Title: Off to College

Brief Overview:

This learning unit is designed for students to investigate the linear, quadratic, and exponential models using TI-83 Plus. Students will be able to analyze and graph data and relate their model to real-life situations using the APPS: Interactive Graphing (Transformation Graphing).

NCTM Content Standard:

Algebra

- Understand patterns, relations and functions.
- Represent and analyze mathematical situations and structures using algebraic symbols.
- Use mathematical models to represent and understand quantitative relationships.

Data Analysis and Probability

- Develop and evaluate inferences and predictions that are based on data.

Grade/Level:

Grades 9-12, Algebra 2.

Duration/Length:

Three class periods, 50-60 minutes in length.

Student Outcomes:

- Students will be able to identify linear, quadratic, and exponential models.
- Students will be able to find the equation that best fits the data.
- Students will be able to understand why and how the model fits the real-world situation.

Materials and Resources:

- TI-83 Plus with Interactive Application (Transformation Graphing)
- Student worksheets
- Teacher notes

Development/Procedures:

Lesson 1

Preassessment – The students should be able to identify the graphs and equations of linear models. Have students work on the worksheet **Opening Activity 1**. Then discuss the graphs and equations of linear models.

Launch – Review the terms slope and y-intercept in the equation y = mx + b. Example: i) slope = $\frac{rise}{run}$, ii) slope = difference of y values/ difference of x values. The y-intercept is the point where the graph crosses the y-axis; m represents 'slope' and b represents 'y-intercept' in the equation y = mx + b. The teacher should have the students use the equation y = ax + b in the Transformation Graphing. So, 'a' refers to 'slope' and 'b' refers to 'y-intercept'. Demonstrate and explain interactive graphing on the TI-83. Demonstrate and explain how one can change the value of the slope and/or the value of the y-intercept. The effects of these changes can be immediately seen on the calculator. See **Instructions for Creating a Scatter Plot on TI-83 Plus** and **Instructions for Using Transformation Graphing for Linear Models on the TI-83 Plus.**

Teacher Facilitation – The teacher will demonstrate an example of using interactive graphing of a linear model. Use **Classwork 1-Basketball Camp.** The teacher will have the students work on the example using Transformation Graphing. This tool enables the students to visualize the effects of changing the slope and y-intercept in the context of a real world situation. Discuss rounding and ask students how many digits should be rounded off for a reasonable answer.

Student Application – Use the problem **Off to College: A Summer Job Investigation (Student Worksheet)** to have them practice the linear model using Transformation Graphing. This worksheet will be completed for homework.

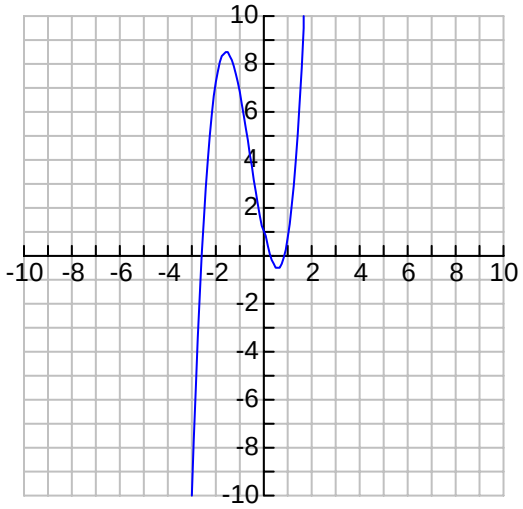
Embedded Assessment – The teacher should check each student's progress in completing the worksheet.

Reteaching/Extension –

- Review the meanings of slope and y-intercept.
- What does the y-intercept represent in our specific model?

OPENING ACTIVITY 1

1) Which of the following graphs describes a **linear** model?



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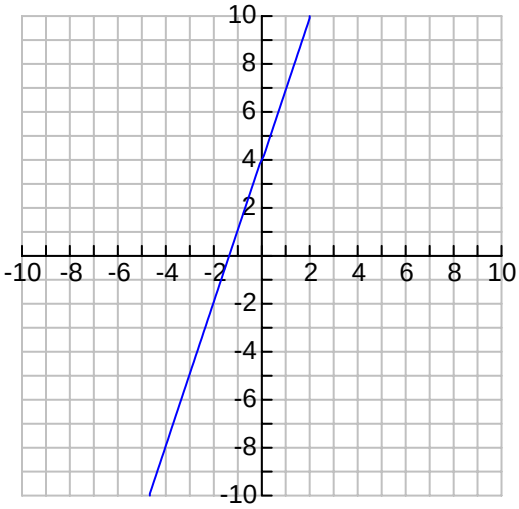
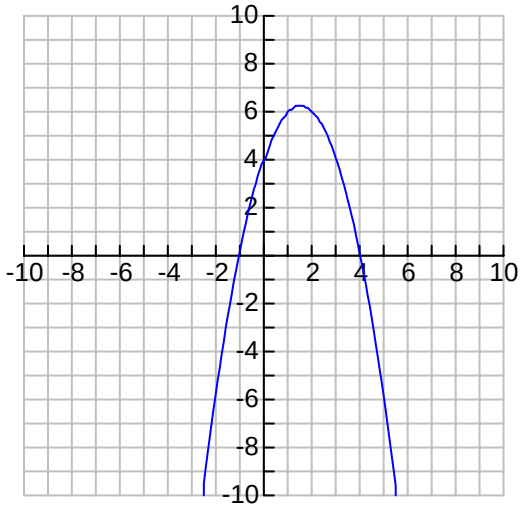
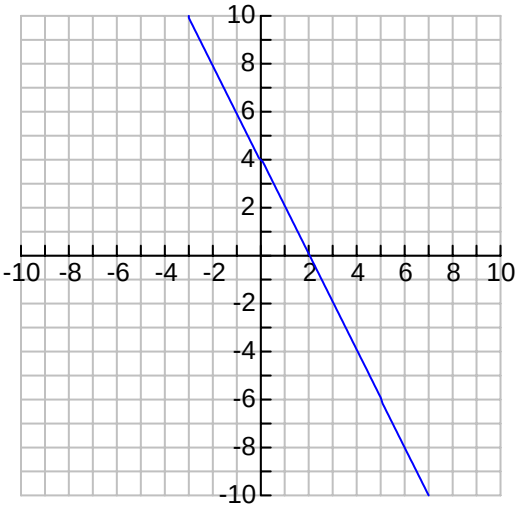
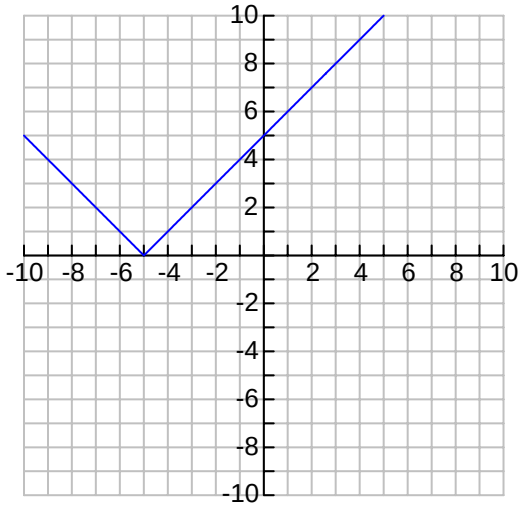




d)





2) Which of the following equations describes a **linear** model?

f) $y = -x^2 + 3x + 4$

g) $y = |x + 5|$

h) $y = 3x + 4$

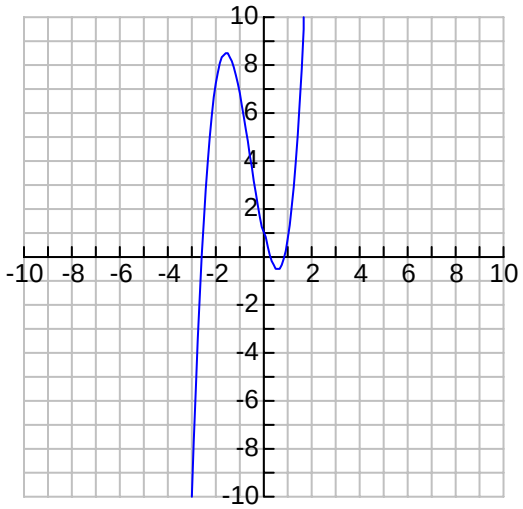
i) $y = 2x^3 + 3x^2 - 5x + 1$

j) $y = -2x - 4$

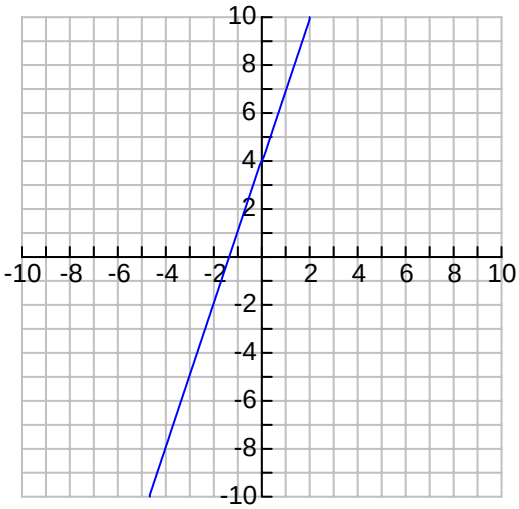
3) Match the equations in Question 2 with its graph in Question 1.

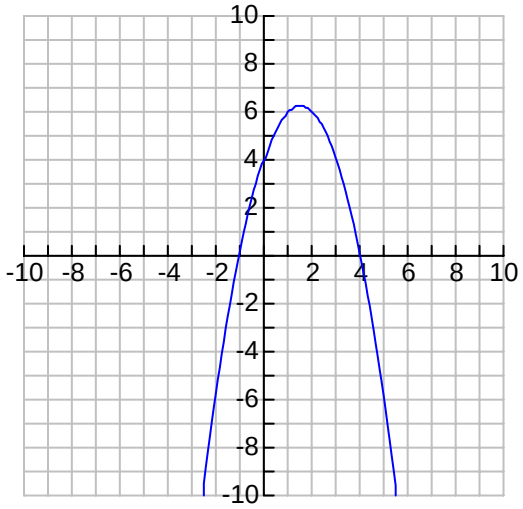
OPENING ACTIVITY 1
TEACHER'S NOTES

1) Which of the following graphs describes a **linear** model?



b)





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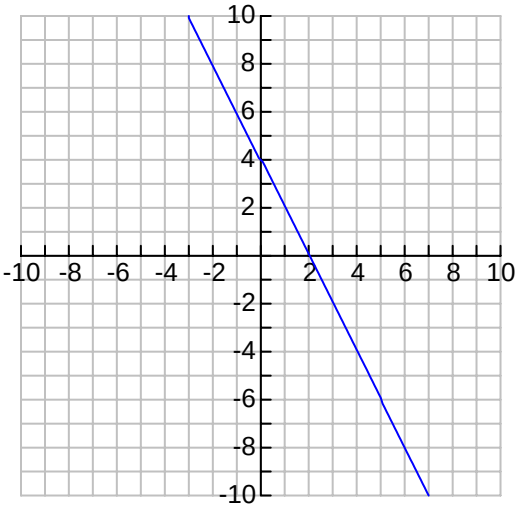
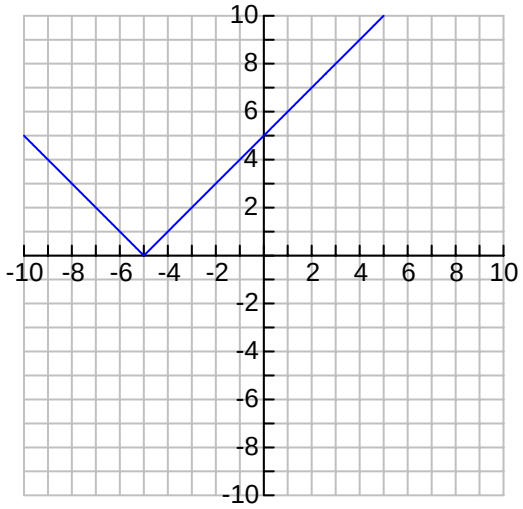




Answer: b and d

2) Which of the following equations describes a **linear** model?

f) $y = -x^2 + 3x + 4$

g) $y = |x + 5|$

h) $y = 3x + 4$

i) $y = 2x^3 + 3x^2 - 5x + 1$

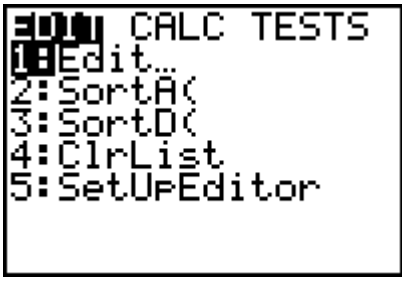
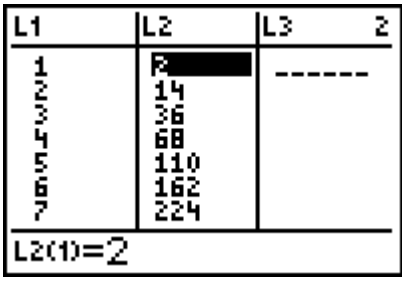
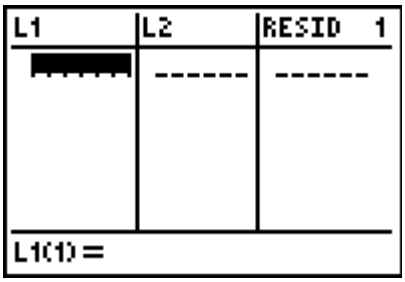
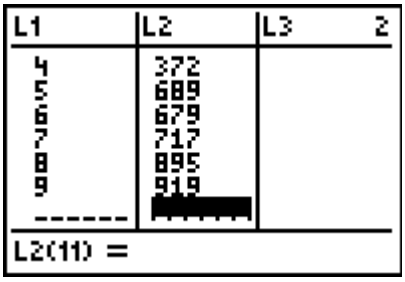
j) $y = -2x - 4$

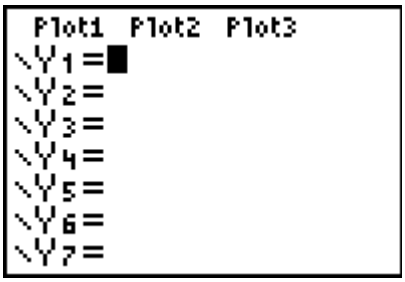
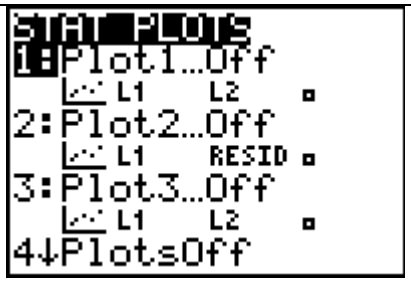
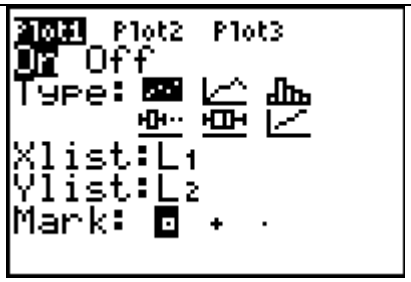
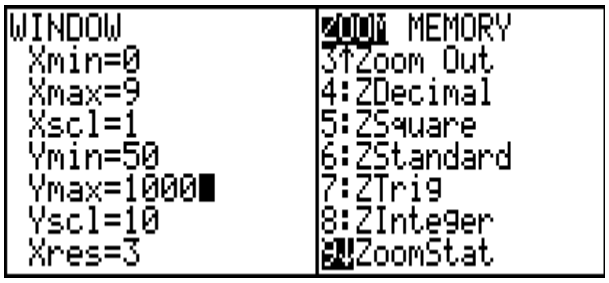
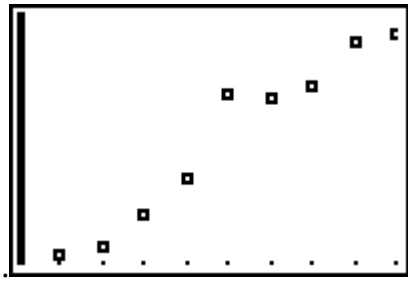
Answer: h and j

3) Match the equations in Question 2 with its graph in Question 1.

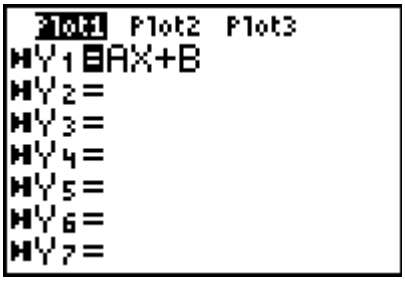
Answer: a and i
b and h
c and f
d and j
e and g

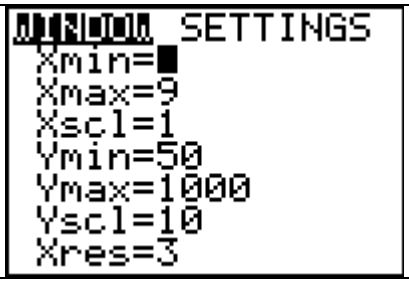
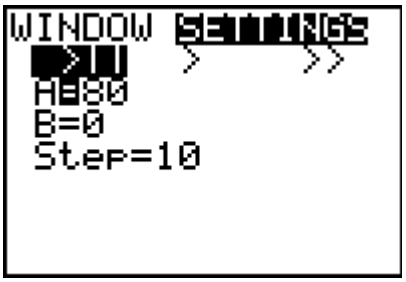
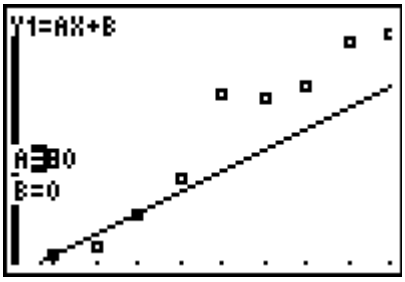
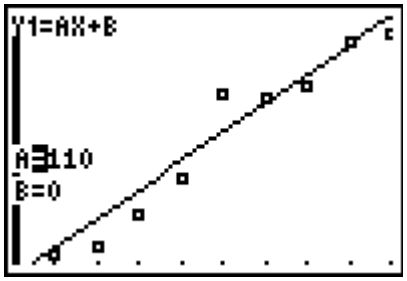
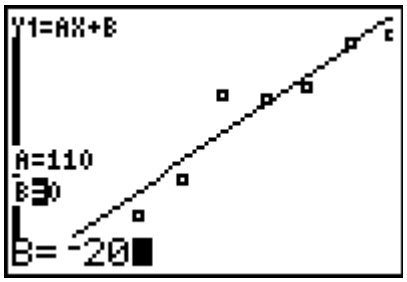
Instructions for Creating a Scatter Plot on the TI-83 Plus Graphing Calculator

Procedure	TI-83 Plus screen
Press STAT Highlight 1:Edit. Press ENTER	
If there is data in your lists, press the up arrow ▲ until L1 or L2 is highlighted. Then press CLEAR and ENTER If you are missing a list, ask your teacher to fix it with SetUpEditor.	
When your screen looks like this, you are ready to enter new data.	
Enter the independent data (X) into L1. Enter the dependent data (Y) into L2.	

Procedure	TI-83 Plus screen
Open the Y= screen. If there is an equation next to "Y_1=", press CLEAR If there are any other equations, move your cursor to the right of the "=" and press CLEAR If the first row (Plot1, Plot2, Plot3) is highlighted, use the up arrow ▲ to move your cursor over the highlighted plot, and click ENTER	
Press 2nd Y= o go to STAT PLOT.	
Press ENTER Choose Plot1 On, Type: Scatter plot, Xlist: L_1, Ylist:L_2, and either Mark.	
Now press WINDOW By examining your data, choose appropriate values for Xmin, Xmax, Xscl, Ymin, Ymax, and Yscl. Alternatively, you can choose ZOOM 9	
Now you can GRAPH your scatter plot.	

Instructions for Using Transformation Graphing for Linear Models on the TI-83 Plus Graphing Calculator

Procedure	TI-83 Plus screen
Press the blue APPS key. Using the blue down arrow ▼ move your cursor to the line that reads "Interact".	
Press ENTER. The screen on the right should appear. If instead the screen looks like the bottom screen, transformation graphing is already installed; just press 2 to continue.	 
Press the blue Y= button, and enter the equation for a line: $Y_1 = AX+B$. To enter letters, you need to use the green ALPHA key, so type ALPHA MATH for A and ALPHA APPS for B. Notice the symbol before the Y_1. That indicates that you are in Transformation Graphing mode. (Note: Transformation graphing was previously called Interactive graphing.)	

Procedure	TI-83 Plus screen
Quit the equations screen by pressing 2nd MODE Then press the WINDOW key.	
Press the up arrow key ▲ to go to SETTINGS. Press the blue ▼ key and enter a value for the slope A. Press the blue ▼ key and enter a value for the y-intercept B. Press the blue ▼ key and enter a value for the step.	
Press GRAPH	
Would you increase or decrease the slope to get a better fit? To change your slope, use the ▲ key so the "A=80" is highlighted. Press ◀ to decrease your slope by the step amount, ▶ to increase your slope. Does increasing your slope improve the fit?	
Move the cursor down to B to adjust the y-intercept. You can use ◀ and ▶ to change the B value. Alternatively, you can just enter a new value. Then press ENTER to view the new graph. When the line approximates the points, you are finished!	

CLASSWORK 1 – BASKETBALL CAMP

The table below gives the enrollment for Sunshine County Basketball Camp program from 1995 to 2004.

Years since 1995	Enrollment
0	58
1	77
2	108
3	235
4	372
5	689
6	679
7	717
8	895
9	919

a) Approximate the line of best fit for the data.

b) If the pattern continues, how many people will enroll in Basketball Camp in 2010?

CLASSWORK 1 – BASKETBALL CAMP
TEACHER'S NOTES

The table below gives the enrollment for Sunshine County Basketball Camp program from 1995 to 2004.

Years since 1995	Enrollment
0	58
1	77
2	108
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5	689
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7	717
8	895
9	919

a) Approximate the line of best fit for the data.

Answer: y = 110x - 20

b) If the pattern continues, how many people will enroll in Basketball Camp in 2010?

Answer: about 1630

Off To College

A Summer Job Investigation
Investigation of Linear Functions

JoJo has just graduated from high school. For the summer, he is working at Uncle Joe's Glass Shop. He realizes that he will have to buy some of the following items for his first year of college. How many hours will JoJo have to work in order to earn enough money to buy the needed items?

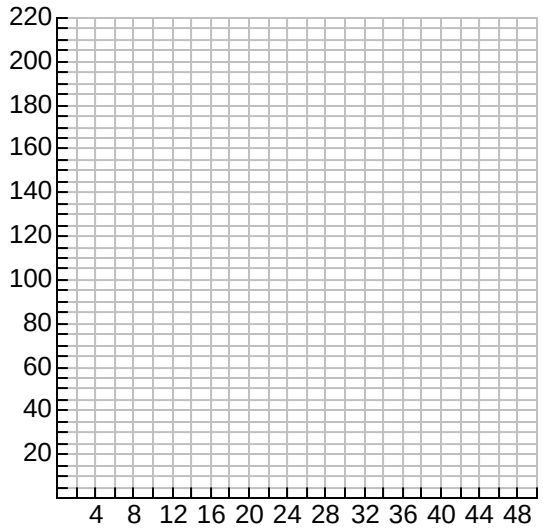
Items	Price
iPod	$375
MP3 Player	$256
DiscMan	$79
Digital Camera	$275
Cell phone	$199
Laptop computer	$1100
Refrigerator	$99
Microwave	$69

JoJo now needs to figure out how many hours he must work in order to make enough money to buy the items he wants.

1. The table below shows the total amount JoJo will make based on the number of hours he works.

Number of hours	**0**	**1**	**3**	**5**	**10**	**20**	**32**	**40**
Amount of pay	0	$5.15	$15.45	$25.75	$51.50	$103	$164.80	$206

2. Make a scatter plot of your data on the grid provided.



3. Find a linear equation that models your data and write it in slope-intercept form ($y = mx + b$).

4. What number represents the y-intercept? _____ What does the y-intercept mean in terms of this problem. _____

5. What number represents the slope? _____ What does this number represent in the given problem?

Now answer these questions about the problem using the graph, the table, graphing calculator, or the equation.

6. About how many hours will JoJo need to work before he can purchase a DiscMan? Explain how you determined your answer.

7. Using your graph or the equation, what items can JoJo purchase after working 40 hours (about 1 week if he works 8 hours a day)? Explain how you determined your answer. Use words, symbols or both in your explanation.

8. JoJo plans only to work 8 weeks (5 days a week and 8 hours a day). How much money will he earn (excluding taxes)? Show how you determined this answer. Use words, symbols or both in your explanation.

9. After 8 weeks can JoJo buy all of the items on his list? If he can't, he now has a dilemma. Give at least two suggestions that you think would help JoJo.

Off To College

A Summer Job Investigation

Teacher's Answers

JoJo has just graduated from high school. For the summer, he is working at Uncle Joe's Glass Shop. He realizes that he will have to buy some of the following items for his first year of college. How many hours will JoJo have to work in order to earn enough money to buy the needed items?

Items	Price
Ipod	$375
MP3	$256
Disc man	$79
Digital Camera	$275
Cell phone	$199
Laptop computer	$1100
Refrigerator	$99
Microwave	$69

JoJo now needs to figure out how many hours he must work in order to make enough money to buy the items he wants.

2. The table below shows the total amount JoJo will make based on the number of hours he works.

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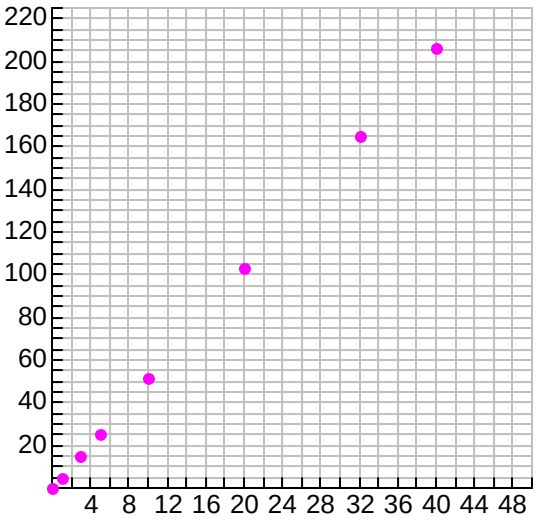
Amount of pay	0	$5.15	$15.45	$25.75	$51.50	$103	$164.80	$206

3. Make a scatter plot of your data on the grid provided.



4. Find a linear equation that models your data and write it in slope-intercept form (y = mx + b).***Answer: y = 5.15x***

5. What number represents the y-intercept? **(0,0)** What does the y-intercept mean in terms of this problem. **Answer: If JoJo does not work, he does not earn any money.**

6. What number represents the slope of the line? ***Answer: 5.15*** What does this number represent in terms of this problem? ***Answer: The number represents the hourly rate.***

Now answer these questions about the problem using the graph, the table, or the graphing calculator.

7. About how many hours will JoJo need to work before he can purchase a DiscMan? Explain how you determined your answer. Use words, symbols, or both in your explanation.
Answer: JoJo would need to work at least 16 hours since the Disc Man costs $79.
16(5.15) = 82.40

8. Using your graph or the equation, what items can JoJo purchase after working 40 hours (about 1 week if he works 8 hours a day)? Explain how you determined your answer. Use words, symbols, or both in your explanation. ***Answer:*** $40 \cdot 5.15 = 206$***; $206 represents his earnings. Therefore he can buy a cell phone; refrigerator and microwave; DiscMan and refrigerator; DiscMan and microwave.***

9. JoJo plans only to work 8 weeks (5 days a week and 8 hrs a day). How much money will he earn (excluding taxes)? Explain how you determined this answer. Be sure to use words, symbols, or both in your explanation. ***Answer: $1648***

10. After 8 weeks can JoJo buy all of the items on his list? If he can't, he now has a dilemma. Give at least two suggestions that you think would help JoJo? **Answer: JoJo cannot buy all of the items. Total of the items is $2452 and he will only earn $1648. Some suggestions: ask parents to buy some of the items, find an additional job, work more hours, etc.**

Lesson 2

Preassessment – The students should be able to identify the graphs and equations of quadratic models. Have students work on the worksheet **Opening Activity 2**.

Launch – Discuss the terms minimum, maximum, zeros (roots) and vertex when applied to parabolas. Review the standard form and vertex form of a quadratic equation. Standard form: $y = ax^2 + bx + c$ (a $\neq 0$), vertex form: $y = a(x-h)^2 + k$ where (h, k) is the vertex. Have the students use $y = a(x-b)^2 + c$ when using Transformation Graphing on their calculators.

Teacher Facilitation – The teacher will demonstrate an example of quadratic model using interactive graphing. Use **Classwork 2-Funneling Water**. Use Transformation Graphing to model the data and discuss the example with the students.

Student Application – Use the **Off to College: A Basketball Investigation (Student Worksheet)** to have the students practice quadratic model using Transformation Graphing.

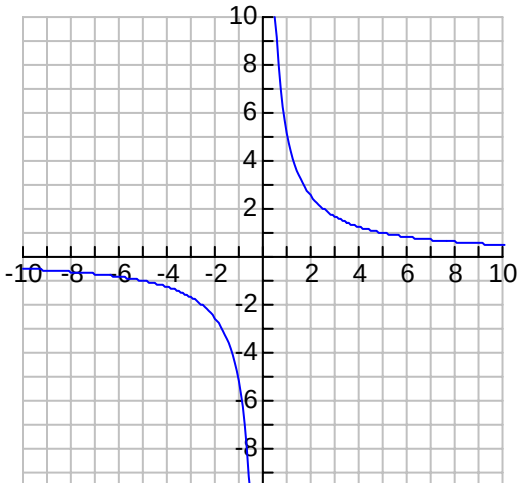
Embedded Assessment – The teacher should check each student's progress in completing the worksheet.

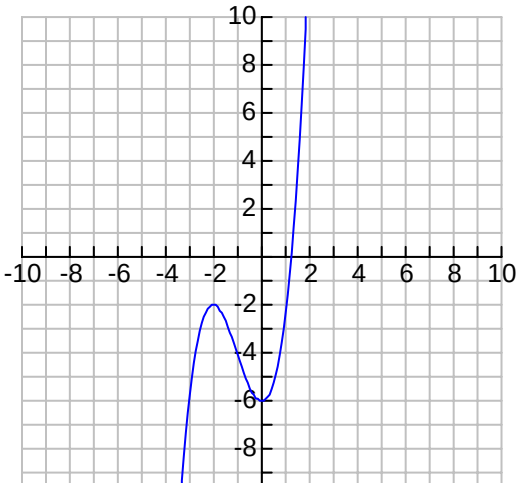
Reteaching/Extension –

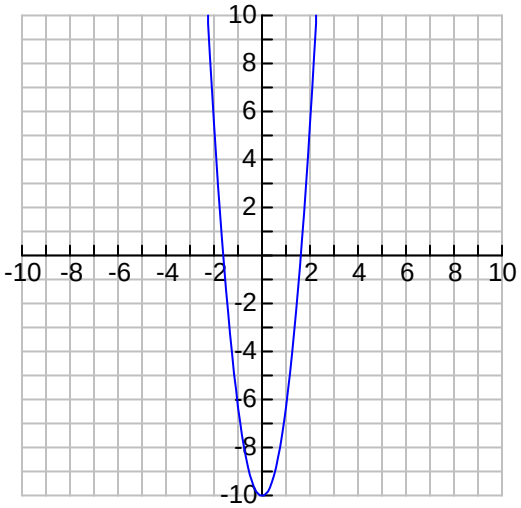
- Review the meanings of maximum and vertex.
- What do the coordinates of the vertex represent in the ball problem?
- What is the minimum number of data you need to find a quadratic model for a set of data?

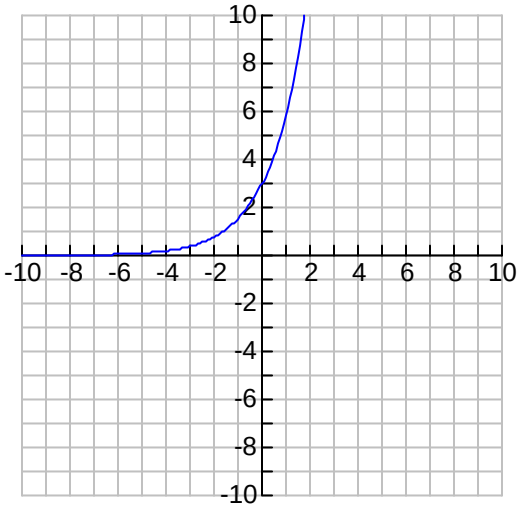
OPENING ACTIVITY 2

1) Which of these graphs describes **quadratic** model?









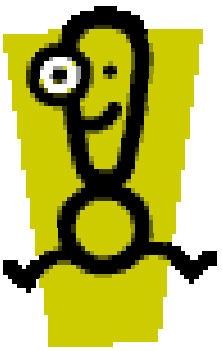
a) $y = 2x^3 + 4x^2 - 5x + 1$

b) $y = \frac{3}{x}$

c) $y = 3x - 1$

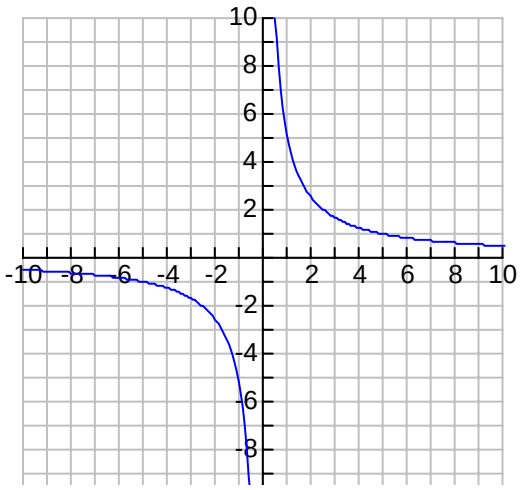
d) $y = 4x^2 + x - 3$

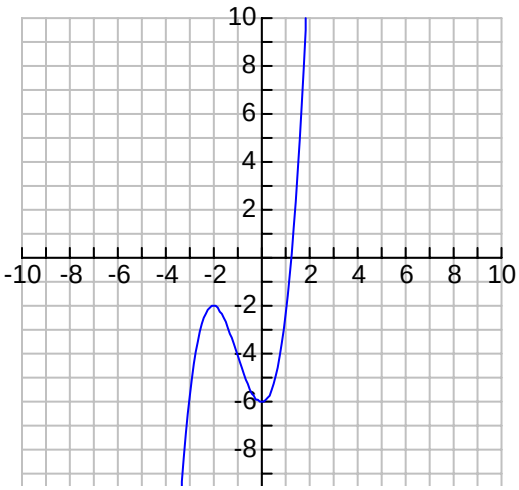
e) $y = 5 \cdot 2^x$

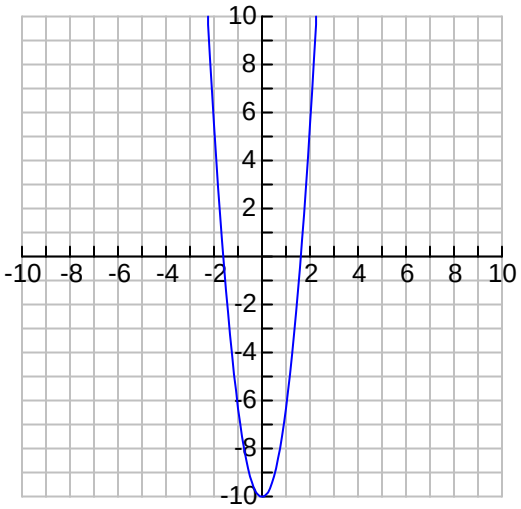


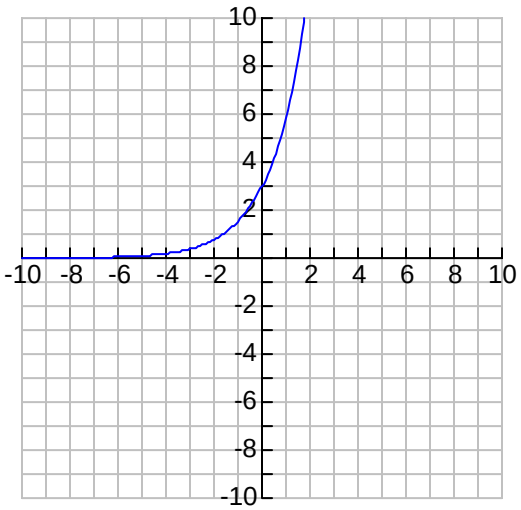
OPENING ACTIVITY 2
TEACHER'S NOTES

1) Which of these graphs describes **quadratic** model?









2) Which of these equations describes a **quadratic** model?

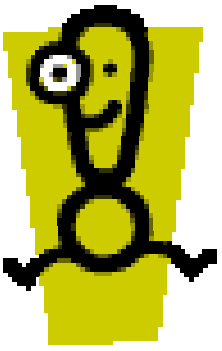
a) $y = 2x^3 + 4x^2 - 5x + 1$

b) $y = \frac{3}{x}$

c) $y = 3x - 1$

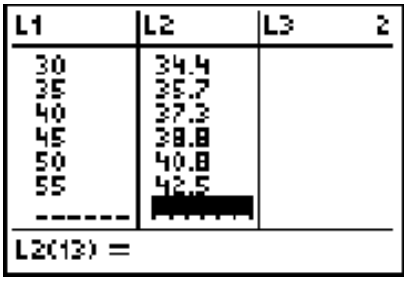
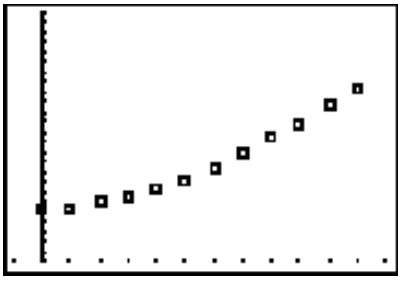
d) $y = 4x^2 + x - 3$

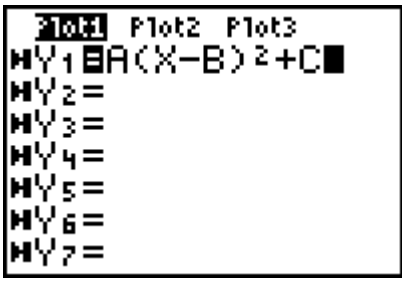
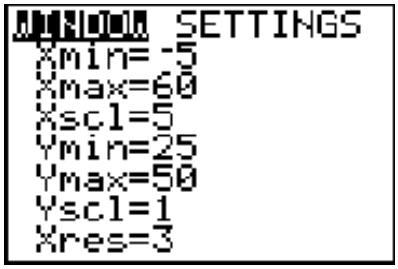
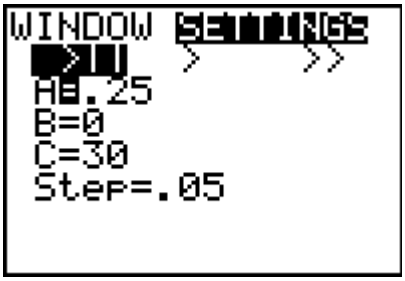
e) $y = 5 \cdot 2^x$

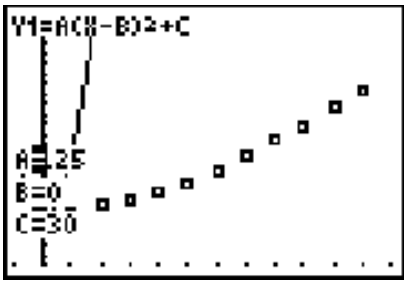
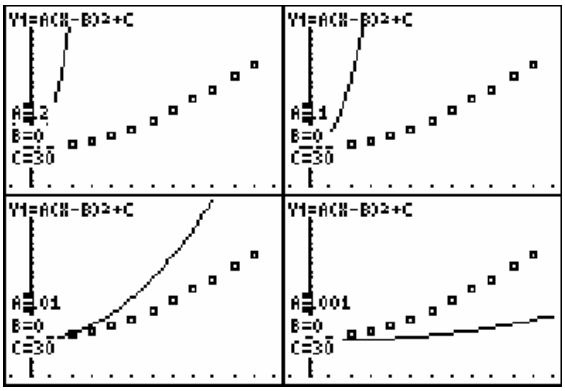
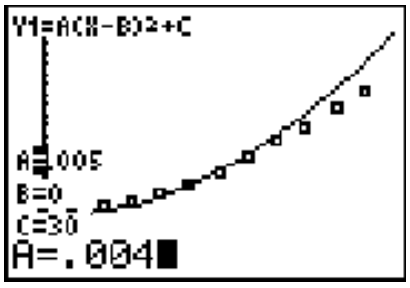
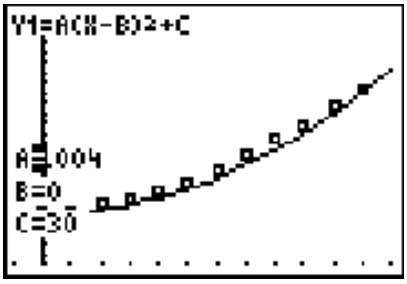


Answer: d

Instructions for Using Transformation Graphing for Quadratic Models on the TI-83 Plus Graphing Calculator

Procedure	TI-83 Plus screen
Enter the independent data (X) into L1. Enter the dependent data (Y) into L2.	
Construct a scatter plot.	
Press the blue APPS key. Using the blue down arrow ▼, move your cursor to the line that reads "Interact".	
Press ENTER . If you calculator is not in transformation graphing mode, the screen on the right will appear.	

If the screen looks like this instead, transformation graphing is already installed, just press [2] to continue.	
Press the blue [Y=] button, and enter the quadratic equation in vertex form. Remember that you need to use the green [ALPHA] key in order to enter letters.	
Quit the equations screen by pressing [2nd] [MODE] Then press the [WINDOW] key, and enter reasonable values.	
Press the up arrow key ▲ to go to SETTINGS. The **A value** indicates if the parabola opens upwards (positive) or downwards (negative), and if the opening is wide (small value for A) or narrow (large A). Press the blue ▼ key and enter values for the **vertex (B, C).** Press the blue ▼ key and enter a value for the step.	

Procedure	TI-83 Plus screen
Press GRAPH Does the vertex (B, C) look about right, or would you move the vertex to get a better fit? Should the parabola be wider (decrease A) or narrower (increase A)?	
Use the ▲ or ▼ keys to highlight the variable you will change. Press ◀ to decrease or ▶ to increase the variable	
Alternatively, you can enter the new value and press ENTER	
When you think the parabola best fits the data, you are finished!	

CLASSWORK 2 – FUNNELING WATER

Water is being funneled into a 2-liter bottle. The table shows the level of the water y measured in centimeters from the bottom of the bottle after x seconds.

Time (s)	0	5	10	15	20	25	30	35	40	45	50	55
Water level(cm)	30.2	30.3	30.9	31.4	32.4	33.2	34.4	35.7	37.3	38.8	40.8	42.5

a) Use Transformation Graphing to find a line of best fit.

b) Use Transformation Graphing to find a quadratic curve of best fit.

c) Determine which equation is a better fit for the data. Justify your answer.

CLASSWORK 2 – FUNNELING WATER
TEACHER'S NOTES

Water is being funneled into a 2-liter bottle. The table shows the level of the water y measured in centimeters from the bottom of the bottle after x seconds.

Time (s)	0	5	10	15	20	25	30	35	40	45	50	55
Water level(cm)	30.2	30.3	30.9	31.4	32.4	33.2	34.4	35.7	37.3	38.8	40.8	42.5

a) Use Transformation Graphing to find a line of best fit.

Possible answer: y = 0.228X + 28.6

b) Use Transformation Graphing to find a quadratic curve of best fit.

Possible answer: $y = .003(x - 07.12)^2 + 30$

c) Determine which equation is a better fit for the data. Justify your answer.

Quadratic model is a better fit for this data.

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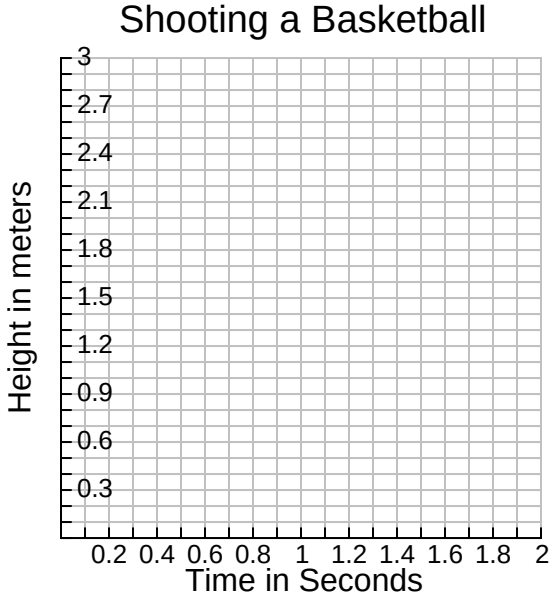
Off to College Part 2:

The Basketball Investigation of Quadratic Functions

JoJo has worked in Uncle Joe's Glass Shop for the last two weeks. During that time he has made tables and graphs of his earnings. He has decided he needs to take a break from all of this thinking, working, and planning for his future. He decides to spend his free time shooting a few hoops. JoJo is making some good shots. Using the data below find the equation that models the path the basketball takes.

1. Make a sketch of the graph.



Time in seconds	0	0.25	0.3	0.4	0.65	0.8	0.9	1.0	1.1
Height in meters	1.5	2.9	3.2	3.5	4	4	3.8	3.6	3.2

2. What type of equation do you predict would best fit your data? Justify your answer.

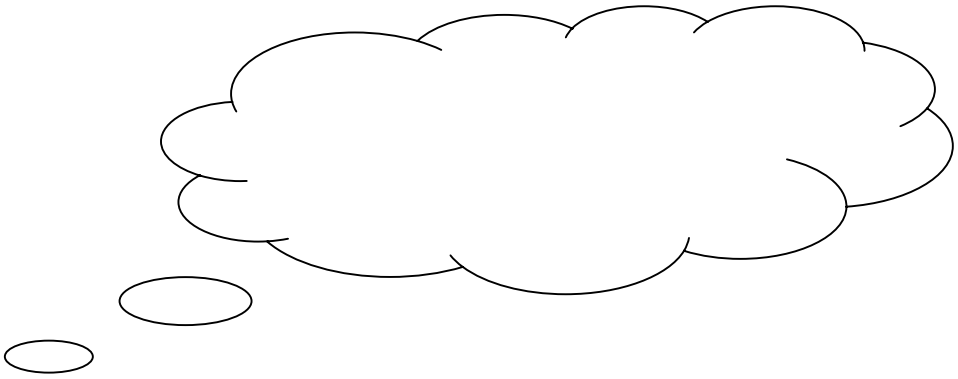
3. Find the equation that best fits the model by using the graphing calculator and directions for *Interactive Graphing (Transformation Graphing).* Write the equation you found. _____ What type of equation did you write? How does the type of equation compare with the type in your prediction?

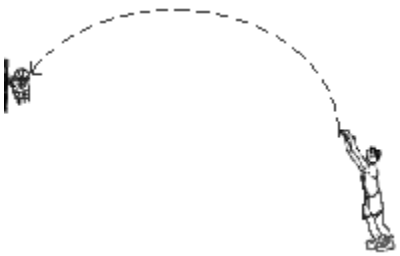
4. Write the coordinates of the vertex _____ What does the vertex represent in this problem _____

5. In approximately how many seconds did it take the ball to reach its highest height?

6. Describe the situation when x = 0 or the time = 0.

7. Describe the situation when y= 0 or the height = 0.





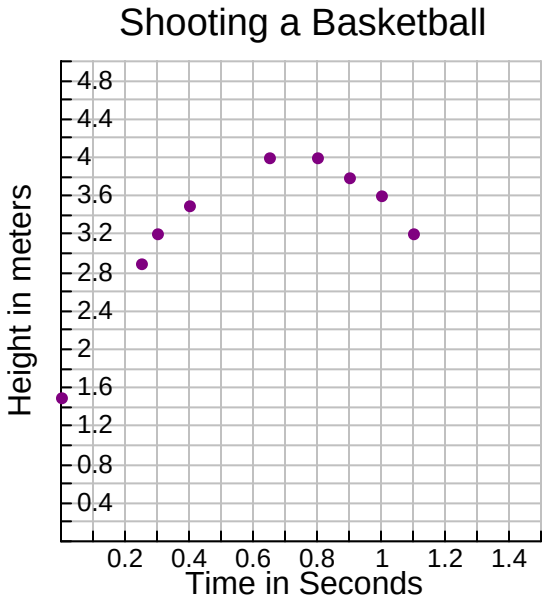
Off to College Part 2:

The Basketball Investigation of Quadratic Functions

Teacher Notes

JoJo has worked in his uncle's glass shop for the last two weeks. During that time he has made tables and graphs of his earnings. He has decided he needs to take a break from all of this thinking, working, and planning for his future. He decides to spend his free time shooting a few hoops. JoJo is making some good shots. Using the data below find the equation that models the path the basketball takes.

8. Make a sketch of the graph.
   Answer: Student's graph should resemble the one below.



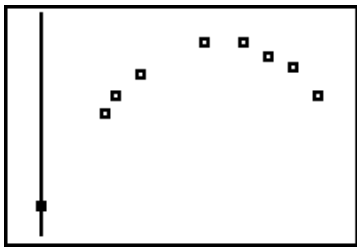
Time in seconds	0	0.25	0.3	0.4	0.65	0.8	0.9	1.0	1.1
Height in	1.5	2.9	3.2	3.5	4	4	3.8	3.6	3.2

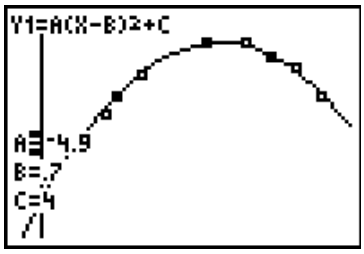
meters									

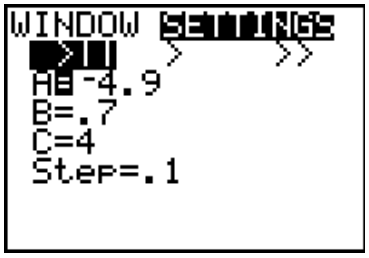
9. What type of equation do you predict would best fit your data? Justify your answer. ***Answer: Quadratic equation because the data points resemble a parabola.***

10. Find the equation that best fits the model by using the graphing calculator and directions for *Interactive Graphing (Transformation Graphing).* ***Answer: See examples below.*** Write the equation you found. **Answer: one possible equation** $y = -4.9(x - .7)^2 + 4$ What type of equation did you write? ***Answer: quadratic equation*** How does the type of equation compare with the type in your prediction? ***Answer: the same***

Possible Screens







11. Write the coordinates of the vertex. ***Answer: (0.7, 4)*** What does the vertex represent in this problem? ***Answer: The vertex represents the maximum height is 4 meters at 0.7 seconds.***

12. In approximately how many seconds did it take the ball to reach its highest height? ***Answer: 0.7 seconds***

13. Describe the situation when x = 0 or the time = 0. ***Answer: Jack has the ball in his hand approximately 1.7 meters above ground level.***

14. Describe the situation when y = 0 or the height = 0. ***Answer: The ball hits the ground.***

Lesson 3

Preassessment – The students should be able to identify the graphs and equations of exponential models. They should also identify the exponential growth and decay. Have students work on the worksheet **Opening Activity 3.**

Launch – Discuss the properties of exponential models using the **Investigation of Exponential Functions.** Emphasize the concepts of increasing and decreasing function, asymptote, exponential growth and exponential decay, domain and range.

Teacher Facilitation – The teacher will discuss the properties of exponential models and their use in real-life. See **Investigation of Exponential Functions. Discussion of Opening Day Activity 3.** It can be used as a guide for a teacher-led discussion.

Student Application – Use the **Off to College: A Graduation Gift Investigation (Student worksheet)** to have the students practice exponential model. The teacher will check problems 1 and 2 with the students. Then the students will be expected to complete the worksheet with their group.

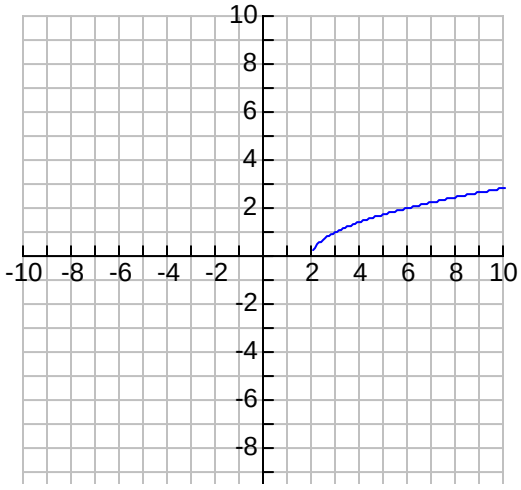
Embedded Assessment – The teacher should check each student's progress in completing the worksheet.

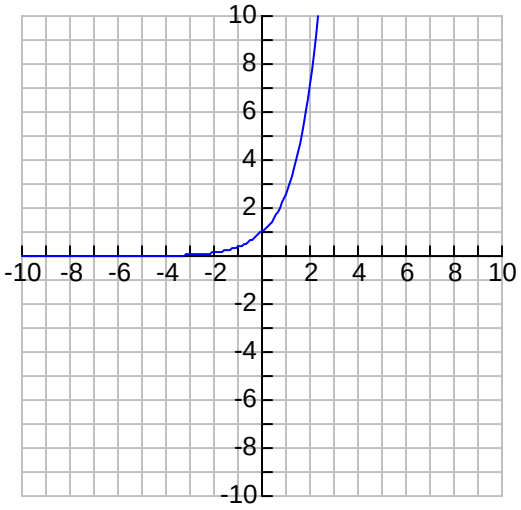
Reteaching/Extension

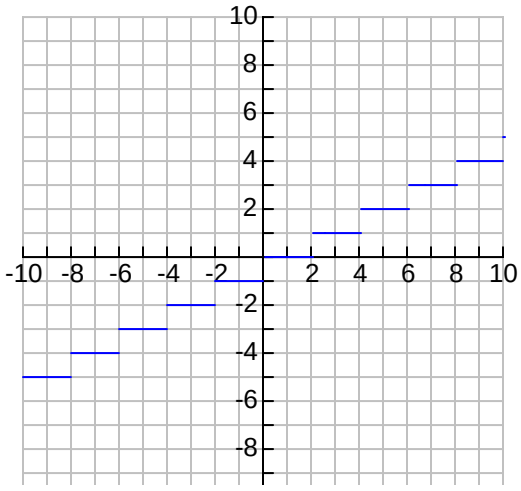
- Review the properties of exponential functions, exponential growth and exponential decay.
- Change how frequently the interest is compounded.

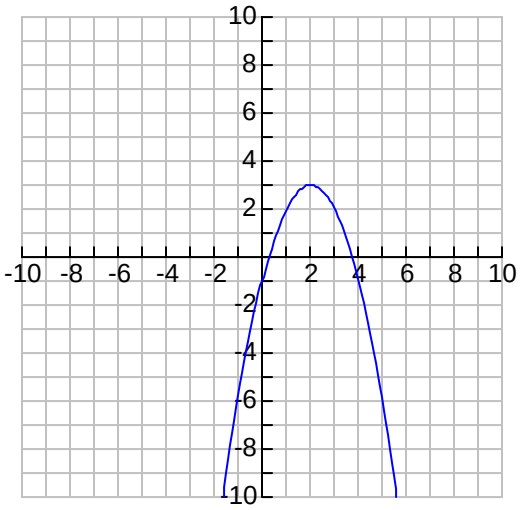
OPENING ACTIVITY 3

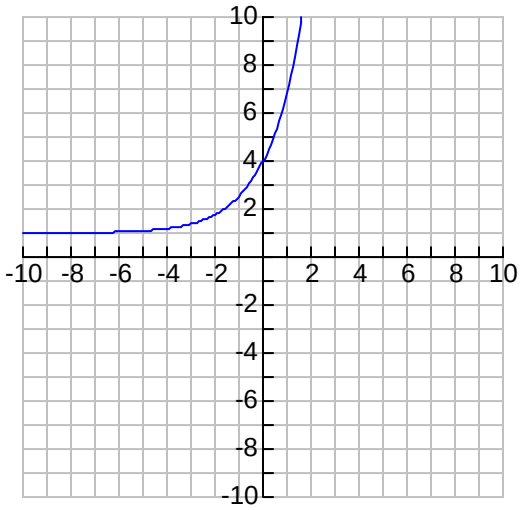
1) Which of the following graphs describes a











2) Determine whether each function is an **exponential growth** or an **exponential decay**.

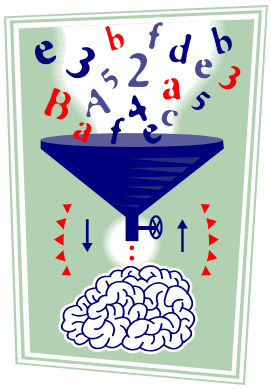
a) $y = \left(\frac{1}{4}\right)^x$

b) $y = 2 \cdot 4^x$

c) $y = 2 \cdot (2.4)^x$

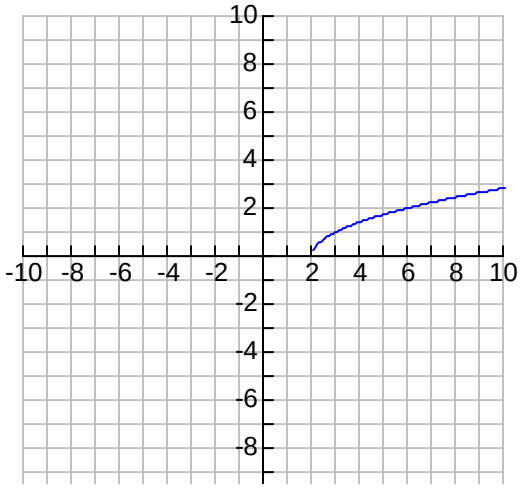
d) $y = 5 \cdot (0.3)^x$

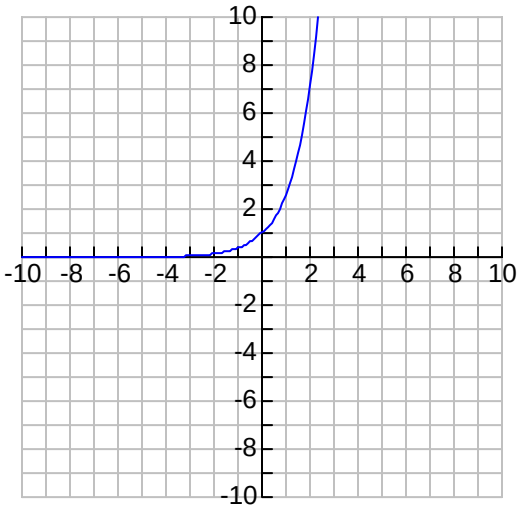
3) Graph the function $y = 3^x$ on your calculator. State the **domain** and **range**.

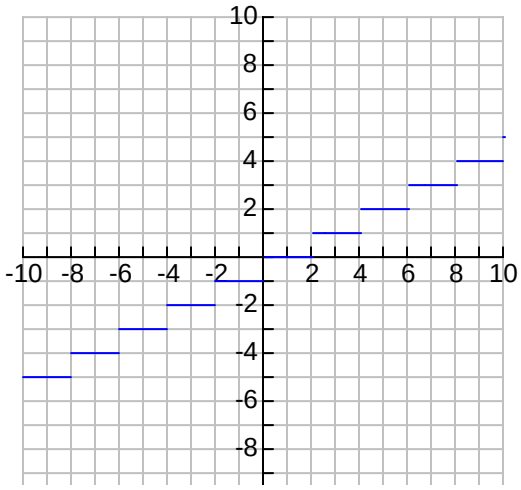


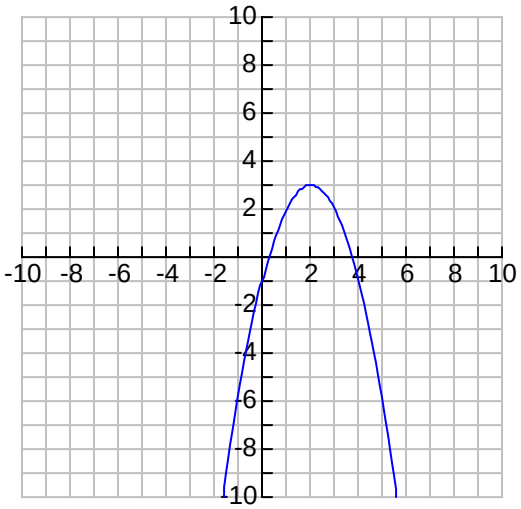
OPENING ACTIVITY 3
TEACHER'S NOTES

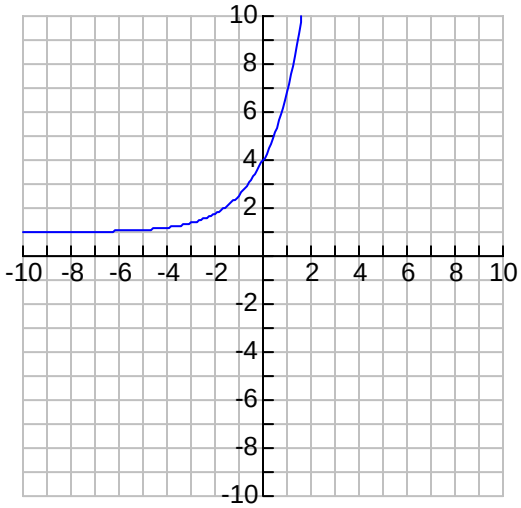
1) Which of the following graphs describes a











Answer: b and e.

2) Determine whether each function is an **exponential growth** or an **exponential decay**.

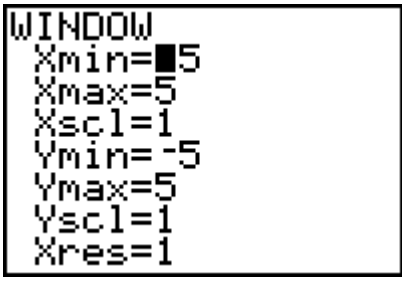
a) $y = \left(\frac{1}{4}\right)^x$ **exponential decay**

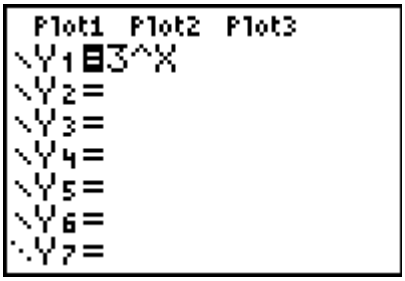
b) $y = 2 \cdot 4^x$ **exponential growth**

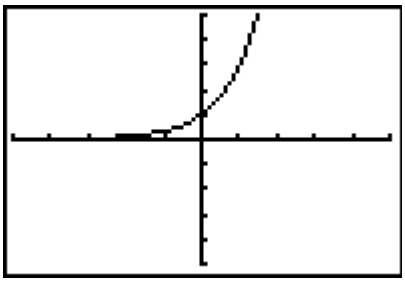
c) $y = 2 \cdot (2.4)^x$ **exponential growth**

d) $y = 5 \cdot (0.3)^x$ **exponential decay**

3) Graph the function $y = 3^x$ on your calculator. State the **domain** and **range**.

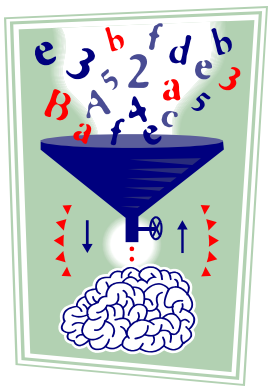






Domain: All real numbers

Range: All positive numbers



Off to College Part 3

A Graduation Gift Investigation

Investigation of Exponential functions

Teacher Notes

Discussion of Opening Activity 3

Let's review the graphs we just looked at. Two of the graphs were for exponential models.

What do the exponential graphs have in common*?*
Possible answers:

- *The domain is all real numbers ($-\infty \leq x \leq \infty$).*
- *The range is all positive values ($y > 0$),*
- *They are bounded from below by the x-axis. (The x-axis is an asymptote.)*
- *The equations are of the form $y = a(b)^x$ where a is positive. If b is positive, the graph increases and is called exponential growth. If b is negative, the graph decreases and is called exponential decay.*

How are the exponential models different from the linear models?
Possible answers:

- *The exponential models do not form a straight line.*
- *The exponential models are not defined for 0 and negative values of the independent variable y.*

How are the exponential models different from the quadratic models?
Possible answers:

- *The exponential models always increase (exponential growth) or decrease (exponential decay). The quadratic models decrease and then increase (for $a > 0$) or increase and then decrease (for $a < 0$).*
- *The exponential models do not have a minimum or a maximum value. The quadratic functions have either a minimum value (for $a > 0$) or a maximum value (for $a < 0$).*

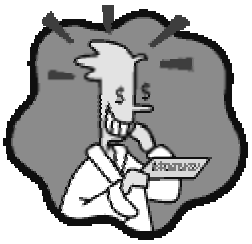
Can you think of some real-life situations that can be modeled exponentially?
Possible answers:

- *Interest*
- *Population growth*
- *Carbon dating*
- *Bacteria growth*

Off to College Part 3

A Graduation Gift Investigation

Teacher Notes



Let the student do problems 1 and 2 on student worksheet. Then follow with a class discussion using the guidelines given below.

- Compare the amounts at the end of four years for the three different options. Although JoJo started out with the most money in option A, he ended up with the least. However, in option C, where he started out with the least money, JoJo ended up with more money than the other two options. How do you explain the results? *Answers may vary.*
- What is the amount of interest JoJo earns for each option? *Answers: $124.86 for option A, $153.68 for option B, $81.47 for option C*

Now let's derive the equations to find the amount for each option after x years.

First look at Option A.

How much did JoJo start with? $1000. How did you determine how much he would have in the bank after one year? *Possible answer: Multiply $1000 by .04 to find the interest and add to $1000.* Notice that this is the same as multiplying $1000 by 1.04. Point out the distributive law. ($1000 x.04) + ($1000 x 1.0) = $1000 x 1.04 .

JoJo has $1040 at the end of 1 year. How do you find how much he'll have after 2 years? *Possible answer: Multiply by 1.04 again.* Show that you have
$1.04 \times \$1040 = 1.04 \times (1.04 \times \$1000) = \$1000\ (1.04)^2$.
The general form is $1000\ (1.04)^x$.

What does the 1000 represent? *Possible answer: How much money JoJo started with*

What does the 1.04 represent? *Possible answer: the interest rate plus what he started with*

Now graph the equation $Y_1 = 1000\ (1.04)^{x.}$
For the WINDOW, use Xmin = -10,
Xmax = 50
Xscl = 1
Ymin = 0
Ymax = 5000
Yscl = 100

What type of graph is this? *Answer: Exponential*
Observe several properties, such as always increasing (money grows), y-intercept at 1000 ($1000 at year 0), etc.

Now let's look at option B.

How much did JoJo start with? *Answer: $975.* How would you determine how much he would have in the bank after one year? *Possible answers: multiply $975 by 1.05; multiply 975 by .05 to find the interest and add to $975*. Show that you get the same result ($1023.75) from both methods, and ask a student to use the distributive property to show why.

JoJo has $1023.75 at the end of 1 year. How do you find how much he'll have after 2 years? *Possible answers: multiply $975 by* $(1.05)^2$*; multiply $1023.75 by 1.05; multiply $1023.75 by .05 and add the result to $1023.75.* Show that all three methods are equivalent.

The general form is $975\ (1.05)^x$

What does the 975 represent? *Possible answer: The amount of money JoJo started with*
What does the 1.05 represent? *Possible answer: the interest rate plus what he started with*

Graph the result using the same window as for the previous equation.

Now look at option C.

How much money does JoJo start with? *Answer: $950*
What is the interest rate? *Answer: 6%*
What would the formula be for the amount JoJo has in the bank after x years? *Answer: $950 (1.06)^x$*

Graph the result using the same window as for the previous equations.

Now graph all three equations on the same screen and compare.

What observations do you notice? *Possible answers:*

- *All three are exponential models.*
- *Option C is steeper than option B, and option A is the least steep.*
- *For a given value of x to the right of where the plots intersect, draw a vertical line, and option C will be on top. Option A will be on the bottom. What does this signify? Possible answer: That at any time after they intersect, option C will result in the most money in the bank and option A will result in the least.*
- *For a given value of y above the point where the plots intersect, say y = $2000 draw a horizontal line, and option A will be on right. Option C will be on the left. What does this signify? Possible answer: That if you want to get $2000 in the bank, option A will take the longest time to get it, and C will take the least amount of time to get $2000.*

Suppose JoJo decided to go to graduate school for two years, and left the money in the bank to accumulate interest. How much would he have at the end? Remember that the money will have been in the bank for a total of six years. *Answer: $1265.32 for option A, $1306.59 for option B, $1347.59 for option C.*

Note that we have an extension for Algebra 2 that changes how often the interest is compounded.

Off To College Part 3

A Graduation Gift Investigation

After graduating from high school, JoJo receives a graduation gift from his grandmother. There is a catch to his grandmother's gift. She will give him the money along with any interest on the day he graduates from a four-year college. However, she will let him choose the investment option.

Option A $1000 at 4% interest rate compounded annually
Option B $975 at 5% interest rate compounded annually
Option C $950 at 6% interest rate compounded annually

Now JoJo must decide which option to choose?

1. Complete the table.

Option A 4% interest rate

Year	Amount
0	1000
1	
2	
4	

Option B 5% interest rate

Year	Amount
0	975
1	
2	
4	

Option C 6% interest rate

Year	Amount
0	950
1	
2	
4	

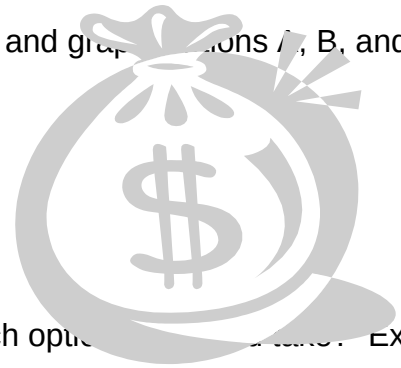
2. On his graduation from college, state the amount of money JoJo can receive for each option. Did any of the amounts surprise you?

3. What amount of interest will JoJo earn for option A? _____ option B? _____ option C? _____

4. Write the equations for each option and graph the equations A, B, and C. Option A _____

   Option B _____

   Option C _____



5. How can the graph show JoJo which option to take? Explain your answer.

6. After graphing the three options what observations can you make? List at least two observations. Be sure to include an explanation.

7. Suppose JoJo decides to go to graduate school. How much would he have after two years for each option?

Extension

8. Suppose the same options are available to JoJo but the interest is compounded quarterly. Would JoJo make the same choice?

9. Suppose the same options are available to JoJo but the interest is compounded monthly. Would JoJo make the same choice?

Off To College Part 3

A Graduation Gift Investigation

Teacher Answers

After graduating from high school, JoJo receives a graduation gift from his grandmother. There is a catch to his grandmother's gift. She will give him the money along with any interest on the day he graduates from a four-year college. However, she will let him choose the investment option.

Option A $1000 at 4% interest rate compounded annually
Option B $975 at 5% interest rate compounded annually
Option C $950 at 6% interest rate compounded annually

Now JoJo must decide which option to choose?

1. Complete the table.

Option A 4% interest rate

Year	Amount
0	1000
1	**1040**
2	**1081.60**
4	**1169.86**

Option B 5% interest rate

Year	Amount
0	975
1	**1023.75**
2	**1074.94**
4	**1185.12**

Option C 6% interest rate

Year	Amount
0	950
1	**1007**
2	**1067.42**
4	**1199.35**

2. On his graduation from college, state the amount of money JoJo can receive for each option. Did any of the amounts surprise you?

See Table

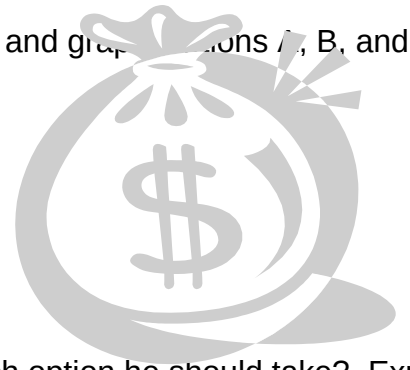
1. What amount of interest will JoJo earn for option A? ***$124.86*** option B? ***$153.68*** option C? ***$81.47***

2. Write the equations for each option and graph equations A, B, and C.

Option A ***$1000(1.04)^x$***

Option B ***$975(1.05)^x$***

Option C ***$950(1.06)^x$***



3. How can the graph show JoJo which option he should take? Explain your answer.

See Teacher Notes

4. After graphing the three options what observations can you make? List at least two observations. Be sure to include an explanation.

See Teacher Notes

5. Suppose JoJo decides to go to graduate school. How much would he have after two years for each option?

Answer: Option A $1265.32, Option B $1306.59; Option C $1347.59

Extension

6. Suppose the same options are available to JoJo but the interest is compounded quarterly. Would JoJo make the same choice?

Answer: Option A $1172.58, Option B $1189.39; Option C $1205.54

7. Suppose the same options are available to JoJo but the interest is compounded monthly. Would JoJo make the same choice?

Answer: Option A $1173.20, Option B $1190.37; Option C $1206.,96

Summative Assessment:

Use MATHEMATICAL MODELS to determine student's progress towards understanding the concept of linear, quadratic and exponential models.

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MATHEMATICAL MODELS
LINEAR, QUADRATIC, EXPONENTIAL

1) The table below gives the stopping distance for a car under certain road conditions.

Speed(mi/h)	20	30	40	50	55
Stooping distance(ft)	16	37	66	104	126

a) Find a **linear** model for the data.

b) Find a **quadratic** model for the data.

c) Compare the models. Which is better? Explain.

2) JoJo throws a ball off the top of a school building. The table shows the height of the ball at different times.

Height of a Ball

Time	Height
0 s	47 ft
1 s	64 ft
2 s	49 ft
3 s	2 ft

a) Find a **quadratic** model for the data.

b) Use the model to estimate the height of the ball at 2.5 seconds.

3) Determine whether the following are **linear**, **quadratic** or **exponential** equations.

a) $y = 2 \cdot 6^x$

b) $y = 3x - 2.5$

c) $y = 2 \cdot (x-2)^2 + 5$

d) $y = -\frac{1}{2}x + 5$

e) $y = -3x^2 + 4x - 1$

f) $y = 3 \cdot (0.6)^x$

4) Which of the following is the **vertex** of the graph of $y = 2(x+5)^2 - 3$?

a) (5, 3) b) (-5, 3) c) (5, -3) d) (-5, -3)

5) Which of the following is an **asymptote** of the graph of $y = 5 \cdot 2^x + 1$?

a) x = 1 b) y = 1 c) x = 2 d) y = 2

6) The table below gives the number of bacteria in a colony growing over time.

Time (hr)	Number of bacteria
0	52
1	312
2	1872
3	11232
4	67392

a) Make a scatter plot using your graphing calculator. What type of function best models this data set?

b) Find the equation for the curve of best fit.

c) What will be the total number of bacteria after 8 hours? Explain how you determined your answer.

MATHEMATICAL MODELS
LINEAR, QUADRATIC, EXPONENTIAL
TEACHER'S NOTES

1) The table below gives the stopping distance for a car under certain road conditions.

Speed(mi/h)	20	30	40	50	55
Stooping distance(ft)	16	37	66	104	126

d) Find a **linear** model for the data.
Answer: y = 3.16x – 53.3

e) Find a **quadratic** model for the data.
Answer: $y = 0.04x^2 - 0.04x - 0.1$

f) Compare the models. Which is better? Explain.
Answer: Quadratic model best fits the data.

2) JoJo throws a ball off the top of a school building. The table shows the height of the ball at different times.

Height of a Ball

Time	Height
0 s	47 ft
1 s	64 ft
2 s	49 ft
3 s	2 ft

c) Find a **quadratic** model for the data.
Answer: $y = -16x^2 + 33x + 47$

d) Use the model to estimate the height of the ball at 2.5 seconds.
Answer: 29.5 ft

3) Determine whether the following are **linear**, **quadratic** or **exponential** equations.

a) $y = 2 \cdot 6^x$ **exponential**

b) $y = 3x - 2.5$ **linear**

c) $y = 2 \cdot (x-2)^2 + 5$ **quadratic**

d) $y = -\frac{1}{2}x + 5$ **linear**

e) $y = -3x^2 + 4x - 1$ **quadratic**

f) $y = 3 \cdot (0.6)^x$ **exponential**

4) Which of the following is the **vertex** of the graph of $y = 2(x+5)^2 - 3$?

a) (5, 3) b) (-5, 3) c) (5, -3) d) (-5, -3)

Answer: d

5) Which of the following is an **asymptote** of the graph of $y = 5 \cdot 2^x + 1$?

a) x = 1 b) y = 1 c) x = 2 d) y = 2

Answer: b

6) The table below gives the number of bacteria in a colony growing over time.

Time (hr)	Number of bacteria
0	52
1	312
2	1872
3	11232
4	67392

a) Make a scatter plot using your graphing calculator. What type of function best models this data set?

Answer: exponential model

b) Find the equation for the curve of best fit.

Answer: $\boldsymbol{y = 52 \cdot 6^x}$

c) What will be the total number of bacteria after 8 hours? Explain how you determined your answer.

Answer: $\boldsymbol{8.73 \times 10^7}$ **Use the TABLE feature of your calculator**.