Title: Functions in Motion

Link to Outcomes:

- **Communication** Students will communicate the results throughout the lesson by discussion and writing.
- **Reasoning** Students will make conjectures, graph polynomial functions, and discuss the outcomes of their graphs.
- **Connections** Students will relate their conclusions about polynomial graphs to a variety of functions that they use in math and science classes.
- **Algebra** Students will demonstrate their ability to graph polynomial functions on paper as well as graphing calculators and Mathematica.
- **Patterns/ Relationships** Students will discover patterns relating the degree of the equation to the polynomial's graph.

Brief Overview:

Graphing of polynomial functions incorporates a wide variety of algebraic levels and skills. The students will begin by sketching algebraic functions using graph paper and pencil. By analyzing their graphs, the students will determine what they think each variable in an equation does to the graph. Technology will then be used to look at a larger range of similar polynomial equations and determine any relationships between these functions.

Grade/Level:

Pre-Algebra, Algebra I, and/or Algebra II

Duration/Length:

Depending on how much exploration time the teacher wants to allow the students, the lesson can run from 2 to 5 class periods. It also depends on the familiarity of the students with the technology chosen.

Prerequisite Knowledge:

Basic algebraic graphing of functions, familiarity with using a graphing calculator, and if possible, previous work with Mathematica.

Objectives:

The students will:

- explore polynomial functions using graph paper and a graphing calculator or Mathematica.
- draw conclusions about how the graphs should look.
- use technology to confirm ideas on why certain variables make a graph look the way it does.
- develop variable equations which represent the relationships they have discovered in polynomials.

Materials/Resources/Printed Materials:

- Pencils
- Graph paper
- Graphing calculators
- Mathematica (optional)

Development/Procedures:

- Graph the function **y=x** on graph paper by plotting points. What shape results from this equation? Add or subtract a value to the function and graph the new equation; for example, **y=x-6** . Discuss what happened to the original graph and make hypothesis on what different values will do to the original graph. Multiply the original function by a value and graph this new equation; for example, **y=4x**. Discuss your results and what will happen with other values, both positive and negative when you multiply. In your discussion, make sure the students identify the equation **y=mx+b** and the importance of "**m**" being the slope and "**b**" the y-intercept.

- Graph **y=x^2** on your graphing calculator (See attached page for directions on using the TI-82 graphing calculator to graph this). What shape is the result of this equation? Add or subtract a value to the function outside of the "x^2"; for example, **y=x^2+4** , and discuss the results of doing this with various numbers. Add or subtract a value to the "x" before squaring; for example, **y=(x-3)^2** , and discuss your results and what different numbers will do to your graph in this position.

- Graph **y=x^2** on your graphing calculator and multiply the right-hand side by a variety of numbers. Begin with **y=3x^2** and then compare it with **y=(1/3)x^2** and **y= -3x^2** . Discuss with the students what multiplying by a number larger than 1, by a fraction less than 1, and by a negative value will do to the graph of **y=x^2.**

- Combine all of the above changes to your original parabola graph and explore what happens. Use your conclusions to determine the standard equation for a parabola, **y=a(x-h)^2+k** . Be sure to discuss each variable in the formula.

- Complete the chart titled “Odd, Even, Up, or Down” using your graphing calculator or preferably Mathematica At this point, allow plenty of time for the students to “play around” with different polynomial functions and their graphs.

- Discuss the results of your chart and brainstorm with the students on ways to remember what the different graphs look like using hand signals or arm signals. Use this summary to help the students organize the results.

Degree	**# of changes between up or down**	**Basic Sketch of Polynomial**
1	0	
2	1	
3	2	
4	3	
5	4	
6	5	

Evaluation:

The students will develop their own complex equation, hypothesize how the graph will look based on the parameters given, and sketch the graph of the equation on graph paper to see if the hypothesis was correct. Depending on the outcome of their graph compared to their original ideas, the students will write a paragraph explaining the results of their project. Grading of this assignment is best accomplished with the use of a rubric (see attached example rubric).

Extension/Follow Up:

Complete the same basic lesson using Mathematica. Change your polynomial equations so that the students will have to alter the range to much larger numbers. Also increase the degrees of the polynomials and see if the same rules apply.

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Odd. Even, Up or Down

DIRECTIONS: Given the equation, decide on the degree of the polynomial. Using your graphing calculator, make a sketch of your graph and count the number of changes in directions. For the blank cells, make up equations of your own to test your conclusions.

Equation	Sketch of the Equation	**Degree of the equation**	**Changes between up and down**
$x^6-14x^4+49x^2-36$			
x3+3x2-2x-5			
$3x^2-2x+4$			
-2x+4			
$x^4+x^3-13x^2-x+12$			
$x^5+3x^4-5x^3-15x^2+4x+12$			

TI-82 Graphing Calculator Instructions
For Graphing Polynomials

(1) To Clear Memory:

EDIT
4:ClrList
Enter (twice)

(2) To Enter Equation:

Y= (top left corner)
x^2 (x is the x,t,theta key)

(3) To Graph Equation:

GRAPH (top right comer)

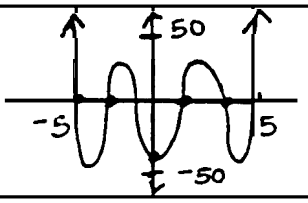
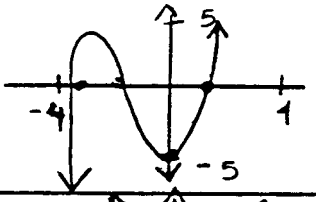
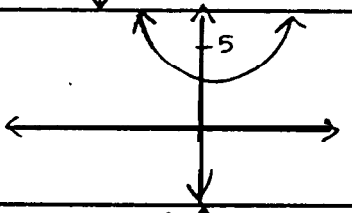
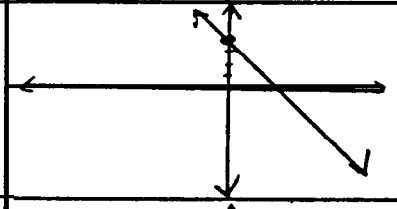
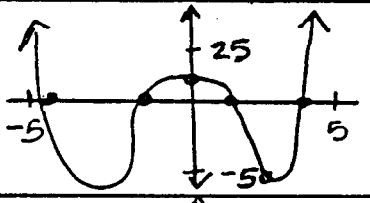
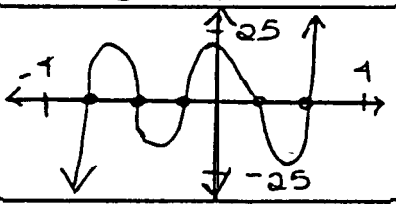
(4) To Adjust the Range:

WINDOW (next to y= key)
Xmin=-5
Xmax=5
Xscl= 1
Ymin=-10
Ymax=lO
Yscl=l

ANSWER KEY

Odd, Even, Up or Down

DIRECTIONS: Given the equation, decide on the degree on the polynomial. Using your graphing calculator, make a sketch of your graph and count the number of changes in directions. For the blank cells, make-up equations of your own to test your conclusions.

Equation	Sketch of the Equation	**Degree of the equation**	**Changes between up and down**
$x^6-14x^4+49x^2-36$		6	5
x^3+3x^2-2x-5		3	2
$3x^2-2x+4$		2	1
$-2x+4$		1	0
$x^4+x^3-13x^2-x+12$		4	3
$x^5+3x^4-5x^3-15x^2+4x+12$		5	4