and the linear regression equation. Are they similar? How? Which one do you think is the best?

Answers may vary. Students generally decide that the linear regression equation is best since it is calculator derived.

Evening Activity:



Enter the data into your calculator.

Create a scatter plot of the data.

What type of correlation does this data have, if any? Explain.

This data represents a negative correlation. As sugar increase, the cost decreases.

Find a line of best fit for the data. Choose any of the three methods we used in class.

Equation: $y = -0.09x + 4.46$

What does the y-intercept mean in the context of the problem?

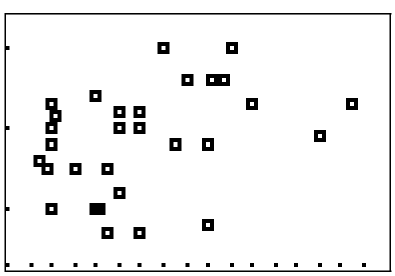
Cost of the cereal when sugar content is zero grams.

What does the slope mean in the context of the problem?

Decrease in cost per gram of sugar.

Extension:

Here is a picture of the REAL data collected from our local grocery store.



Does there appear to be a correlation between weight of the cereal and the price? Why or why not?

Not really, the scatter plot is literally scattered everywhere which makes it difficult to discern any kind of relationship.

Let's say we have a box of Cheerios for \$4.99 and Lucky Charms for \$4.39. Which one would you buy if this is the only information you have?

Lucky Charms since it's cheaper.

What if I tell you the Cheerios is a 20 oz. box and the Lucky Charms is a 14 oz. box. Does
that affect your decision?

Yes/Maybe, it appears that Cheerios may be the better choice.

How can I mathematically determine which one is the better buy?

Divide the price by the weight. Cheerios are approximately 25 ¢ per ounce
Lucky Charms are approximately 31 ¢ per ounce. Lucky Charms is the best buy based on unit price.

Grocery stores are required to provide this information to the consumers. Watch out!! They frequently give these numbers in different units making it very difficult to make an accurate comparison. General rule of thumb….store brand is typically cheaper than name brand and the ingredients are almost always the same!!! Check it out for yourself!

Summative Assessment Lesson 1

Several physics students are performing an experiment that involves rolling marbles down a four foot long tube and measuring how far the marble rolled from the end of the tube. The tube is being raised at a constant rate after each trial. The students have collected data and are trying to figure out the equation they might use to model the data. Can you help them?

Height of Book Stack (cm)	Distance Marbles Rolled (cm)
3.75	40.23
7.5	100.7
11.25	151.1
15	192.97
18.75	236.2
22.5	290.7

Enter the data in your calculator.

Create a scatter plot of the data.

What type of correlation does this data have, if any? Explain.

Find a line of best fit for the data. Choose any of the three methods we used in class.

Equation: ____

What does the *y*-intercept mean in the context of the problem?

What does the slope mean in the context of the problem?

Summative Assessment Lesson 1 **(KEY)**

Several physics students are performing an experiment that involves rolling marbles down a four foot long tube and measuring how far the marble rolled from the end of the tube. The tube is being raised at a constant rate after each trial. The students have collected data and are trying to figure out the equation they might use to model the data. Can you help them?

Height of Book Stack (cm)	**Distance Marbles Rolled (cm)**
3.75	40.23
7.5	100.7
11.25	151.1
15	192.97
18.75	236.2
22.5	290.7

Enter the data in your calculator.

Create a scatter plot of the data.

What type of correlation does this data have, if any? Explain.

As the height of the book stack increases, the distance the marble rolls increases.

Find a line of best fit for the data. Choose any of the three methods we used in class.

Equation: $y = 12.96x - 1.422$

What does the *y*-intercept mean in the context of the problem?

When the height of the book stack is zero, the distance the marble rolls is -1.422 cm. Since negative distance is not possible in this situation, students should add that this means that the marble did not roll at all.

What does the slope mean in the context of the problem?

The marble rolls approximately 13 centimeters for each centimeter that the book stack is raised.
Development/Procedures:

Lessons 2A,2B

Preassessment

Students will review understanding of concepts of perimeter, area, and volume.
Students will practice using *Trace* and *Calc* on the graphing calculators.

Launch

Ask students to tell you what they remember about perimeter, area and volume and discuss, focusing on rectangles and rectangular solids.

Lesson 2A

Teacher Facilitation

Distribute Activity 2A.1 to students and have them take turns reading the instructions. Review elementary area and perimeter problems with students; these are necessary for our understanding of volume and the optimization of the box volume. Complete Activity 2A.1.

Check to make sure all students have found at least the rectangular possibilities and assist slower students to find missing figures. Calculate perimeter, area, and perimeter/area with common sets of measurements. Invite comments and encourage their discovery of fact that the "squarer" the dimensions are the smaller the perimeter. Solicit ideas on what to do if dimensions were not integers.

Conduct a discussion of approaches to the 36 square unit problem and the final answer they might get. Have students proceed on their own, filling in the table. The completion of this table can serve as an embedded assessment.

Hand out Activity 2A.2. Develop the same algebraic material that appears in the student material on the blackboard or an overhead. Expressing the length as a function of width, given an area, may require some review. Make sure students can explain the formula and identify the meaning of the numbers and the variables.

Direct students to enter the formula and check their graphs. Make sure everyone can find the minimum with **Trace**, and then use the **Calc** and **Minimum** functions to find the exact minimum. (The most common problem in using the **Minimum** is failing to bracket the x’s where the minimum occurs). Check that everyone has found the minimum perimeter of 24 units when $x = 6$ and that the ratio of perimeter to area is 1.5 for this example.

Now have students complete the table in Activity 2A.2. (For each case, they should find a widths and lengths that are identical and equal to the square root of the area; see teacher's guide for answers). *Proper completion of this table can serve as an embedded assessment.*

Share some of the "Your Choice" results and have class compare answers, then discuss the questions following the table. Make sure all students now arrive at the idea that width and length must be the same, and therefore the optimal shape is a square whose side is the square root of the area. As an advanced extension, have strong students try to prove the result algebraically. (See Teacher's Guide Activity 2A.3 for one approach to lead them through).

Student Application

This activity can be conducted as a whole class activity or be done in small groups. Students will relate area and volume calculations to real life objects, calculate perimeter and area using calculators, and perform simple optimizations on the graphing calculator.

Re-teaching/Extension

Students with difficulty with the concepts may require physical objects and coloring of graph paper to solidify area concepts. They may need to review basic algebraic manipulations to work with new examples with whole number answers.
In addition, they may need to review calculator procedures.

For an extension, have advanced students try to prove algebraically that the minimum ratio of perimeter to area for a rectangle is achieved when the shape is a square. (See Teacher's Guide Activity 2A.3 for one approach to lead them through).

Embedded Assessments

Teacher will circulate around room to verify completion of tables, check results on calculators, and see that questions have been answered.

Lesson 2B

Teacher facilitation

Ask students to recall the original problem with the Puffos™ box. Hand out activities 2B.1. Review what was covered in the lesson 2A, especially the concept of a square being the rectangular shape that produces minimal perimeter for a given area. Review volume calculations with simple board examples.

Read Activity 2B.1 together. Go through the example given for the first row together, and check answers. Have students complete the table. Completion of this table is an embedded assessment. Discuss questions after the table. Make sure students understand the idea of solving for the height from the base area; practice with examples at the board. Check student completion of table, and discuss answers to questions after the table.

First develop the algebra found in Activity 2B.2 on the board or overhead, then hand out to students and re-read together. This algebraic reasoning is the critical element of the whole exercise.

Hand out Activity 2B.3, and have students complete questions and tables. Check that all students arrive at a final answer for the last question.

Student Application

This activity can be conducted as a whole class activity or be done in small groups. Students will relate area and volume calculations to real life objects, calculate perimeter and area using calculators, and perform simple optimizations on the graphing calculator.

Reteaching/Extension

For re-teaching, use arrangements of physical stacks of cubes to illustrate volume concepts, cut out a shoebox and open it up to illustrate the lateral area concept, And review algebra and try new examples on paper and on the calculator, emphasizing cases that are perfect cubes.

For an extension, have advanced students prove algebraically that the minimum ratio of area to volume is achieved when the shape is a cube. See Teachers Guide Activity 2B.4. Advanced students can also try to determine the radius and height of a cylindrical box with a specified volume that has a minimal surface area. Finally, consider that no cereal boxes are actually shaped like cubes. Have students discuss, research and apply factors that manufacturers might consider more important that the amount of cardboard used in the box.

Embedded Assessments

Teacher will circulate around room to verify completion of tables, check results on calculators, and see that questions have been answered.

Additional assessment activities by level

Basic

- Students recall and apply area and volume formulas.
- Students calculate sides of rectangle of specific area that minimize perimeter.
- Students calculate dimensions of rectangular solid of specific volume that minimizes surface area.

Standard (in addition to behaviors above)

- Students demonstrate algebra necessary to set up minimization problems on calculator.
- Students solve minimization problems on calculator.
- Students reproduce and apply general formulas for minimization problems.

Advanced (in addition to behaviors above)

- Students derive perimeter minimizing dimensions using algebra.

Highly advanced (in addition to behaviors above)

- Students derive the volume minimizing dimensions using algebra

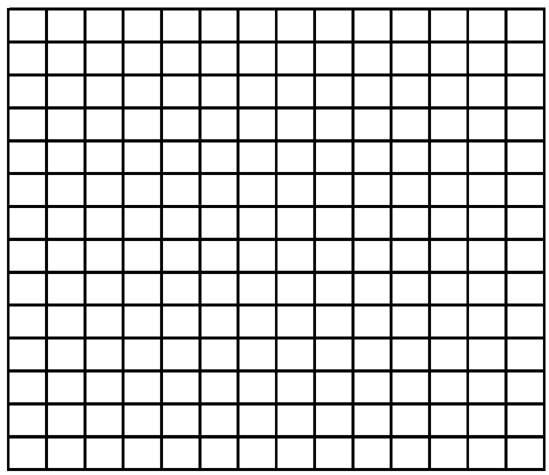
Activity 2A.1 Review of Perimeter and Area

Our ultimate goal is to find the best dimensions for a box of Puffos™ cereal with a volume of 150 cubic inches. When manufacturers decide what shape and size to make their boxes, they need to consider both manufacturing costs and consumer preferences. We're going make the simplified assumption that the best box minimizes the total surface area for a given volume.

First, we need to think about perimeter and area.

Use the graph paper below to lay out all the possible rectangles with *integer* dimensions that have an area of 12 square units.

Enter your results in the table below. Then finish the table by calculating the ratio of perimeter to area.



Width	*Length*	*Perimeter*	*Area, Units*^{2}	$\frac{\textit{Perimeter}}{\textit{area}}$
			12	
			12	
			12	
			12	
			12	
			12	
			12	

Suppose this is a fencing problem. We might want to enclose a specific area with the shortest possible fence.

What rectangle would enclose an area of twelve square units with the smallest perimeter?

____.

Now consider an area of 36 square units. What dimensions do you predict will minimize the perimeter?

____.

Write all possible sets of integer dimensions and calculate the ratio as before to check your answer.

Width	*Length*	*Perimeter*	*Area Units*^{2}	$\frac{Perimeter}{Area}$
			36	
			36	
			36	
			36	
			36	
			36	
			36	

Now we'll use algebra and the calculator to investigate the problem more carefully.

Activity 2A.2 Applying algebra and using the calculator for perimeter minimization

Let w be the width, l the length, P the perimeter, and A the area of a rectangle.

$A = wl$, so $l = \frac{A}{w}$. $P = 2w + 2l$. Substitute for l and get $P = 2w + 2\frac{A}{w}$.

Perimeter is now a function of area and width. For a *given area*, perimeter will be a function only of w. Now we'll use the calculator to minimize the perimeter.

We will use an area of 36 square units from Activity 2A.1

Using the **"*y*="** function on your calculator, enter **"*y* = 2*x* + 2*36/*x*".** ***y*** is the perimeter, ***x*** is the width, and ***36*** is the area. This formula will calculate the perimeter (***y***) for all choices of width (***x***) for a rectangle with area 36.

Use the trace function to find the value of x that gives the smallest perimeter (the lowest point on the graph) and record the value. ___________________.

Now find a precise value by using the "Calc" and "Minimum" functions. Did this result match your initial conception? _____________________________.

Practice using the same procedure with the following areas. Calculate the perimeter and the ratio of area to perimeter. (Put each new area into the formula instead of the 36).

Area, Units^2	*Best Length*	*Best Width*	*Best Perimeter*	$\frac{Perimeter}{Area}$
16				
25				
100				
2				
3				
1000				
(Your choice)				

How would you describe the best solution in words?

____.

Can the proper dimensions be calculated directly from the area?_______________.

Try your idea with an area of 144 square units. _____________________________.

Write a formula for the optimal dimensions given any area A, and check your formula with your teacher.

Activity 2B.1 Boxes with minimum surface area

Suppose the manufacturer of Puffos™ want to pack cereal in the box with least surface area.

For a rectangular box, we can multiply the width by length by height. Consider a box with width, length, and height of 2, 3, and 8 units. The volume of this box is 2 x 3 x 8 = 48 cubic units. To determine the surface area of this box, we need to add up the areas of all six sides.

We will have two sides that are 2 by 3, two sides that are 3 by 8, and two sides that are 2 by 8. The total area will be 12 + 48 + 32 = 92 square units.

Find as many combinations of *integer* dimensions as you can, calculate the total surface area and ratio of area to volume. Don't duplicate dimensions!

Width	*Length*	*Height*	*Area Units^2*	*Volume Units^3*	$\frac{Area}{Volume}$
2	**3**	**8**	**92**	**48**	**1.917**
1	**1**	**48**		**48**	
				48	
				48	
				48	
				48	
				48	
				48	
				48	
				48	

Which box dimensions provide the best (highest) ratio of volume to area?

____.

Can you predict the exact (perhaps non-integer!) dimensions that would provide the smallest area for this box of 48 cubic units?

____.

Write an equation for the best dimension(s) of a box with volume *V*.

____

Activity 2B.2. Preliminary algebra (study this after your teacher's presentation)

To put our problem into a form that works well on the calculator, we'll need to do a little algebra.

For a rectangular solid with width w, length l, and height h, volume V = $V = wlh$. If we pick a specific volume V for the box, we can solve for the height: $h = V/wl$. However, wl is the area (A) of the base of the box, so we can say that, for a given volume V, $h = V/A$. In other words, if we pick the volume and the area, the height of the box is determined.

The surface area of a rectangular box can be looked at as 3 pairs of rectangular areas, as in activity 2B.1, but there is another way. Imagine the box with the top and bottom removed. How could we determine the surface area of the walls (the lateral area)?

Imagine cutting the "walls" down one corner and then flattening out this sheet. This sheet would be as *long* as the perimeter of the base, and as *high* as the height of the cube.

So, the lateral area equals the perimeter of the base times the height. The total surface area (TA) of the box is that of two bases plus the lateral area.

$$TA = 2wl + (2w + 2l)h$$

Suppose we pick a base area B and a volume V for the box. Then the height, h, is determined by $h = V/B$. The total area of the box is therefore that of two bases plus the perimeter times the height.

$$TA = 2B + Ph = 2B + \frac{V}{B}$$

From Activity 2A.2, we already know that the smallest perimeter for a rectangle of a given area occurs when the width and height are the same. In other words, the base of the box will be a square. Let x equal the side of this square base. Then the area of the base is x^2 and the perimeter of the base is $4x$. We can write the total area as

$$TA = 2x^2 + 4x\frac{V}{x^2}$$

Now we have the total surface area of the box written as a function of our choice for the edge of the (square) bottom. On the calculator, $\boldsymbol{x}$ will be the length of this edge, and $\boldsymbol{y}$ will be the total area.

Now we are ready to solve this problem on the calculator.

Activity 2B.3 Minimizing surface area for a given volume on the calculator

Enter "y = 2* x^2 + 4x*48/ x^2" on your graphing calculator.

The 48 is the volume of the box, the $2x^2$ is the area of the bases, the $4x$ is the perimeter of the base, and the $\frac{48}{x^2}$ is the height. This formula gives us the total surface area for any choice of x for the base of box a rectangle with volume 48 cubic units.

Now, use the trace function to find the value of x that gives the smallest area (the lowest point on the graph). ____.

Find a precise value by using the Calc and Find Mininum function. _______________.
You just found the side of the base that minimizes the volume of a rectangular box!

Did these values match your initial conception? ______________________________.

Practice using boxes with the following volumes, and calculate the ratio of area to volume.
Change the 48 in your formula to the new volume. Calculate the height by dividing the volume by x^2.

Volume Cubic units	*Best Base Side length*	*Best Height*	*Smallest Surface area Units*^{2}	$\frac{\textit{Area}}{\textit{Volume}}$
8				
27				
81				
125				
3				
10				
100				

Without resorting to the calculator, can you predict the width, length, and height, surface area, and ratio to volume for the rectangular box with a volume of 64 cubic inches?

____.

For a volume of 10,000 cubic inches? ________________________.

What formula expresses the dimensions of a minimal surface box with given volume V?

____.

And now, what should the dimensions of the Puffos™ box (volume of 343 cubic inches) be?

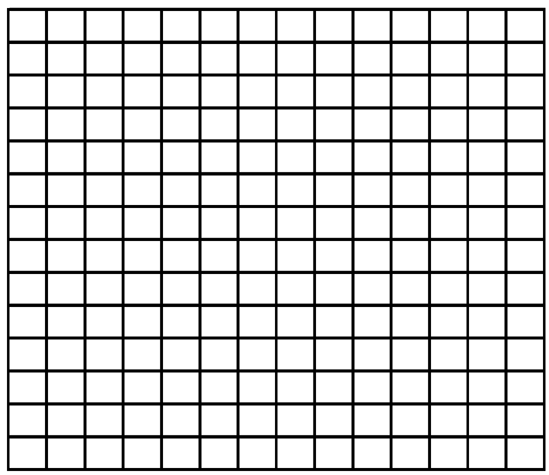
Teacher's Guide
Activity 2A.1 Review of Perimeter and Area

Our ultimate goal is to find the best dimensions for a box of Puffos™ cereal with a volume of 150 cubic inches. When manufacturers decide what shape and size to make their boxes, they need to consider both manufacturing costs and consumer preferences. We're going make the simplified assumption that the best box minimizes the total surface area for a given volume.

First, we need to think about perimeter and area.

Use the graph paper below to lay out all the possible rectangles with *integer* dimensions that have an area of 12 square units.

Enter your results in the table below. Then finish the table by calculating the ratio of perimeter to area.



Width	*Length*	*Perimeter*	*Area, Units*^{2}	$\frac{\textit{Perimeter}}{\textit{area}}$
1	**12**	**26**	**12**	**2.17**
2	**6**	**16**	**12**	**1.33**
3	**4**	**14**	**12**	**1.17**

Suppose this is a fencing problem. We might want to enclose a specific area with the shortest possible fence.

What rectangle would enclose an area of twelve square units with the smallest perimeter?

integer solution is 3 units by 4 units; non-integer optimum is 3.464 (= $2\sqrt{3}$) units

Now consider an area of 36 square units. What dimensions do you predict will minimize the perimeter?

6 units x 6 units

Write all possible sets of integer dimensions and calculate the ratio as before to check your answer.

Width	*Length*	*Perimeter*	*Area Units^2*	$\frac{Perimeter}{Area}$
1	36	74	36	2.056
2	18	40	36	1.111
3	12	30	36	0.833
4	9	26	36	0.722
6	6	24	36	0.667

Now we’ll use algebra and the calculator to investigate the problem more carefully.

Now we’ll use algebra and the calculator to investigate the problem more carefully.

Teacher's Guide

Activity 2A.2 Applying algebra and using the calculator for perimeter minimization

Let w be the width, l the length, P the perimeter, and A the area of a rectangle.

$A = wl$, so $l = \frac{A}{w}$. $P = 2w + 2l$. Substitute for l and get $P = 2w + 2\frac{A}{w}$.

Perimeter is now a function of area and width. For a *given area*, perimeter will be a function only of w. Now we'll use the calculator to minimize the perimeter.

We will use an area of 36 square units from Activity 2A.1

Using the ***"y="*** function on your calculator, enter ***"y = 2x + 2*36/x".*** ***y*** is the perimeter, ***x*** is the width, and ***36*** is the area. This formula will calculate the perimeter (***y***) for all choices of width (***x***) for a rectangle with area 36.

Use the trace function to find the value of x that gives the smallest perimeter (the lowest point on the graph) and record the value.<u> around 6 </u>**.**

Now find a precise value by using the "Calc" and "Minimum" functions. Did this result match your initial conception?<u> 6 (Discuss) .</u>

Practice using the same procedure for finding the best length (and width) for minimizing the perimeter of rectangles with the following areas. Calculate the perimeter and the ratio of area to perimeter. (Put each new area into the formula instead of the 36).

Area, Units^2	*Best Length*	*Best Width*	*Best Perimeter*	$\frac{Perimeter}{Area}$
16	4	4	16	1.0
25	5	5	20	0.8
100	10	10	40	0.4
2	1.414	1.414	5.657	2.828
3	1.732	1.732	6.928	2.309
1000	31.623	31.623	126.491	126.491
(Your choice)				

How would you describe the best solution in words?

<u>*Smallest perimeter occurs when length and width are the same* .</u>

Can the proper dimensions be calculated directly from the area?<u> *Yes .*</u>

Try your idea with an area of 144 square units. <u>*Best size is 12 units by 12 units .*</u>

Write a formula for the optimal dimensions given any area A, and check your formula with your teacher. $w = l = \sqrt{A}$

Teacher's Guide

Activity 2A.3 Advanced extension! Finding the minimum perimeter using algebra

Often, minima and maxima of functions are found using calculus. However, we can do some simple algebra to prove the result we discovered experimentally, above.

Suppose we have a rectangle with width w and length l. These two values will have an average, $\frac{w+l}{2}$. Suppose we let x equal this average.

The length and the width are each the same distance, k, (above and below, respectively) from the average. So $w = x - k$ and $l = x + k$.

Area = wl, so $A = (x-k)(x+k)$. By FOIL, $A = x^2 - k^2$.

So we can make area A a maximum by making k as small as possible. This will obviously happen when k = 0, in which case the length and width are the same. So the rectangle with the smallest perimeter with given area A has width = length. It must be a square!

The perimeter minimizing shape for a rectangle is a square with side s, and $A=s^2$ and $s = \sqrt{A}$.

Teacher's Guide

Activity 2B.1 Boxes with minimum surface area

Suppose the manufacturer of Puffos™ want to pack cereal in the box with least surface area.

For a rectangular box, we can multiply the width by length by height. Consider a box with width, length, and height of 2, 3, and 8 units. The volume of this box is 2 x 3 x 8 = 48 cubic units. To determine the surface area of this box, we need to add up the areas of all six sides.

We will have two sides that are 2 by 3, two sides that are 3 by 8, and two sides that are 2 by 8. The total area will be 12 + 48 + 32 = 92 square units.

Find as many combinations of *integer* dimensions as you can, calculate the total surface area and ratio of area to volume. Don't duplicate dimensions!

Width	*Length*	*Height*	*Area Units^2*	*Volume Units^3*	*Ratio of Area/Volume*
2	**3**	**8**	**92**	**48**	**1.917**
1	**1**	**48**	**194**	**48**	**4.041**
1	**2**	**24**	**148**	**48**	**3.083**
1	**3**	**16**	**134**	**48**	**2.792**
1	**4**	**12**	**128**	**48**	**2.667**
1	**6**	**8**	**124**	**48**	**2.583**
2	**3**	**8**	**92**	**48**	**1.917**
2	**4**	**6**	**88**	**48**	**1.833**
3	**4**	**4**	**80**	**48**	**1.667**
4	**6**	**2**	**88**	**48**	**1.833**

Which box dimensions provide the best (highest) ratio of volume to area?

3 units x 4 units x 4 units .

Can you predict the exact (perhaps non-integer!) dimensions that would provide the highest area for this box of 48 cubic units?

3.634 units .

Write an equation for the best dimension(s) of a box with volume *V*.

$w = l = h = \sqrt[3]{V}$ but emphasize we only a *theory* so far, we haven't proved it

Teacher's Guide

Activity 2B.3 Minimizing surface area for a given volume on the calculator

Enter "y = 2* x^2 + 4x*48/ x^2" on your graphing calculator.

The 48 is the volume of the box, the $2x^2$ is the area of the bases, the $4x$ is the perimeter of the base, and the $\frac{48}{x^2}$ is the height. This formula gives us the total surface area for any choice of x for the base of box a rectangle with volume 48 cubic units.

Now, use the trace function to find the value of x that gives the smallest area (the lowest point on the graph). Around 3.6 units .

Find a precise value by using the Calc and Find Mininum function. 3.634 units **.**
You just found the side of the base that minimizes the volume of a rectangular box!

Did these values match your initial conception? They should! But discuss.

Practice using boxes with the following volumes, and calculate the ratio of area to volume.
Change the 48 in your formula to the new volume. Calculate the height by dividing the volume by x^2.

Volume Cubic units	*Best Base Side length*	*Best Height*	*Smallest Surface area Units^2*	$\frac{\textit{Area}}{\textit{Volume}}$
8	**2**	**2**	**24**	**3**
27	**3**	**3**	**54**	**2**
81	**4.327**	**4.327**	**112.325**	**1.387**
125	**5**	**5**	**150**	**1.2**
3	**1.442**	**1.442**	**12.481**	**4.160**
10	**2.154**	**2.154**	**27.85**	**2.785**
100	**4.642**	**4.642**	**129.266**	**1.293**

Without resorting to the calculator, can you predict the width, length, and height, surface area, and ratio to volume for the rectangular box with a volume of 64 cubic inches?

4 inches x 4 inches x 4 inches 96 sq inch surface area ratio = 1.5

For a volume of 10,000 cubic inches? 21.544 inches .

What formula expresses the dimensions of a minimal surface box with given volume V?

$$w = l = h = \sqrt[3]{V}$$

And now, what should the dimensions of the Puffos™ box (volume of 343 cubic inches) be?
7 x 7 x 7 inches

Teacher's Guide

Activity 2B.4 Advanced extension! Finding the minimum surface area using algebra

Though it is more difficult, the result for the box can also be proven with algebra. Let x be the average of the dimensions of width, length, and height, and let $x+k$ represent the width of the box. For minimum area, the length must be the same size, $x+k$, as we then proved in 2A.3.

Since average of the dimensions is x, the height must be equal to $x-2k$.
The volume is given by the product of all the dimensions:

$$V = \left(x+k\right)\left(x+k\right)(x-2k)$$

The total surface area is given by the sum of the top and bottom areas and the perimeter of the base times the height, or twice the base area plus the lateral area.

$$A = 2(x+k)(x+k)+4(x+k)(x-2k)$$

Expanding this, we obtain $A = 2x^2+4xk+2k^2+4x^2+4kx-8kx-8k^2$

Combining terms and then factoring, $A = 6x^2-6k^2 = 6(x+k)(x-k)$

So we can write the ratio of volume over area as

$$\frac{V}{A} = \frac{(x+k)(x+k)(x-2k)}{6(x+k)(x-k)} = \frac{(x+k)(x-2k)}{6(x-k)}$$

Multiplying out, we get

$$\frac{V}{A} = \frac{1}{6}\frac{(x^2-kx-2x^2)}{(x-k)} = \frac{1}{6}\frac{x(x-k)-2k^2}{(x-k)} = \frac{1}{6}(x-\frac{2k^2}{(x-k)})$$

For what value of k will this function be the largest? If positive, the value of k must be smaller than x**,** because otherwise we have a 0 or negative dimension for the width. Then $x-k$ is positive, and $\frac{2k^2}{(x-k)}$ is positive, so subtracting it reduces the ratio of $\frac{V}{A}$. To subtract the smallest possible amount, $\frac{2k^2}{(x-k)}$ = 0, and so k must be zero. On the other hand, if k is negative, $x-k$ is positive and k^2 is positive, so $\frac{2k^2}{(x-k)}$ is also positive. Again, the minimum occurs when $k=0$. Thus the best choice of dimensions for minimizing the surface area of a box is that they are all equal!

$$V = x^3 \text{ and } x = \sqrt[3]{V} \ !$$

Summative Assessments Lessons 2A, 2B

1. Calculate the perimeter and area of a rectangle with sides 4 inches and 9 inches.

   ____.

2. Calculate the dimensions and perimeter of the rectangle that contains an area of 81 square inches that has the smallest possible perimeter.

____.

3. Calculate the dimensions and perimeter of the rectangle that contains an area of 30 square inches that has the smallest possible perimeter.

____.

4. Using the graphing calculator, demonstrate finding the answer to 3. using Trace and Calc Minimum, show to instructor.

5. Calculate the surface area and volume of a cube with dimensions 10 cm by 30 cm by 40 cm.

____.

6. Calculate by any means the dimensions and surface area of a box with a volume of 9000 cm^3 that minimize the surface area.

   ____.

Answer key
Summative Assessments Lessons 2A, 2B

1. Calculate the perimeter and area of a rectangle with sides 4 inches and 9 inches.

   Perimeter = 26 inches, area = 36 inches

2. Calculate the dimensions and perimeter of the rectangle that contains an area of 81 square inches that has the smallest possible perimeter.

   9 inches x 9 inches; perimeter = 81 square inches

3. Calculate the dimensions and perimeter of the rectangle that contains an area of 30 square inches that has the smallest possible perimeter.

   5.477 inches x 5.477 inches; perimeter = 21.909 square inches

4. Using the graphing calculator, demonstrate finding the answer to 3. using Trace and Calc Minimum.

   Students will show instructor curve and location of point.

5. Calculate the surface area and volume of a cube with dimensions 10 cm by 30 cm by 40 cm.

surface area = 2 x 10 x 30 + (2 x 10 + 2 x 30) x 40 = 3800 cm^2; vol = 10 x 30 x 40 = 12,000 cm^3.

6. Calculate by any means the dimensions and surface area of a box with a volume of 9000 cm^3 that minimize the surface area.

   all dimensions are 20.801cm, and surface area is 2596.09 cm^2

Lesson 3

Preassessment – Prior to starting the activity, students should have an understanding of graphing linear inequalities and systems of linear inequalities. At the beginning of the class, give the students the “Revisiting Linear Inequalities” worksheet that reviews these concepts. Allow the students 5 minutes to complete the assignment, and discuss the exploration questions as a group.

Launch – Ask students what kind of breakfast foods they like and if they are aware of the nutritional and dietary value of the breakfast foods that they eat. Point out that sometimes, healthy foods may be more expensive than unhealthy foods but that there is a way to eat healthy and pay less.

Teacher Facilitation – Hand the worksheets to the class and ask a student to read the problem aloud. Before answering the questions discuss the information that is given and what is being asked. Go through the questions in the worksheets with the class but allow the students to fill in their own answers before discussing them and moving on.

When you reach the graphing part of the worksheet, you may have the students graph the inequalities using the TI-84+ Inequalities Graphing Application. If the technology is not available to you, or you would rather have the students practice graphing, then have them do it by hand. Although there is a section in which to draw the graph, you may want to provide scrap paper for student work.

Have the students identify the vertices and help them understand that the best options occur at the vertices. Have the students optimize the cost and assure them that although their solution may be strange, it still meets all the restrictions and minimizes the cost. Have the students complete the “Analyzing the Solution” section and then discuss the results as a class. At this point, give students the homework assignment and explain that it is based on today’s lesson and the example problem.

Student Application – Linear programming uses and reinforces the concepts of graphing and solving linear inequalities and evaluating a function at a given point. Students will use these concepts to solve the optimization problem.

Embedded Assessment – Student progress will be gauged by their responses to the questions in the worksheet, both written and verbal. In addition, completion of the homework assignment will be used to measure understanding.

Reteaching/Extension

- The homework assignment is a good opportunity for the students who need help and need practice in applying the concepts.

- The summative assessment serves as an extension since the feasible region of the constraints is unbounded.

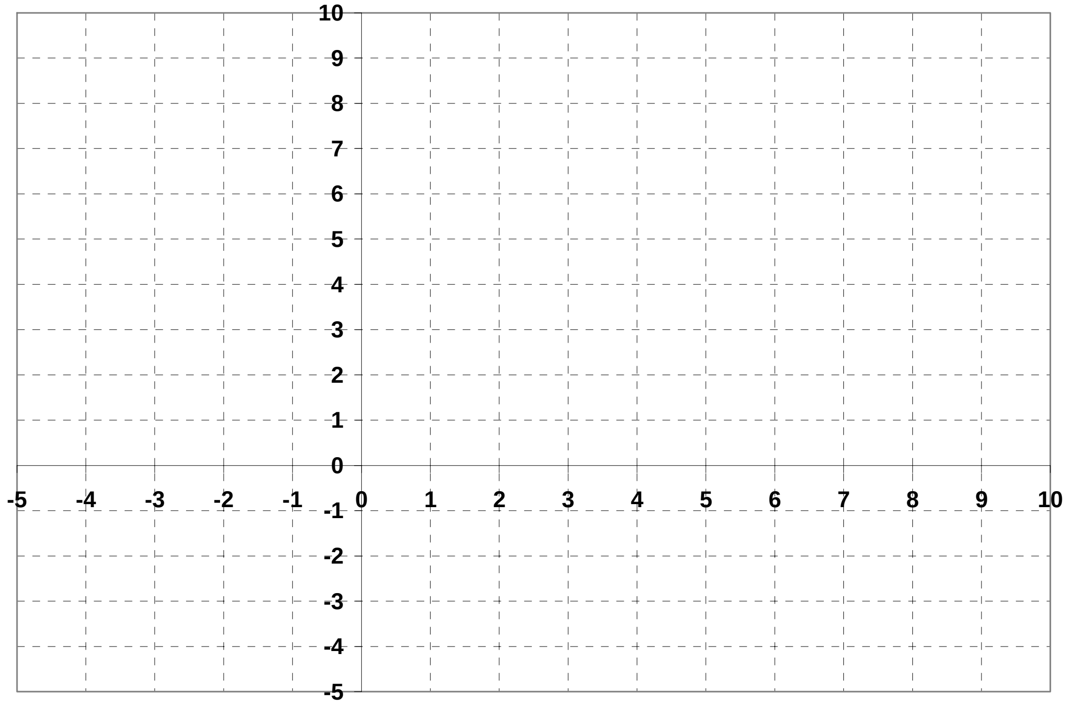
Revisiting Linear Inequalities

Directions: Graph the following system of inequalities on the coordinate plane. Then, answer the questions that follow.

$$\begin{cases} -2x + y \le 3 \\ x + y \le 5 \end{cases}$$

Remember:

- ❖ The easiest way to graph a linear inequality is to convert it into slope-intercept form. In addition, graphing calculators require that the equations be in this form.
- ❖ Remember to darken the intersection of the inequalities since they can easily be confused.



? What does the darkest shaded region indicate?

? Does this region have a vertex? If so, identify the coordinate(s)?

Milk for Breakfast?

The Problem

Jennifer is planning a light breakfast. Since she is nutrition conscious, Jennifer wants to make sure her meal has no more than 250 calories and no more than 5 grams of fat. Also, she wants to consume at least 600mg of calcium in this meal. If each serving of cereal is $.72 and each serving of milk is $.90, how much of each should she eat to minimize her cost?

	Brand A Cereal **1 Serving**	Milk **1 Serving**
Calories	150	100
Fat (g)	0g	4g
Calcium (mg)	240mg	480mg

Directions: Follow along with your teacher and complete any questions preceded by a ?

Analyzing the Problem

? What are the two unknown variables that Jennifer wants to find in this problem?

We can use this information to define our variables.

? Let $\boldsymbol{x}$ = the number of servings of ________________

$\boldsymbol{y}$ = the number of servings of ________________

? Remember that Jennifer would like to minimize the cost of her breakfast. So, let's define a function that represents the cost of the meal using our variables. Write your function below:

$$C(x, y) = \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$$

Constraints

There are several restrictions placed on these variables, which we can express in terms of inequalities. For example, Jennifer's complete meal can contain at most 250 calories. The following inequality illustrates this relationship:

$$150x + 100y \le 250$$

The number of calories in *x* servings of cereal

The number of calories in *y* servings of milk

The total number of calories in the meal

? What two other r plac bles?

a.____

b.____

? Write two inequalities based on your assumptions in the previous question.

a.____

b.____

Also, we must consider the problem logically. Since we are dealing with quantities, it would not make sense to have negative values for either variable. So two additional constraints can be included and are defined by the following inequalities:

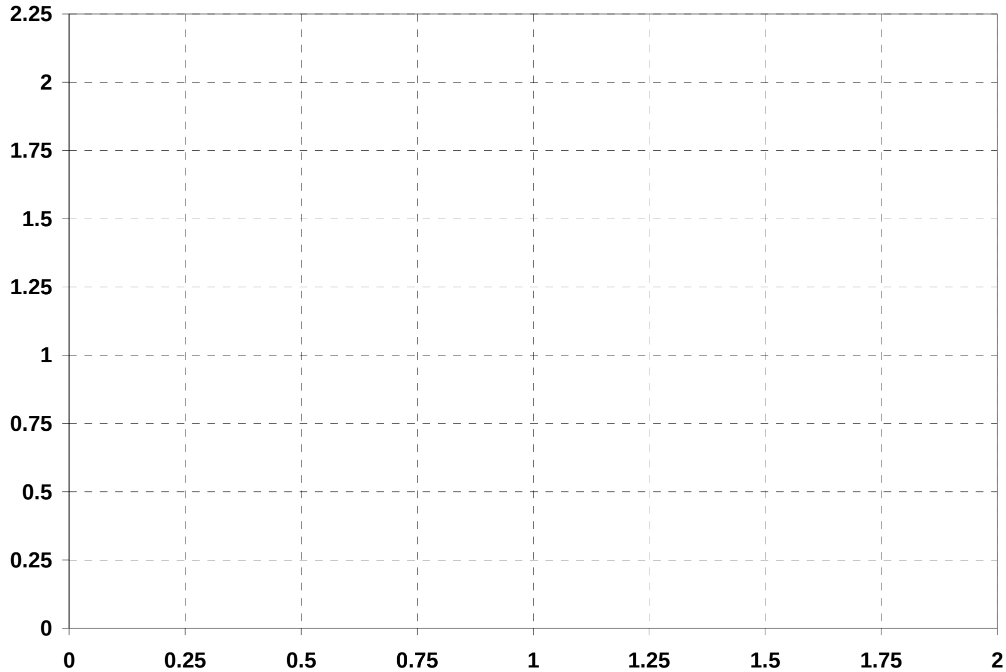
$$x \geq 0$$
$$y \geq 0$$

We end up with a system of linear inequalities that will help us find possible solutions for the problem. These inequalities define the constraints of our variables. Write the resulting system in the space below.

?

Restriction	**Inequality**
Calories	
Fat	
Calcium	
x is non-negative	
y is non-negative	

Graph these inequalities in the following plane. (Remember to put the all the equations in slope-intercept form.)



The solution of the system of linear inequalities is known as the **feasible region**. The feasible region in a linear programming problem represents all the possible solutions for the system. Any ordered pair within this region would guarantee that we don't violate any of our restrictions.

The best option will occur at one of the vertices of the feasible region.

? Use your graph to identify the vertices and record each below.

A(,)
B(,)
C(,)

Optimizing the Cost

? Now, evaluate the cost function at each of the vertices, record the values in the chart below and indicate the minimum value.

(x,y)		$C(x, y)$

? How much of each serving would give Jennifer the most inexpensive meal?

Analyzing the Solution

? What is strange about our solution?

? What option should Jennifer choose if she wants to eat cereal as well as milk if she still wishes to minimize her cost?

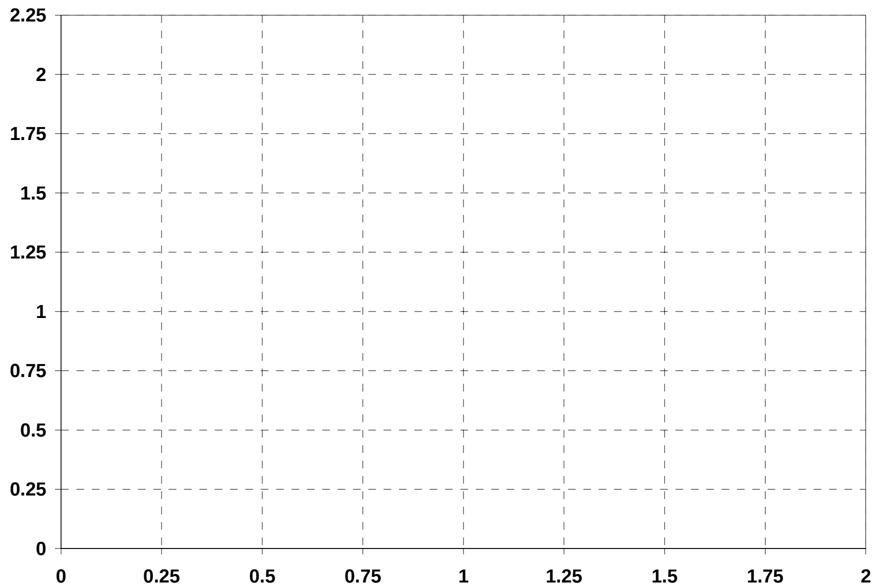
? Are there any other restrictions that we can include to ensure that Jennifer has both milk and cereal? If so, can you make any suggestions?

Homework Assignment

Assume that in addition to the constraints that Jennifer first identified, she wanted to add a constraint that guaranteed that she ate at least ½ serving of cereal.

? What inequality expresses this restriction?

? Combine this inequality with the ones we used in class and graph the feasible region on the following plane. (Remember, the graph will be the same with exception to one line, so you may end up with some of the same vertices as before.)



? Re-evaluate your cost function for this new set of points. Then, indicate the choice of servings that has the lowest cost.

(x,y)	$0.72x+.90y$	$C(x,y)$

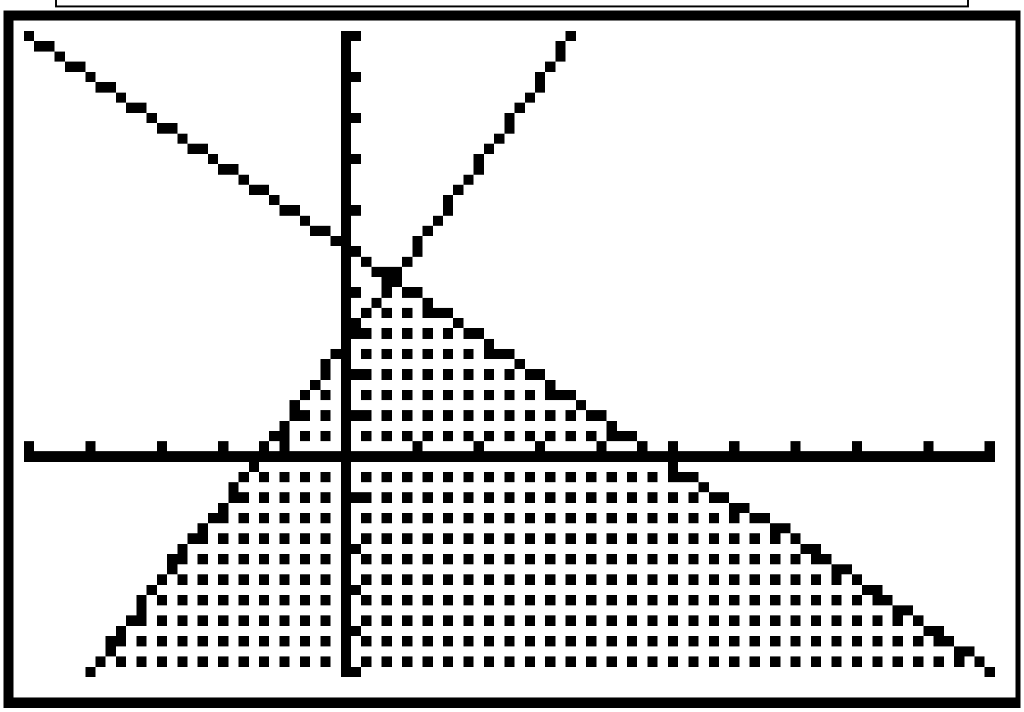
? Describe how this restriction changes the results.

TEACHER'S KEY
<u>Revisiting Linear Inequalities</u>

Directions: Graph the following system of inequalities on the coordinate plane. Then answer the questions that follow.

$$-2x + y \le 3$$
$$x + y \le 5$$

Remember:
- ❖ The easiest way to graph a linear inequality is to convert it into slope-intercept form. In addition, graphing calculators require that the equations be in this form.
- ❖ Remember to darken the intersection of the inequalities since they can easily be confused.



? What does the darkest shaded region indicate? The ordered pairs that are solutions to both inequalities.

? Does this region have a vertex? If so, identify the coordinate(s)? Yes the region has a vertex. It is located at (1,3)

TEACHER'S KEY

Milk for Breakfast?

The Problem

Jennifer is planning a light breakfast. Since she is nutrition conscious, Jennifer wants to make sure her meal has no more than 250 calories and no more than 5 grams of fat. Finally she wants to consume at least 600mg of calcium in this meal. If each serving of cereal is $.72 and each serving of milk is $.90, how much of each should she eat to minimize her cost?

	Brand A Cereal **1 Serving**	Milk **1 Serving**
Calories	150	100
Fat (g)	0g	4g
Calcium (mg)	240mg	480mg

Analyzing the Problem

? What are the two unknown variables that Jennifer wants to find in this problem? The number of servings of cereal and milk that she should use to minimize her cost.

? We can use this information to define our variables.

Let x = the number of servings of cereal
y = the number of servings of milk

? Remember that Jennifer would like to minimize the cost of her breakfast. So, let's define a function that represents the cost of the meal using our variables. Write your function below:

$$C(x, y) = 0.72x + .90y$$

Constraints

There are several restrictions placed on these variables, which we can express in terms of inequalities. For example, Jennifer's complete meal can contain at most 250 calories. The following inequality illustrates this relationship:

$$150x + 100y \le 250$$

The number of calories in x servings of cereal

The number of calories in y servings of milk

The total number of calories in the meal

TEACHER'S KEY

? What two other restrictions are placed on the variables?

a. Jennifer's complete meal can contain at most 5g of fat.

b. Jennifer's complete meal must contain at least 600mg of calcium.

? Write two inequalities based on your assumptions in the previous question.

a. $4y \leq 5$

b. $240x + 480y \geq 600$

Also, we must consider the problem logically. Since we are dealing with quantities, it would not make sense to have negative values for either variable. So two additional constraints can be included and are defined by the following inequalities:

$$x \geq 0$$
$$y \geq 0$$

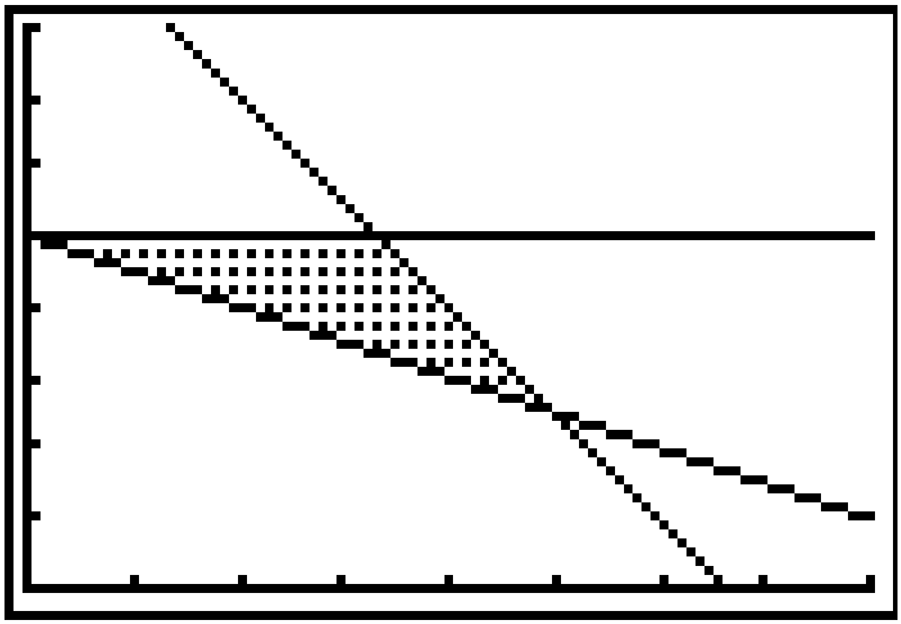
We end up with a system of linear inequalities that will help us find possible solutions for the problem. These inequalities define the constraints of our variables. Write the resulting system in the space below.

?

Restriction	**Inequality**
Calories	$150x + 100y \leq 250$
Fat	$4y \leq 5$
Calcium	$240x + 480y \geq 600$
x is non-negative	$x \geq 0$
y is non-negative	$y \geq 0$

TEACHER'S KEY

Graph these inequalities in the following plane. (Remember to put the all the equations in slope-intercept form.)



The solution of the system of linear inequalities is known as the **feasible region**. The feasible region in a linear programming problem represents all the possible solutions for the system. Any ordered pair within this region would guarantee that we don't violate any of our restrictions.

The best option will occur at one of the vertices of the feasible region.

Use your graph to identify the vertices and record each below. (Again, you may use the *TI-84+ Inequality Graphing Application*) $(0.8\overline{3}, 1.25), (1.25, 0.625), (0, 1.25)$

TEACHER'S KEY
Optimizing the Cost

? Now, evaluate the cost function at each of the vertices, record the values in the chart below and indicate the minimum value.

(x,y)	$0.72x+.90y$	$C(x,y)$	
$(0.8\overline{3},1.25)$	$0.72(0.8\overline{3})+.90(1.25)$	\$1.725	
$(1.25,0.625)$	$0.72(1.25)+.90(0.625)$	\$1.4625	
$(0,1.25)$	$0.72(0)+.90(1.25)$	\$1.125	←minimum

How much of each serving would give Jennifer the most inexpensive meal? The solution to the problem recommends that Jennifer have no servings of cereal and 1.25 servings of milk.

Analyzing the Solution

What is strange about our solution? It requires that Jennifer only drink milk and not have any cereal.

What option should Jennifer choose if she wants to eat cereal as well as milk if she still wishes to minimize her cost? She should choose 1.25 servings of cereal and .625 servings of milk.

Are there any other restrictions that we can include to ensure that Jennifer has both milk and cereal? If so, can you make any suggestions? Answers may vary.

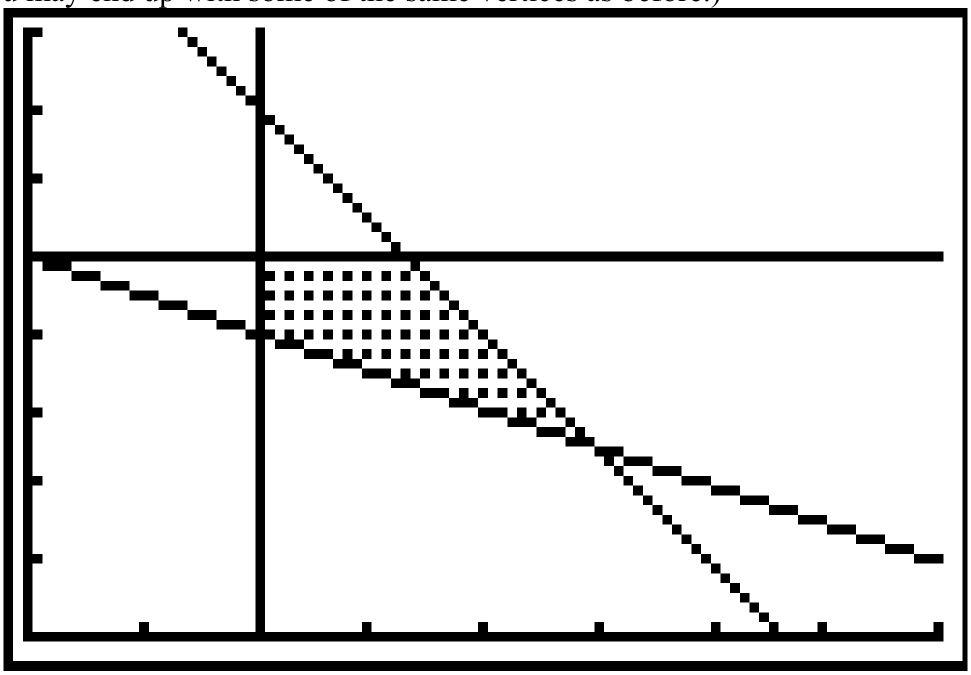
TEACHER'S KEY

Homework Assignment

Assume that in addition to the constraints that Jennifer first identified, she wanted to add a constraint that guaranteed that she ate at least ½ serving of cereal.

? What inequality expresses this restriction? $x \geq \frac{1}{2}$

? Combine this inequality with the ones we used in class and graph the feasible region on the following plane. (Remember, the graph will be the same with exception to one line, so you may end up with some of the same vertices as before.)



? Re-evaluate your cost function for this new set of values. Then, indicate the choice of servings that has the lowest cost.

(x,y)	$0.72x + .90y$	$C(x, y)$
$(0.8\bar{3}, 1.25)$	$0.72(0.8\bar{3}) + .90(1.25)$	1.725
$(1.25, 0.625)$	$0.72(1.25) + .90(0.625)$	1.4625
$(0.5, 1)$	$0.72(0.5) + .90(1)$	1.26
$(0.5, 1.25)$	$0.72(0.5) + .90(1.25)$	1.485

? Describe how this restriction changes the results. It guarantees that Jennifer has cereal in as part of her breakfast, and it minimizes the final cost of the meal.

Summative Assessment

The dietitian at Bayside High School wants to prepare a meal of meat and vegetable that meets the Food and Drug Administration recommended daily allowances of iron and protein. The RDA minimum for iron is 20mg and protein is 35g. If 1 serving of meat contains 4g of fat and 1 serving of vegetables contains 2g of fat, how many servings of each should the dietician use in order to minimize fat content?

	Meat **1 Serving**	Vegetable **1 Serving**
Protein Iron (mg)	35g 10mg	7g 6mg

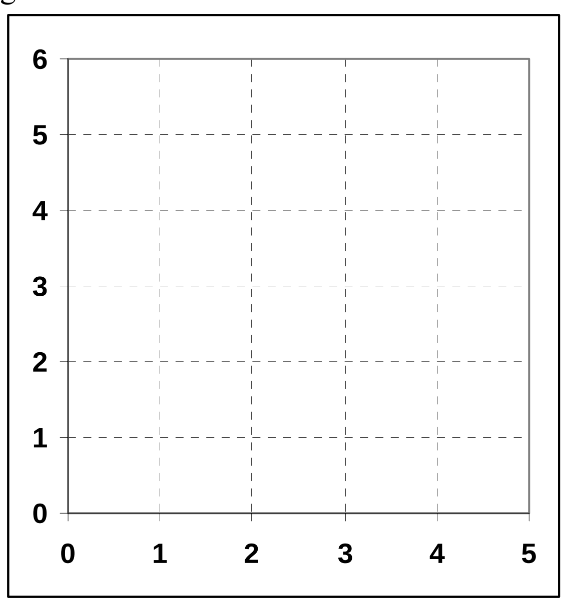
? Write the objective function that represents the number of fat grams in the meal.

$F(x,y) =$___________________

? Write a system of linear inequalities to represent the constraints on protein and iron.

Restriction	**Inequality**

? Graph the feasible region.



? Use your graph to identify the vertices, evaluate the objective function at these points and identify the minimum value.

(x,y)		$F(x,y)$

? How many servings of meat and vegetables should the dietitian use for the meal?

? How many grams of fat does this meal contain?

TEACHER'S KEY

Summative Assessment

The dietitian at Bayside High School wants to prepare a meal of meat and vegetable that meets the Food and Drug Administration recommended daily allowances of iron and protein. The RDA minimum for iron is 20mg and protein is 35g. If 1 serving of meat contains 4g of fat and 1 serving of vegetables contains 2g of fat, how many servings of each should the dietician use in order to minimize fat content?

	Meat **1 Serving**	Vegetable **1 Serving**
Protein	35g	7g
Iron (mg)	10mg	6mg

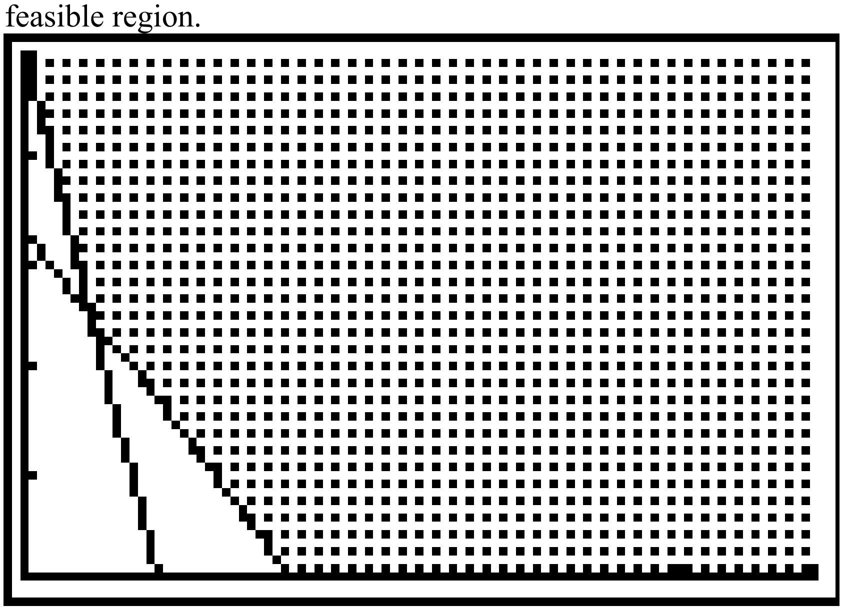
? Write the objective function that represents the number of fat grams in the meal.

$$F(x,y) = 4x + 2y$$

? Write a system of linear inequalities to represent the constraints on protein and iron.

Restriction	**Inequality**
Protein	$35x + 7y \geq 35$
Iron	$10x + 6y \geq 20$
x is non-negative	$x \geq 0$
y is non-negative	$y \geq 0$

? Graph the feasible region.



? Use your graph to identify the vertices, evaluate the objective function at these points and identify the minimum value.

(x,y)	$4x+2y$	$F(x,y)$	
$(0,5)$	$4(0)+2(5)$	10	
$(0.5, 2.5)$	$4(0.5)+2(2.5)$	7	←minimum
$(3.\overline{3},0)$	$4(3.\overline{3})+2(0)$	13.3	

? How many servings of meat and vegetables should the dietitian use for the meal? The dietitian should use 0.5 servings of meat and 2.5 servings of vegetables.

? How many grams of fat does this meal contain? This meal contains 7g of fat.