Title: Relating Distance and Velocity Over Time

Link to Outcomes:

- **Communications** Students will demonstrate the ability to express physical phenomena and mathematical ideas orally and in writing.
- **Reasoning** Students will make and test conjectures to interpret data.
- **Statistics** Students will express data as data plots.
- **Technology** Students will demonstrate the ability to use the TI-82 calculator and the TI Calculator Based Laboratory (CBL) System.
- **Connections** Students will demonstrate the ability to make connections among various mathematical topics and applications to other disciplines.
- **Cooperation** Students will demonstrate the ability to solve mathematical problems in a cooperative atmosphere.
- **Algebra** Students will demonstrate the ability to apply algebraic concepts in problem-solving situations involving equations, tables, and graphs.

Brief Overview:

Students will collect data using a CBL and motion detector. The data will be graphed using distance-time and velocity-time. Teachers will have the option of using this investigation at three increasingly difficult skill levels (data collection, line of best fit, and comparison of slope and velocity).

Cooperative learning groups will be formed consisting of two to four students Each will be assigned a specific task in the collection of data. Students will study linear MOTION.

Grade/Level:

Grades 10–12: Appropriate for Algebra II, Trigonometry, Pre-calculus, and Calculus.

Duration/Length:

Minimum of two class sessions.

Prerequisite Knowledge:

- Students should have observational skills and some familiarity with the TI-82 and CBL motion detector. (This may best be facilitated if the students have completed the learning unit, “An Introduction to the CBL Systems Linked to the TI-82 Calculator by Observing MOTION and Interpreting Data Plots,” 1994 High School Summer Institute for Mathematical Teachers.)

- Students should have the ability to read and interpret coordinate graphs.

- Students must be familiar with the slope-intercept form of the line and be able to determine slope given two points.

- Students should be familiar with quadratic equations and be able to find quadratic regression equations on the TI-82 for Level II.

Objectives:

- Observe MOTION.
- Use CBL system to collect MOTION data.
- Connect physical MOTION to data lists and to data plots.
- Use TI-82 to examine data.
- Interpret data plots.
- Use TI-82 lists to find two data points and determine equation for line of best fit.
- Graph equation of a line on TI-82.
- Relate rate of a moving object over time to the slope of the line graphing distance over time.

Materials/Resources/Printed Materials:

- TI-82 with MOTION and PLOTS programs installed (One per group)
- CBL systems set up at work stations
- motion detector (One for each CBL System used)
- Lab activity worksheet
- Technology Set-up Procedures

Development/Procedures:

1. The TI-82 calculator must have the MOTION and PLOTS programs. It is left to the discretion of the teacher to decide whether to link the programs prior to class or during the class period.

2. The teacher may model the data collection on the calculator overhead with sample graphs.

3. Organize class into groups of two or three.

4. Distribute Lab Activity Worksheet. Have students complete sheet following the directions. The Worksheet has five sections. The teacher may choose to stop at the end of any section.

   Section 1 Data Collection — Use the CBL and Motion Detector to collect data.

   Section 2 Data Interpretation — Find the line of best fit algebraically.

   Section 3 Velocity Extension — Predict velocity/time graph from observations and verify by using the Stat Plot feature on the TI-82. Compare velocity value to the slope value in the distance-time graph.

Section 4 Distance and Velocity Practice — Review sections 1–3 using a different distance-time graph.

Section 5 Summary and Evaluation.

NB: The graphs used in this activity use time as the independent variable on the horizontal axis. The MOTION and PLOTS programs name these as distance-time or velocity-time, however, time is always on the x-axis.

Ideally, students should be working in pairs, so that both can see the calculator screen.

For the purpose of this activity, positive velocity is being used to describe the speed of an object moving away from the Motion Detector and negative velocity is the speed of an object moving toward the Motion Detector

The CBL sometimes gives readings with a negative distance which will give a distance-time line below the x-axis. It is not clear why this happens at this time, but it will not interfere with the worksheet questions.

Each section of the worksheet increases in difficulty. Teachers may choose to stop at any section. Section 5, however, can be used as an assessment at any level.

Evaluation:

Teacher will supervise groups and grade worksheets. The summary on the Lab Activity Worksheet includes an evaluation of the students understanding. As an alternative assessment, the instructor can give students four distance-time graphs and four velocity-time graphs and ask them to match them appropriately.

Extension/Follow Up:

The same activities could be used with parabolas and sine curves produced by the motion detector. A walking MOTION produces a parabola and a ball dropped from a bungee cord suspended over the motion detector produces a sin curve. For Calculus students the Lab Activity Worksheet could be used as an introduction to first derivatives.

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TECHNOLOGY SETUP PROCEDURE FOR VELOCITY-DISTANCE PROBLEM

1. The CBL System comes with a programmed TI-82 calculator. You will need to transmit the programs MOTION and PLOTS to your TI-82 calculator for the overhead.

2. Connect the CBL unit to the TI-82 overhead calculator with a link cable using the I/O ports located on the bottom edge of each unit.

3. Connect the Vernier Ultrasonic Motion Detector to the Sonic port on the left side of the CBL unit.

4. Turn on the CBL unit, the TI-82 calculator, and the overhead.

5. Place the Vernier motion detector on a flat surface parallel to the floor so it will detect the movement of a student walking away from or towards the motion detector. The detector should be high enough that its beam hits the student.

The CBL system is now ready to receive commands from the calculator.

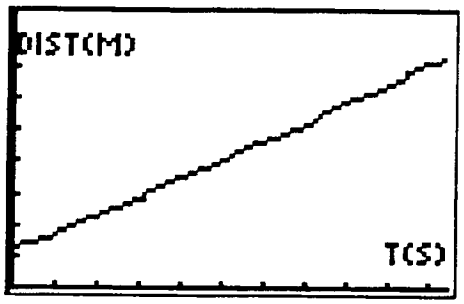
LAB ACTIVITY WORKSHEET LEVEL 1

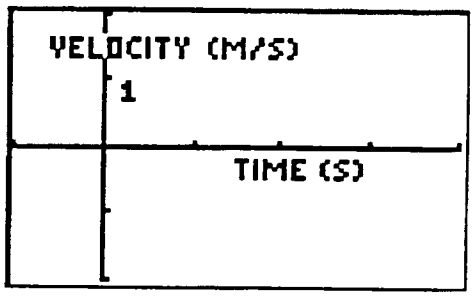
MATERIALS: You will need a TI-82 which contains the MOTION and PLOTS programs and CBL. There are five different sections of this worksheet. The work is best completed with a partner.

I. DATA COLLECTION USING THE CBL

1. When your group is ready, move to the Workstation where the CBL is set up and turned on. Link the CBL to your TI-82 with the link cable. Push in connectors firmly.

2. To create a walking MOTION which matches the data plot shown below, use these steps:
   a. Press [PRGM], cursor down to MOTION, press [ENTER], [ENTER].
   b. When the TI screen reads, "HIT ENTER TO ZERO," hit [ENTER].
   c. After "ENTER COLLECTION TIME IN SECONDS?," press [4], [ENTER].
   d. Position student in front of detector, ready to walk. One student counts down "3, 2, 1."
   e. "Hit ENTER to start collection data" as student walks. You will know the motion detector is registering data because a red light goes on and there is a clicking noise.
   f. Wait until the CBL says DONE and the PLOT OPTIONS appear.
   g. Choose PLOT option 1, "DISTANCE-TIME."





3. You may need several tries to create a similar data plot. When you are satisfied with your data, disconnect your calculator. All the CBL data is now stored in your calculator and will be used shortly. The calculator stores the time (seconds) in list L_2, the distance (meters) in list L_4, and the velocity (meters per second) in list L_5.

4. Return to your seats so the next group may use the CBL.

II. DATA INTERPRETATION
ANSWERS ALL QUESTIONS IN SENTENCES.

5. The calculator has graphed a set of DISTANCE-TIME data points that lie on a line. Look at the coordinates of the line's data points under [STAT], [EDIT], [1], [ENTER]. The x coordinates are in L2 and the y coordinates are in L4. Choose one pair of coordinates. (____,____)

   What does the x value mean?_____

   What does the y value mean?_____

6. Using the lists, find an ordered pair for a time near two seconds. (____,____) Find another ordered pair for a time near three seconds. (____,____) Use these two ordered pairs to find the slope of your line. Show your work. Round to the nearest thousandth.

7. You found the slope in #6 by finding the change in y values (distance) over the change in x values (time). What does this slope represent in our walking MOTION? (Here are two hints: You know d = rt or distance = rate x time, so what's d/t? For our purposes, another word for rate is velocity)

8. Use your lists to find the approximate y-intercept of your line. Explain what this represents in terms of your walking MOTION.

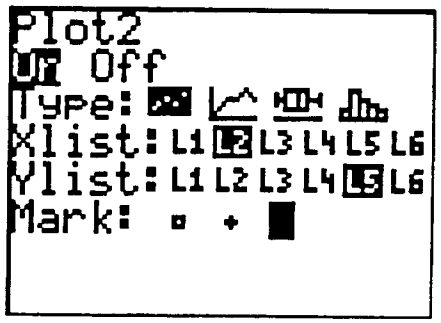
9. Give the equation of your DISTANCE-TIME line in y = mx + b form.

10. Press [Y=] and enter your equation as y1. Then press graph. Does your line fit the data? If not, check steps 6 through 9 until you have an approximate best-fit line. Write the equation of the line on the DISTANCE-TIME plot in #2 on page 1.

PART III VELOCITY EXTENSION

11. Now we'll study your walking MOTION a different way. This time we'll consider your walking speed or velocity. Suppose you had been asked to make a graph using Time on the horizontal axis and ***Velocity*** on the vertical axis. What do you think that graph would look like? Explain and make a sketch.

12. The calculator can show a VELOCITY-TIME graph of your data points. To see its graph, press [2nd], [Y=], [4], [ENTER], and press [2nd], [Y=] again. Then complete these steps:
    a. Cursor to Plot2. [ENTER]
    b. Highlight these choices:



    c. Press [WINDOW] and key in these values: Xmin=-1, Xmax=4, Xscl=1, Ymin=-2, Ymax=2, Yscl=1.
    d. Press graph.

13. Be sure everyone in your group sees the data plot. This is the calculator's plot of VELOCITY-TIME points. It should look like a cluster of data points. How does the calculator's graph of VELOCITY-TIME compare to yours from step 11?

14. The VELOCITY-TIME data plot appears basically horizontal. Use [TRACE] to view the y coordinates of these points. What do these y values represent?

Note that these y values are not the same, but are relatively close. Why?

15. Find the equation of a best-fit horizontal line for this plot:
    a. Your screen should show your VELOCITY-TIME scatter plot.
    b. Press [2nd] [PRGM].
    c. Press [3]
    d. Use the up and down arrow keys until you see a horizontal line on your scatter plot. Use the arrows to position this line as a best-fit line, then press [ENTER].

What is the equation of the horizontal line?

What does this line represent?

Sketch this line and write its equation on the VELOCITY-TIME graph on page 1 in #2.

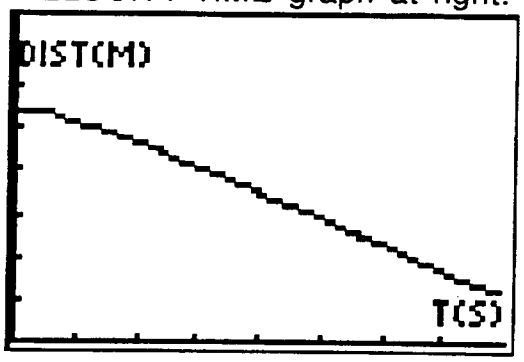
16. In #2 there are now two different graphs of the same MOTION. Explain why they are different.

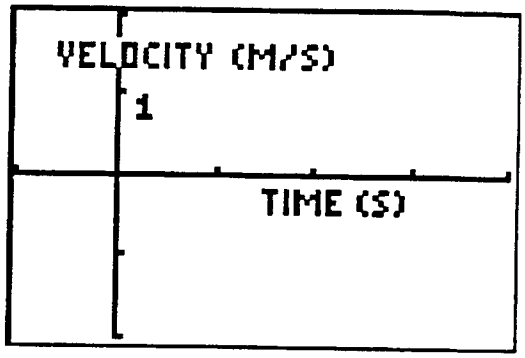
17. There is an important connection between the ***slope*** of the DISTANCE-TIME line and the ***constant*** of the VELOCITY-TIME line.
    a. What do both these numbers have to do with your speed?

    b. Why are they appoximately the same?

IV MORE DISTANCE AND VELOCITY PRACTICE

18. Repeat **ALL** the steps 1–15 to create the new DISTANCE-TIME graph shown below. As you work, answer the questions below and complete the VELOCITY-TIME graph at right:





19. What is the equation of the line shown in the DISTANCE-TIME graph?

20. What is the equation of the line in the VELOCITY-TIME graph?

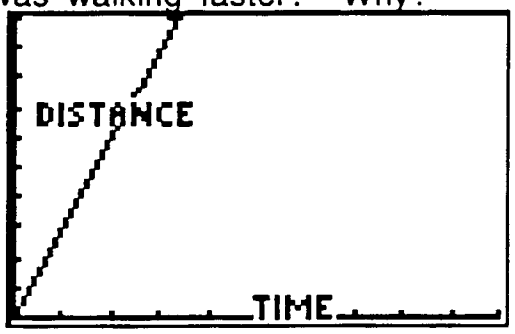
21. What is the connection between the ***slope*** of the DISTANCE-TIME line and the ***constant*** of the VELOCITY-TIME line.

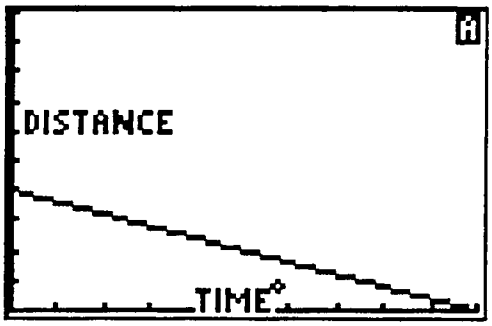
V. SUMMARY

22. Suppose you gave a friend the two graphs in #18. Explain two ways he or she could use your graphs to find the velocity of your walking MOTION.

23. Suppose two friends show you their DISTANCE-TIME graphs below. Who was walking toward the motion detector? Why?

Who was walking faster? Why?





24. For each graph above, describe the graph of the VELOCITY-TIME line and sketch it below.