

Title: Incredible Integers!

Brief Overview:

Students of sixth grade mathematics often come to middle school lacking a firm grasp on the concept of positive and negative numbers. The integer unit is critical for the development of this understanding and serves as a vital springboard for future work with negative numbers. The unit is divided into three main components, each involving a full, ninety-minute lesson. An overhead projector should be available for all lessons. The first lesson reviews key vocabulary and builds a fundamental understanding of positive and negative whole numbers and their absolute values. The second lesson moves to the application of those integers in addition and subtraction. The final lesson applies integers to situations requiring multiplication and division.

NCTM Content Standard:

Number and Operations Standard for Grades 6-8

Understand numbers, ways of representing numbers, relationships among numbers, and number systems.

- Develop meaning for integers and represent and compare quantities with them.

Grade/Level:

Grade 6, advanced, and Grade 7.

Duration/Length:

Three (3) ninety-minute lessons.

Student Outcomes:

Students will be able to:

Lesson 1:

- Identify and plot integers on a number line.
- Determine the absolute value of any integer.

Lesson 2:

- Evaluate sums and differences of integers.

Lesson 3:

- Evaluate products and quotients of integers.
- Demonstrate mastery of unit principles through summative assessment.

Materials and Resources:

Lesson 1:

- Painter's Tape for number line
- Tissue Box – empty – for absolute value visual aid
- Centimeter Cubes (class set)
- “Coach and Player” Handout (one per pair)
- Student Integer Cards (class set)
- Crossword Puzzle Handout (one per student)

Lesson 2:

- Painter's tape for number line
- Integer Bingo Template (one per student)
- Algebra Tiles (class set)
- Centimeter Cubes (class set)
- Counters (approx. 24 per students)
- Worksheets (three) (one each, per student)
- Index Card set of random Bingo “draws”

Lesson 3:

- Dry Erase White Boards (class set)
- Spinner Template (copy two per student – to assemble front and back)
- String for hanging spinners
- Poem / Song Handout (two per team)
- Student Integer Cards (class set)
- Teacher list of Integer expressions
- Hula-hoop or string to create circle

Vocabulary:

- Absolute Value - The distance of a number from zero.
- Associative - A property of integers pertaining to both addition and multiplication, whereby the manner in which three integers are grouped does not alter the sum or product of the integers.
- Commutative - A property of integers pertaining to both addition and multiplication, whereby the order in which three integers are arranged does not alter the sum or product of the integers.
- Difference - The answer to a subtraction problem.
- Distributive - The property of multiplication over addition which demonstrates that for all numbers a , b , and c ; $a(b + c) = ab + ac$, and $ab + ac = a(b + c)$.
- Integers - The set of numbers containing zero, the natural numbers, and all the negatives of the natural numbers.

- Inverse - Opposite. The additive inverse of a number is the opposite sign of that number.
- Irrational Numbers- Numbers that cannot be expressed as the ratio of two integers.
- Natural Numbers - The counting numbers.
- Negative Numbers - Real numbers that are less than zero.
- Number Line - A line on which every point represents a real number.
- Positive Numbers - Real numbers that are greater than zero.
- Product - The answer to a multiplication problem.
- Quotient - The answer to a division problem.
- Rational Numbers - Numbers that can be expressed as the ratio of two integers; denominator cannot equal zero.
- Sum - The answer to an addition problem.
- Whole Numbers - The set of numbers that includes zero and all of the natural numbers.

Development/Procedures:

Lesson 1

Preassessment – Students should have prior knowledge of basic integer vocabulary. To verify / review this, perform “The Hot Seat” activity: Student chairs are organized in a single row, all facing the same direction. Students are seated. First student in the row is on “The Hot Seat” to define a randomly selected vocabulary word from the list. If correct, student remains seated, and “The Hot Seat” moves to the next seated student. If incorrect, student stands and moves to the end of the row. All other students move up one seat. Instructional objective is to reinforce required unit vocabulary. Motivational objective is to be in one of the first “X” number of seats in the row at the end of the activity, depending on class size. Reward those students with a class-appropriate reward.

Launch – Have painter’s tape tacked to floor in view of all students for large-scale representation of number line. Remind students of positive number sequencing to the right of zero on number line. Introduce / remind students of negative number sequencing to the left of zero. Explain that these are opposites – inverses – of the positive numbers. This can segue to the concept of absolute value. Show “BrainPop” video available at www.brainpop.com (subscription required) on absolute value.

Teacher Facilitation – Use tape number line to illustrate absolute value – distance from zero – in connection with negative integers. Introduce the “car wash” idea to illustrate absolute

value. “Clean” is positive; “dirty” is negative. No matter which way the car goes into the car wash, it comes out clean! No matter whether a number starts out “clean” or “dirty” it comes out of the absolute value operation “clean”. Include issue of negative **outside** parentheses: if you “dirty” the car after the car wash, it will always be “dirty”.

Use centimeter cubes to reinforce opposite integers: set one color as negative and another as positive. Have students display opposites and others stand on the number line to represent those integers. Address the issue of ordering integers. Which is “greater”: $+4$ or -12 ? Place students on number line and teach rule: “**L**eft is **L**ess” (greater values are always to the right of lesser values). Illustrate several examples using centimeter cubes and students standing on number line.

Student Application – Verifying that the concepts of negative integers and absolute value are being understood and mastered is critical prior to moving forward to operations with integers. Assign or allow students to find a partner. Give each pair a “Coach and Player” handout. Students take turns in each role from question to question. The “Coach” tells the “Player” every step to solving a problem. The “Player” must write **only** what the “Coach” says. Once problem is completed, if “Player” feels answer is wrong, he or she then suggests the correction to the “Coach” and the pair arrive at an agreement for that problem. Pairs solve the next problem with roles reversed.

Following paired practice, arrange class into two teams and pass out randomly arranged integer cards to all students. At the teacher’s signal, students look at their integers and arrange their team in a row from least to greatest. First team to arrange themselves correctly wins. Play several rounds.

Embedded Assessment – Both the “Coach and Player” activity and the team activity provide embedded assessment tools for the instructor. The crossword puzzle handout for homework is also a means of assessment.

Reteaching / Extension – For those who have not completely understood the lesson, students can be aided one-on-one during the “Coach and Player” activity. Work with these students using the number line.

Coach	and	Player
1.	Order from least to greatest: 18, 30, -6, -28, 15, -45, 23	
2.	Solve: $ -11 + 7 $	
3.	Determine the integer: Sixty-two feet below sea level.	
4.	Order from least to greatest: -14, 6, 21, 13, -8, -4, -12, 7, -5	
5.	Solve: $- 3 + -9 $	
6.	Determine the integer: A profit of \$150.00	
7.	Order from least to greatest: 19, $ -16 $, 44, -23, -21, $ -4 $, -2	
8.	Order from least to greatest: 0, 39, $ -36 $, 34, -33, -31, $- 38 $	

Integer Cards for Lessons One and Three

Directions: Laminate integer cards for long-term use and then cut out each card. Please photocopy and laminate as many stacks as needed – Remember one stack per group and its best if you color code each group a different color. Color makes it easier to quickly spot which group has correct answer.

-20	-19	-18
-17	-16	-15
-14	-13	-12
-11	-10	-9

-8	-7	-6
-5	-4	-3
-2	-1	0
1	2	3

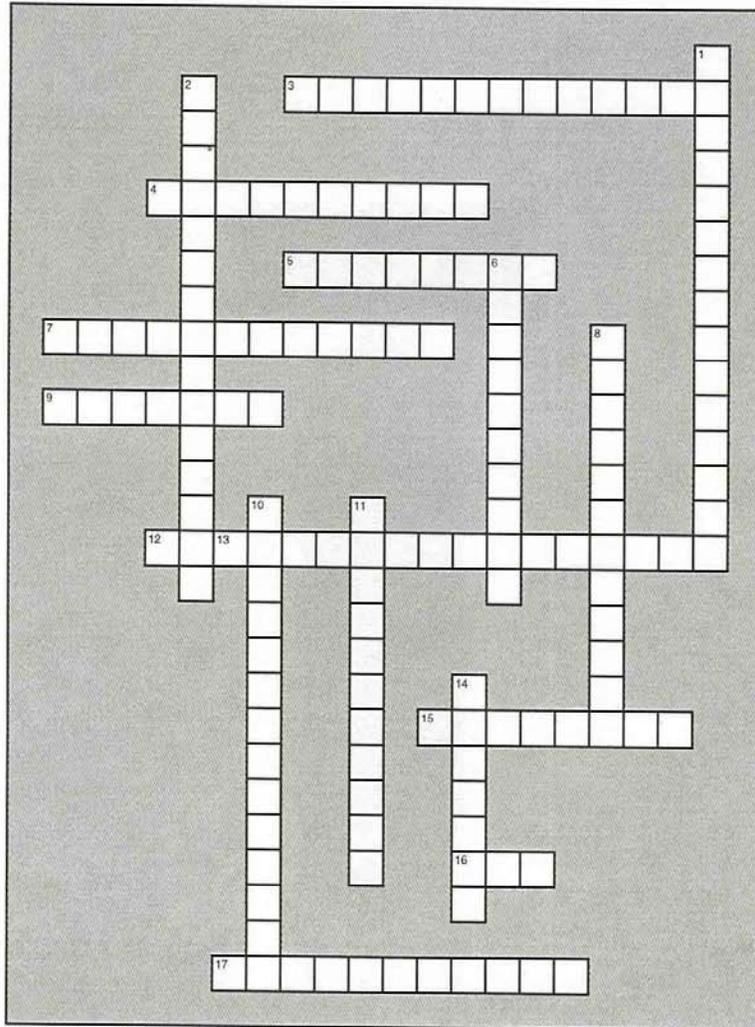
4	5	6
7	8	9
10	11	12
13	14	15

16	17	18
19	20	+
-	+	-
-	+	-

Vocabulary Crossword:

Name _____

Crossword



Across

- 3. The distance of a number from zero.
- 4. The answer to a subtraction problem.
- 5. The answer to a division problem.
- 7. The set of numbers that includes zero and all of the natural numbers.
- 9. The answer to a multiplication problem.
- 12. Numbers that cannot be expressed as a ratio of two integers.
- 13. Numbers that can be expressed as the ratio of two integers.
- 15. The set of numbers containing zero, the natural numbers, and all the negatives of the natural numbers.
- 16. The answer to an addition problem.
- 17. A property of integers pertaining to both addition and multiplication, whereby the manner in which three integers are grouped does not alter the sum or product of the integers.

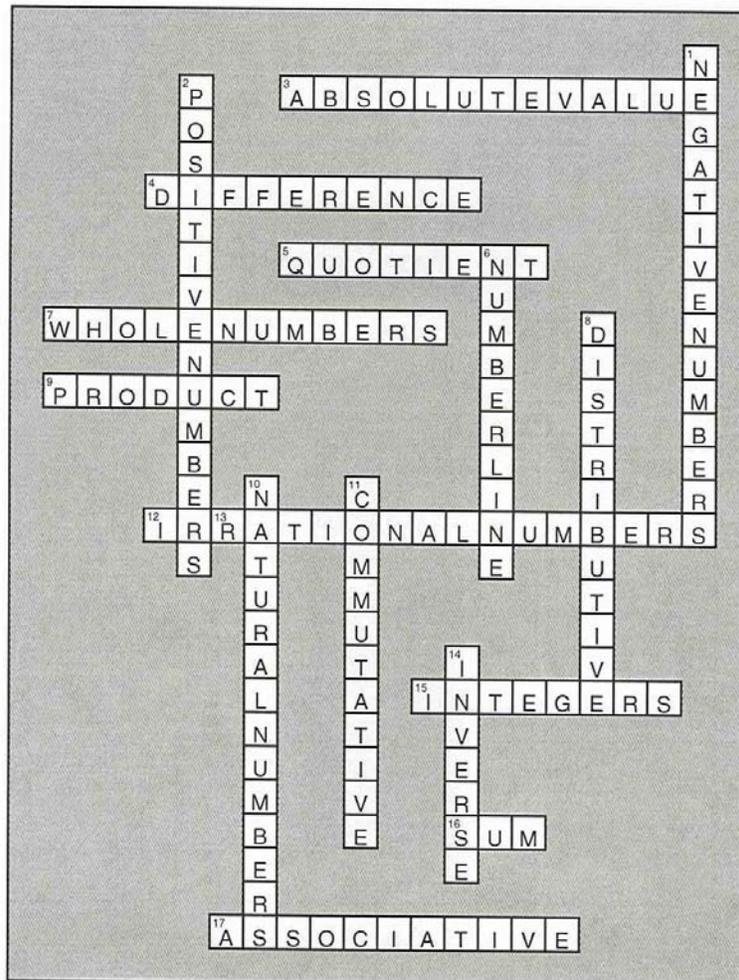
Down

- 1. Real numbers that is less than zero.
- 2. Real numbers that are greater than zero.
- 6. A line on which every point represents a real number.
- 8. A property of integers whereby one integer multiplied by the sum of two other integers is equal to the sum of the product of that first integer and the other two integers.
- 10. The counting numbers.
- 11. A property of integers pertaining to both addition and multiplication, whereby the order in which three integers are arranged does not alter the sum or product of the integers.
- 14. Opposite. The additive inverse of a number is the opposite sign of that number.

Crossword Answer Key:

Name _____

Crossword



Across

3. The distance of a number from zero.
4. The answer to a subtraction problem.
5. The answer to a division problem.
7. The set of numbers that includes zero and all of the natural numbers.
9. The answer to a multiplication problem.
12. Numbers that cannot be expressed as a ratio of two integers.
13. Numbers that can be expressed as the ratio of two integers.
15. The set of numbers containing zero, the natural numbers, and all the negatives of the natural numbers.
16. The answer to an addition problem.
17. A property of integers pertaining to both addition and multiplication, whereby the manner in which three integers are grouped does not alter the sum or product of the integers.

Down

1. Real numbers that is less than zero.
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11. A property of integers pertaining to both addition and multiplication, whereby the order in which three integers are arranged does not alter the sum or product of the integers.
14. Opposite. The additive inverse of a number is the opposite sign of that number.

Lesson 2

Preassessment – Students should have prior knowledge on the properties of an integer and their location on a number line.

Warm-up Activity:

Order these numbers from least to greatest and then graph on a number line.

a) 3, -5, 5, 8, 0, -2

b) 4, 9, 2, -4, -1, 0

c) 5, -2, -6, 2, 4, 8

Add and Subtract the following numbers

a) $10 - 9$

b) $7 + 6$

c) $13 + 7$

Launch – Go to www.brainpop.com. Click on Math. Click on Adding and Subtracting Integers. BrainPop gives a brief introduction on what integers are and how to add and subtract integers. Subscription required.

Explain objective to students. At the completion of the lesson the students will be able to evaluate sums and differences of integers.

Teacher Facilitation- You will need an overhead projector. Begin with “unit” Algebra tiles. Write on the board what the tiles represent. Red (dark) tiles are negative, Yellow (light) tiles are positive. Place 4 yellow tiles on the overhead and explain that this represents positive 4. Place 4 red tiles on the overhead and explain that these tiles represent negative 4. Explain how the tiles cancel each other out, also called zero pairs. Present various examples for the students. If students do not have algebra tiles, students can draw integers or use centimeter cubes of two different colors.

Place 4 red tiles and 2 yellow tiles on the overhead. Remind students that zero pairs cancel each other out. Ask them how many tiles are left after removing the cancelled tiles. The answer would be 2 but because they are red the final answer is negative 2. Write the equation on the board. $-4 + 2 = -2$ Try more examples using the algebra tiles. $3 + -2 = 1$, $5 + -3 = 2$.

Model the following three rules prior to stating each: (a) the sum of two positive integers is always positive. $3 + 4 = 7$. (b) The sum of two negative numbers is always negative.

$-7 + (-6) = -13$. (c) The sum of a positive and negative number carries the sign of the integer with the larger absolute value. $6 + (-2) = +4$. $(-7) + 3 = -4$.

Move on to subtraction. Write $-6 - 4$. Place 6 red tiles on the overhead projector. Can't subtract 4 yellow tiles, so we have to add two sets of "zero pairs" (1 red and 1 yellow). So, place 4 yellow tiles and 4 more red tiles. Then subtract the 4 yellow tiles needed to complete the operation. The final answer should be -10 . Explain to the students the process of subtracting. When subtracting signed numbers, write the following rule: Change the operation to addition and change the sign of the second integer. $(-4) - (-5) = (-4) + (+5) = 1$. Provide more examples for the students, modeling with the algebra tiles and the numbers that represent them.

If reteaching or further modeling is needed, then place the painters tape on the ground and write numbers according to how many students are in your class. Rules: The student is to start at 0, facing positive integers, at the beginning of each equation. Students will be given an equation. Ex. $9 - 4$. They will then walk nine in the positive direction – positive is always walking **forward**. The next part is minus 4 so they will step backwards four steps, which would lead them to their answer of 5. Another example would be $5 - (-5)$. They would begin at zero facing the positive direction then walk 5 steps in that direction. Since the number is negative they know that they have to walk **backwards** but they must turn physically around (subtraction always indicates "turn around") and then walk backwards five steps. Their answer would then be 10. Solicit questions as needed.

Student Application – Hand out worksheet "I Can Use Integers For That". Explain the directions for the worksheet. Go over the example that is prepared then pair students up. Give a time frame for the worksheet. Circulate to verify that students are clear on the concept. Help as needed.

Assessment – Hand out Integer Bingo board and counters. Ask students to place the numbers -20 through 20 on their bingo board. Rules are similar to regular Bingo.

Reteaching / Extension – There are 2 levels of worksheets provided for the various levels of learners.

Integer Bingo

		FREE		

Please use the following numbers: -20, -19, -18, -17, -16, -15, -14, -13, -12, -11, -10, -9, -8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20

Integer Bingo Teacher Resource List

Addition/Subtraction

Directions: $A \pm B = C$

Beginner – Play where you call out any integer in column A or column C and the students place their Bingo Chip over the square where they wrote that number.

Advanced – Play where you call out the problem ($A \pm B$) and the student must solve the problem and place their Bingo Chip over the square of the answer that the teacher has in column C.

A	B	C		A	B	C
15	-26	-11		18	1	19
-10	1	-9		0	-8	-8
9	7	16		-17	17	0
-1	-12	-13		-4	0	-4
-1	2	1		6	-17	-11
-12	5	-7		-6	-10	-16
-17	25	8		13	6	19
-6	25	19		9	-20	-11
6	11	17		-20	6	-14
-15	21	6		-5	23	18
7	1	8		-5	0	-5
-20	0	-20		-11	6	-5
4	-20	-16		-13	18	5
-9	21	12		-11	0	-11
-20	15	-5		12	-10	2
17	-17	0		-2	-8	-10
14	-15	-1		-7	17	10
-10	27	17		9	-15	-6
9	-13	-4		18	-9	9
-10	-3	-13		9	-13	-4
-16	13	-3		-15	17	2
9	11	20		-5	-10	-15
-11	0	-11		3	-17	-14
4	-5	-1		3	9	12
13	-31	-18		-7	4	-3

** You may create your own random integer bingo list if you have a TI graphing calculator. I used the TI-84 plus silver edition and entered the following steps.

- Steps:
- (1) STAT
 - (2) 1:EDIT
 - (3) You will see your table for L1, L2, L3, etc.
 - (4) Highlight the L1 (list name)

- (5) **Highlight** the L1 (list name) – hit CLEAR if you already have info. in list
- (6) At the bottom of the screen you will see L1 =
- (7) Hit the MATH key.
- (8) Arrow to the right until PRB is **highlighted**
- (9) Arrow down to 5:randInt(
- (10) Hit Enter.
- (11) Type in the lowest number you want, then place a coma (,) after the lowest number, then type in the highest number you want and enter a coma (,), and finally enter the quantity of numbers you want to generate. For example, if I want a range from -20 to 20. Then my calculator would display this.
L1=randInt(-20,20,50). This would produce 50 random integers from -20 to 20. Hit Enter.
- (12) **Highlight** L3
- (13) At the bottom of the screen you will see L3 =
- (14) Repeat step 7 through 11 for list 3 (L3).
- (15) Highlight L2
- (16) At the bottom of the page you will see L2 =
- (17) Type in the equation you would like for the answer. For example, with the addition and subtraction chart, I typed in L3 – L1. Hit Enter.
- (18) Connect calculator to your computer with an USB cord
- (19) Click on TI Device Explorer to locate your lists
- (20) Copy and Paste your lists into a chart onto a Microsoft Excel spreadsheet
- (21) Design your chart to your liking
- (22) Enjoy!

Name _____

Date _____

I Can Use Integers For That?

Directions: Please read the following questions and apply the appropriate response.

The Washington Redskins played a game against the New York Giants. Help us figure out what happened at the game.

1. After the kickoff the Redskins were at the 20-yard line, they made a play and ended up at the 40-yard line. How many yards did the team advance?
2. After the yards were determined they loss 15 yards, then gained 5 yards. Where are the Redskins now located?
3. The quarterback was tackled on the next play and lost 8 yards, but gained 12 on the next play. What yard line are they on now?

Please show the following equations by drawing algebra tiles. Specify what each tile represents. Include the solution.

4. $-3 + 4 =$

5. $-5 + 8 =$

6. $9 + (-2) =$

7. $-5 - 3 =$

8. The sum of two positive numbers is _____.

9. The sum of two negative numbers is _____.

10. The sum of a positive and a negative number can be one of the following, give examples.

- A)
- B)
- C)

11. Complete the chart. Then create an equation for you partner to solve.

Number	+/-	Number	=	Answer
-3			=	6
	+	4	=	8
		12	=	-3
14	-	20	=	
64			=	-20
			=	

I Can Use Integers For That? ANSWER KEY

The Washington Redskins played a game against the New York Giants. Help us figure out what happened at the game.

1. After the kickoff the Redskins were at the 20-yard line, they made a play and ended up at the 40-yard line. How many yards did the team advance? **20 yards**
2. After the yards were determined they lost 15 yards, then gained 5 yards. Where are the Redskins now located? **10 yard line**
3. The quarterback was tackled on the next play and lost 8 yards, but gained 12 on the next play. What yard line are they on now? **14 yard line**

Please show the following equations by drawing algebra tiles. Specify what each tile represents. Include the solution.

4. $-3 + 4 =$  $+$  $=$ 

5. $-5 + 8 =$
 $+$  $=$ 

6. $9 + (-2) =$
 $+$  $=$ 

7. $-5 - 3 =$
 $-$  $=$ 

8. The sum of two positive numbers is positive.

9. The sum of two negative numbers is negative.

10. The sum of a positive and a negative number can be one of the following, give examples.

A) Zero - ex. $6 + (-6) = 0$

B) Positive - ex. $10 + (-2) = 8$

C) Negative - ex. $-12 + (5) = -7$

11. Complete the chart. Then create an equation for you partner to solve.

Number	+/-	Number	=	Answer
-3	+	9	=	6
4	+	4	=	8
9	-	12	=	-3
14	-	20	=	-6
64	-	84	=	-20
			=	

Name _____ Date _____

Integer Worksheet – level 1

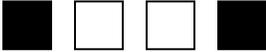
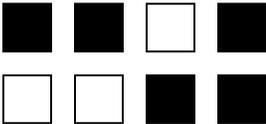
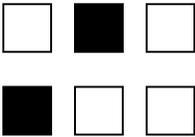
Use tiles to answer the following equations

1. $7 + -6 =$
2. $-5 + 5 =$
3. $9 - -4 =$
4. $20 + -4 =$
5. $-15 + 18 =$

Use a number line to answer these questions.

1. $-2 + 13 =$
2. $4 + -5 =$
3. $23 - -10 =$
4. $-5 + -26 =$
5. $-15 + -9 =$

What integer do the algebra tiles present?

1. 
2. 
3. 
4. 
5. 

Find two integers whose sum is equal to -26 . Are there more examples that you can think of? Give at least 4 examples.

Name _____ Date _____

Integer Worksheet – level 1 ANSWER KEY

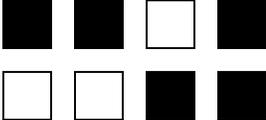
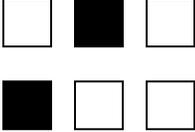
Use tiles to answer the following equations

1. $7 + -6 = 1$
2. $-5 + 5 = 0$
3. $9 - -4 = 13$
4. $20 + -4 = 16$
5. $-15 + 18 = 3$

Use a number line to answer these questions.

1. $-2 + 13 = 11$
2. $4 + -5 = -1$
3. $23 - -10 = -33$
4. $-5 + -26 = -31$
5. $-15 + -9 = -24$

What integer do the algebra tiles present?

1.  0
2.  -2
3.  4
4.  2
5.  -1

Find two integers whose sum is equal to -26 . Are there more examples that you can think of? Give at least 4 examples.

Answers may vary: $-12 + -14$, $-23 + -3$, $30 + -4$, $-6 + -20$, $0 + -26$, $-10 + -16$

Name _____

Date _____

Integer Worksheet – level 2

Please use the number line, algebra tiles, mental math or any other method to solve the following problems. Show your work.

1. Brandon had \$35 on Monday. He went to the store and purchased food that cost \$10. How much money does Brandon have left?

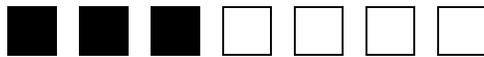
2. James had \$18 to go shopping for a party. He called Devin to go shopping with him. James and Devin went to the supermarket their money totaled \$59. How much money did Devin add to the total?

3. Mary owes you \$13 she gave you \$20, how much money does she get back?

4. Today is Tuesday, what day is -3 days ago?

5. Draw algebra tiles to represent the following equation, include the solution:
 $5 + -8 =$

6. What is the equation and solution for the algebra tiles represented? Black Tiles are negative, white tiles are positive.



7. The chart below lists the five-day temperature for Vostok, Antarctica in late June.
 - a. Find the difference between the warmest day and the coldest day.
 - b. Find the sum of the two coldest days. The two warmest days.
 - c. Which day has the coldest temperature?
 - d. What day has the warmest temperature?
 - e. Create a question using the table provided.

Location	Vostok, Antarctica
Day 1	-67° F
Day 2	-79° F
Day 3	-59° F
Day 4	-47° F
Day 5	-48° F

Integer Worksheet – level 2 ANSWER KEY

1. Brandon had \$35 on Monday. He went to the store and purchased food that cost \$10. How much money does Brandon have left?

$$35-10=25 \quad \$25$$

2. James had \$18 to go shopping for a party. He called Devin to go shopping with him. James and Devin went to the supermarket their money totaled \$59. How much money did Devin add to the total?

$$59-18=41 \quad \$41$$

3. Mary owes you \$13 she gave you \$20, how much money does she get back?

$$20-13=7 \quad \$7$$

4. Today is Tuesday, what day is -3 days ago?

Tuesday, Monday, Sunday, Saturday ***Saturday***

5. Draw algebra tiles to represent the following equation, include the solution:

$$5 + -8 =$$

$$= \quad \blacksquare \quad \blacksquare \quad \blacksquare \quad -3$$

6. What is the equation and solution for the algebra tiles represented? Black Tiles are negative, white tiles are positive.

$$-3+4=1$$

7. The chart below lists the five-day temperature for Vostok, Antarctica in late June.

a. Find the difference between the warmest day and the coldest day. **32**

b. Find the sum of the two coldest days. The two warmest days.

Coldest: -146° Warmest: -95°

c. Which day has the coldest temperature? ***Day 2***

d. What day has the warmest temperature? ***Day 4***

e. Create a question using the table provided. ***Answers may vary***

Location	Vostok, Antarctica
Day 1	-67° F
Day 2	-79° F
Day 3	-59° F
Day 4	-47° F
Day 5	-48° F

Lesson 3

Pre-assessment – Students should have prior knowledge of:
Multiplying all positive numbers; Identifying integers and absolute values; evaluating sums / differences of integers.

Launch – Refresh students on the prior two lessons of integers. Ask one to five random problems reviewing the previous days' concepts. Introduce the idea of multiplying and dividing integers by showing a short video clip on "Brain Pop." Go to www.brainpop.com, click on Math, then click on Multiplying and Dividing Integers. (Subscription Required)

Teacher Facilitation - Direct the students in a discussion about multiplication and division, then lead them into multiplying and dividing using integers. Today's lesson will focus on learning and comprehending the basic rules to multiplying and dividing integers. Adding of integers can be used as a springboard for understanding the rules for multiplication and division:

Rule One: $(3)(4) = 12$ 3 fours are 12.
 $4 + 4 + 4 = 12$ (prior knowledge)

Multiplying or dividing a positive by a positive = positive.

$$\frac{12}{3} = 4 \quad \text{inverse is also true.}$$

Rule Two: $(3)(-4) = -12$ 3 neg. fours are -12.
 $(-4)+(-4)+(-4) = -12$ (prior knowledge)

Multiplying or dividing a positive by a negative = negative.

$$\frac{-12}{3} = -4 \quad \text{inverse is also true.}$$

Rule Three: $(-3)(-4) = 12$ this one requires
observing a pattern based on Rule Two:

$(-3)(3) = -9$; $(-3)(2) = -6$; $(-3)(1) = -3$; $(-3)(0) = 0$;

what's next in the pattern? $(-3)(-1) = 3!$

Multiplying or dividing a negative by a negative = positive.

$$\frac{12}{-3} = -4 \quad \text{inverse is also true.}$$

Write the three rules of multiplying and dividing integers. Model and demonstrate two to three examples of each operation.

Restate the integer rules explained earlier in class. Then have students add that these rules ONLY apply when using

two integers. If multiplying or dividing three or more integers, the students **MUST** multiply or divide two numbers at a time. Group by parentheses.

Multiplying or dividing three or more integers:

Example One: (3) (4) (5)

$$3 \cdot 4 = 12$$

$$12 \cdot 5 = 60$$

Positive 60 is the correct answer

Example Two:

(-3) (4) (-5)

$$-3 \cdot 4 = -12$$

$$-12 \cdot -5 = 60$$

Positive 60 is the correct answer

Example Three:

(-3) (-4) (-5)

$$-3 \cdot -4 = 12$$

$$12 \cdot -5 = -60$$

Negative 60 is the correct answer

Leave all three examples on chalkboard. Then ask the students if they can identify any patterns in the rules. Give hints if necessary. May need to say that the pattern is dealing with the negative signs. Once discovery is complete, write the “Short Cut” *Rule* on the chalkboard. When you are multiplying or dividing three or more integers the students must count the negative signs. If there is an **even** number of integers the answer yields a **positive** response. If there is an **odd** number of integers the answer yields a **negative** response.

Student Application – Divide the class into groups of four (conditional upon class size). Hand out the integer card sets. One set of integers per group. The integer set should include (-20 → 20). The groups need to be placed around the outer edge of the classroom in order to make room to place the hula-hoop in the center of the room. The teacher will have the hula-hoop integer teacher resource list of problems and stand in the front of the room. The students will place all cards face down so that no integers are showing. The teacher will then read off a problem from the list and the students will then flip the cards over and search for the integer that answers the problem. The first team to run to the hula-hoop and stand in the hula-hoop with the correct answer

wins a point. Reward could be offered to the winning team. Once the students have the basic concept down, the teacher will then reverse what he/she is asking. The teacher will call out an answer to a problem that the students will have to create with their integers in their group.

For example:

Beginning Concept – the teacher calls out $(-5)(2)$ and the students search for -10 and one person in the group runs the answer to the hula-hoop.

Advanced Concept– the teacher calls out (-10) and the students search for a problem that yields (-10) as the answer. Possible answers could be $(5)(-2)$ or $(-5)(2)$, etc.

Embedded Assessment – The in-class “Hula-hoop” activity reinforces the Product and Quotient Integer rules. Use informal assessments to assess the students’ comprehension and application of the integer rules taught that day. For an additional assessment use the “Fun with Integers” worksheet as homework. You may either begin in class and finish at home or simply hand out as the homework for the evening. This worksheet was created to reinforce what was learned in lesson three along with a short section for recalling prior knowledge of the previous two-day’s lessons.

Reteaching / Extension – For those who **have not** completely grasped the concept of multiplying or dividing integers you may use the hula-hoop resource list as a guide and continue to go over questions with the students however, you will need to use the smaller digits to reinforce the concept. For those who **have** completely grasped the concept of multiplying or dividing integers, you may use the hula-hoop resource list as a guide and challenge them by using larger numbers than provided. At the end of the re-teaching time, you may want to play one last round of the hula-hoop competition.

Integer Hula-Hoop Teacher Resource List

Multiplication/Division

Directions: $A \bullet B = C$ or $C \div B = A$

Beginner – Play where you call off any integer in column C and the students find the numbers from their integer packet on their desk and run the numbers from the teacher’s column A or B to the hula-hoop.

Advanced – Play where you call out the problem’s product and either the dividend or the divisor from column A and B. Whichever columns you call, the students run the missing column’s integer to the hula-hoop.

A	B	C		A	B	C
13	-12	-156		-7	-13	91
1	-10	-10		18	4	72
7	-13	-91		-8	-14	112
14	7	98		-5	7	-35
-17	3	-51		9	14	126
-6	-1	6		19	-20	-380
13	20	260		-20	-6	120
8	-10	-80		-4	18	-72
12	3	36		16	-18	-288
7	-10	-70		0	7	0
9	0	0		5	12	60
-20	6	-120		-19	-1	19
15	8	120		-15	11	-165
-11	3	-33		-17	14	-238
13	-12	-156		16	20	320
-5	-17	85		18	-8	-144
-20	1	-20		-4	-4	16
-5	12	-60		-6	17	-102
-13	-11	143		-7	6	-42
-16	2	-32		15	3	45
-5	17	-85		-7	5	-35
13	10	130		-12	16	-192
16	-19	-304		-2	-5	10
-8	3	-24		-5	-8	40
7	9	63		16	-13	-208

** When zeros are used, the missing integer may possibly be different from the missing integer in the resource table.

Fun with Integers

Directions: Read each question carefully. Once you have read or examined each question, begin to work. You must SHOW all of your work in order to receive full credit.

Section One: Quick Review

Place a $<$, $>$, or $=$ sign on the line provided.

(1) -4 _____ -9 (2) 5 _____ -7 (3) -3 _____ 6 (4) 8 _____ $|-8|$

Section Two: Multiplying/Dividing Integers

Solve – All work must be shown inside the box provided. [] = multiply

(1) $ -5 \cdot -3$	(2) $[9][2][-3]$	(3) $[8][-2][3]$	(4) $ 15 \div (-3)$
(5) $-6 \cdot -9$	(6) $[-2][-4][-8]$	(7) $[36 \div 6][-4]$	(8) $[-4][-16 \div -2][-3]$
(9) $-42 \div 7$	(10) $[-4 \div -2][-81 \div 9]$	(11) $[-5][2][-3][3][-3]$	(12) $[36 \div 6][10 \div -2]$

(13) In the past 7 plays, the Sligo Lions have received a 5-yard penalty on each play. By how many yards did the Lions' position on the field change?

(14) During the game, the Sligo Lions received 8 penalty violations for a total loss of 40 yards. If they lost the same yardage for each penalty call, by how many yards did the Lion's field position change on each penalty?

Fun with Integers – Answer Key

Directions: Read each question carefully. Once you have read or examined each question, begin to work. You must SHOW all of your work in order to receive full credit.

Section One: Quick Review

Place a $<$, $>$, or $=$ sign on the line provided.

$(2) -4 \underline{>} -9$

$(2) 5 \underline{>} -7$

$(3) -3 \underline{<} 6$

$(4) 8 \underline{=} |-8|$

Section Two: Multiplying/Dividing Integers

Solve – All work must be shown inside the box provided. $[]$ = multiply

(1) $ -5 \cdot -3$ -15	(2) $[9][2][-3]$ -54	(3) $[8][-2][3]$ 48	(4) $ 15 \div (-3)$ -5
(5) $-6 \cdot -9$ 54	(6) $[-2][-4][-8]$ -64	(7) $[36 \div 6][-4]$ -24	(8) $[-4][-16 \div -2][-3]$ 96
(9) $-42 \div 7$ -6	(10) $[-4 \div -2][-81 \div 9]$ -18	(11) $[-5][2][-3][3][-3]$ -270	(12) $[36 \div 6][10 \div -2]$ -30

(13) In the past 7 plays, the Sligo Lions have received a 5-yard penalty on each play. By how many yards did the Lions' position on the field change?

-35 yards

(14) During the game, the Sligo Lions received 8 penalty violations for a total loss of 40 yards. If they lost the same yardage for each penalty call, by how many yards did the Lion's field position change on each penalty?

-5 yards

Summative Assessment:

The spinner activity is a valuable summative assessment in requiring students to, not only formulate their own questions, but to correctly solve the full spectrum of questions developed by their peers. Each student creates twelve unique questions from among all concepts taught in the integer unit. These questions are written onto the 12 sections of one of their two spinners. Then, students must insert all twelve answers to their problems onto their other spinner, being careful to match the sequencing of questions to the upside-down sequencing of the answers, so when the two spinners are glued together, each answer is directly behind its question on the opposite face. Students then hang their spinners from string. Students pair up with another student. Student #1 spins their hanging spinner. Their partner stops the spinner by pinching it. At the point of contact, student #1 reads the question and student #2 must answer the selected question. Check the answer on the back of the spinner. Trade places, using student #2 spinner, which student #1 must answer. Students then rotate to a new partner and repeat the sequence. Continue rotating as time and need for assessment mandates.

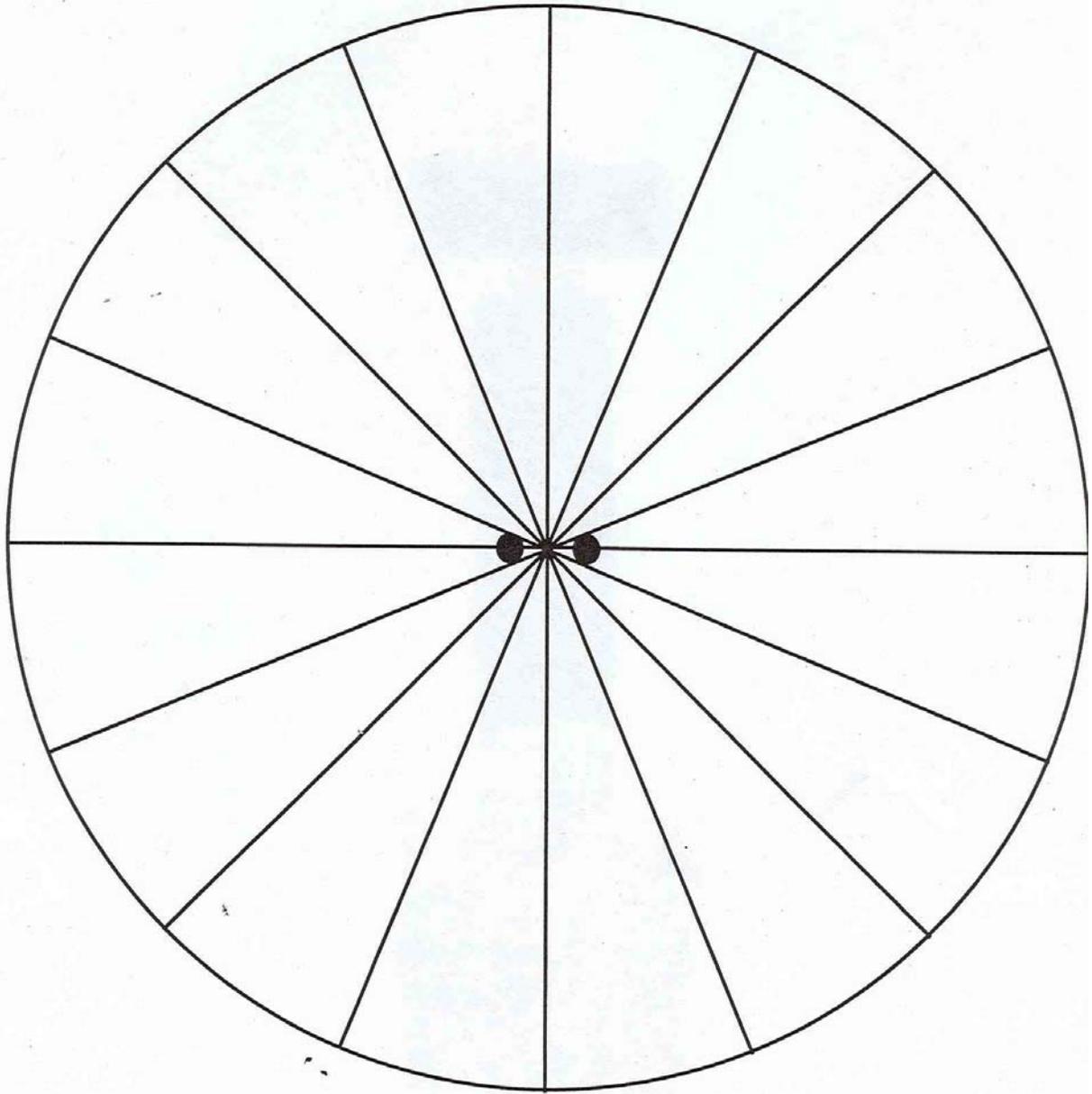
The final summative evaluation is in the form of a performance assessment. Here the students apply what they have learned about integers in an interdisciplinary task involving language arts and/or fine arts. Research confirms that incorporating music or visuals to instruction enhances and reinforces learning.

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Make two for each spinner. Cut out and glue back-to-back.
Write the answers behind the questions.
Punch two holes as marked and attach string.

