Title: Proportional Geometry

Brief Overview:

This unit will allow students to investigate the difference between similar and congruent figures, to apply proportional reasoning to similar figures, and to use the proportional relationship to construct similar figures.

NCTM Content Standard/National Science Education Standard:

Standard 2: Geometry – Apply the properties of similar polygons
Standard 3: Measurement – Use proportional reasoning to solve measurement problems.
Standard 6: Number Relationships or Computation – Solve problems using proportional reasoning

Grade/Level:

Grades 6–8/Standard – Pre Algebra

Duration/Length:

4 –5 days (45 minute blocks)

Student Outcomes:

Students will:

- Differentiate between similar and congruent figures.
- Identify corresponding parts of similar figures.
- Determine proportional relationships among similar figures.
- Calculate missing measurements of similar figures.
- Construct similar figures based on a proportional relationship.

Materials and Resources:

Student Resource Pages (1A – 1D, 2A – 2B, 3A – 3F)
Teacher Resource Pages (1A, 1B, and 1D; 2A and 2B; 3C and 3E/3F)
Overhead transparency of Student Resource Pages as indicated
Cut outs of student resource pages 1A and 1B (possibly copied onto cardstock) in envelopes
Chart paper
Graph paper
Rulers and/or measuring tapes
Large cooking spoon

Prior Knowledge: Ratios

Development/Procedures:

Lesson 1 Similar vs. Congruent figures

Materials: Student Resource Pages 1A – 1D
Teacher Resource Pages 1A, 1B, and 1D

Preassessment and Launch– Group students in twos or threes. Give each group of students an envelope containing ten triangles copied onto cardstock from Student Resource Pages 1A – 1B and a piece of chart paper. Ask them to pair up the triangles that are "related" in some way. Each triangle has a pair.

Each group of students should select one pair to stick on the chart paper and write a brief explanation of why they paired them. Charts should be posted around the room.

Teacher Facilitation –

- Go around to each group's paper and survey the class ("thumbs up" or "thumbs down") to see if they agree or disagree with the pairing.
- Get the students to verbalize why they paired the triangles.
- Talk about the triangles that are exactly the same. Introduce/define the word "congruent."
- Talk about the triangles that are the same shape. Introduce/define the word "similar."
- Use the Student Resource 1C with the Venn Diagram to allow students to summarize and to compare the definitions of similar and congruent.
- Discuss corresponding sides and angles. Ask the students to find the sides that match up in each pair of triangles, even when they are not oriented the same way. Make sure to mention that the angles of both types of pairs are the same (if a student doesn't.) Demonstrate this by placing the corresponding angles on top of each other to prove that they have the same measure.

Student Application – To close the class, pass out Student Resource Page 1D and have the students decide if each pair of figures is similar, congruent, or neither.

Embedded Assessment – Check students' answers to the questions to make sure that they understand the definitions of similar and congruent.

Reteaching/Extension

- Work with the group or student individually and show them how some of the pairs of triangles are exactly the same (congruent) and fit on top of each other, and some have the same shape (similar). For the ones with the same shape, the corresponding sides can be matched against each other to see that one side of the larger triangle may be two or three times the size of the side of the smaller triangle.
- Allow students to trace shapes on centimeter graph paper and compare the side lengths and areas to check for congruency. Congruent triangle's sides and areas are equivalent.
- Have the students figure out how many times bigger the larger of the similar triangles is than the smaller in each pair.

Lesson 2 Discovering the Proportional Relationship Among Sides of Similar Figures

Teacher Set Up – Mark positions on the floor where you would need to place the overhead projector so that the figures on the screen are 3 times smaller, 2.5 times larger, and $\frac{1}{2}$ as large as compared to the original figure placed on the projector.

Materials: Student Resource Pages 2A, 2B
Overhead of Teacher Resource Page 2A
Teacher Resource Page 2B

Preassessment and Launch–

- Place overhead with the named geometric shapes (Teacher Resource Page 2A) on the projector that is placed in such a position as to enlarge the image of the shape to 3 times its actual size.
- Ask students to explain using vocabulary from Lesson 1 how the two shapes (the original and the image) compare.
- Tell students to predict how much larger the image is than the original.

Teacher Facilitation

- Distribute Student Resource Page 2A – Recording Sheet to each student.
- Invite volunteers to the screen and to the projector to measure assigned sides in centimeters. Students record measurements on the charts in Position 1.

- As measuring continues, allow students to verbalize the relationship between the actual measurements and the image measurements. Students should begin to see that the image measures 3 times the original. Once students have recognized the relationship, the students can make predictions prior to measuring and eventually complete the chart for the remaining figures.
- Once all of the measurements for the figures in Position 1 have been completed, move the overhead to a second position, preferably to the $\frac{1}{2}$ position and repeat the experiment.

Student Application

- Move the projector to the third position and allow students to make measurements and record data in their charts as you supervise. Allow student groups to determine the relationship between the original and the image and have each group report their findings once all groups have found the relationship.

Embedded Assessment

- Have students complete Student Resource Page 2B, using the partial chart to identify missing measurements and justifying their choices.

Reteaching/Extension

- Demonstrate to struggling students how to set up an equation to
- represent the relationship. Use the 3 times as large measurements.
  - Ex. Actual Size ×_____= Image Size
    Have the students explain what number would make a true statement.
- Extend by asking students to determine where to place the overhead to make the similar figures 4 times as large as the original images.

Lesson 3 Identifying and Using Proportional Relationships Among Similar Figures

Materials: Student Resource Pages 3A
Teacher Resource Pages 3C and 3E/3F
Centimeter rulers
Large cooking spoon
Plastic teaspoons

Preassessment and Launch

- Distribute Student Resource Page 3A. Students are directed to draw two shapes similar to the original, one larger and one smaller, and to predict how much larger and smaller their shapes are.

Teacher Facilitation

- Distribute the triangles from the Day 1 Launch (Student Resource Pages 1A and 1B), cm rulers, and graph paper (Student Resource Page 3B).
- Model for students how to determine the proportional relationship between the similar triangles.
- Emphasize that proportionality is based on the multiplication of the measurements by a common ratio.
- Model the proportional set up if three measurements are known to determine the unknown using the triangle manipulatives.
- Allow three measurements to be made and then guide students in the proportion set-up to determine the missing measurement. Students can then verify the mathematics by measuring.

Student Application

- Students then return to Resource Page 3A and use their knowledge of proportions and similar shapes to verify that their shapes are similar and to make corrections if the shapes do not demonstrate proportionality.

Embedded Assessment

- Student Resource Page 3C – Given a figure on a grid and one measurement of a similar shape, students are to find the measurements of the remaining sides of the similar shape and draw the similar figure.

Reteaching/Extension

- Use simpler shapes, like rectangles, to help students find the missing measurements.
- Extend by allowing student pairs to draw a shape and give the size change and let the partner determine the missing measurements and draw the similar figure.

Summative Assessment/Project:

Launch

- Bring in a large cooking spoon and ask students why they do not eat with a spoon this size on a regular basis. Ask students to predict what size you would need to be to use that spoon as you would a cereal spoon.
- Distribute cm rulers, Student Resource Page 3D and teaspoons to each group of four students. Pass around the large cooking spoon as student request to allow them to collect their data. Allow students 10 minutes to use what they have learned about proportionality to make valid predictions that answer the question.
- Allow student groups to present the predictions and explain how they arrived at their sizes. If no group has set up a proportion relating the

teaspoon to the size of a student, review how to relate the spoons to student heights proportionally using the measurements of the spoons and an average student.

Assessment/Project

- Distribute Student Resource Pages 3E and 3F to pairs of students for students to complete in pairs as a pair-check activity (25 minutes).
- Partner 1 completes question 1, Partner 2 verifies the solution to question 1 and initials to indicate agreement and verification.
- Partner 2 then completes question 2 and Partner 1 verifies the solution to question 2 and initials to indicate agreement and verification. This procedure continues until all 8 questions are answered.
- Clearly explain that if a student is the verifying partner that they cannot make any corrections to the working partner's work, but they can verbally identify errors for the working partner to correct.

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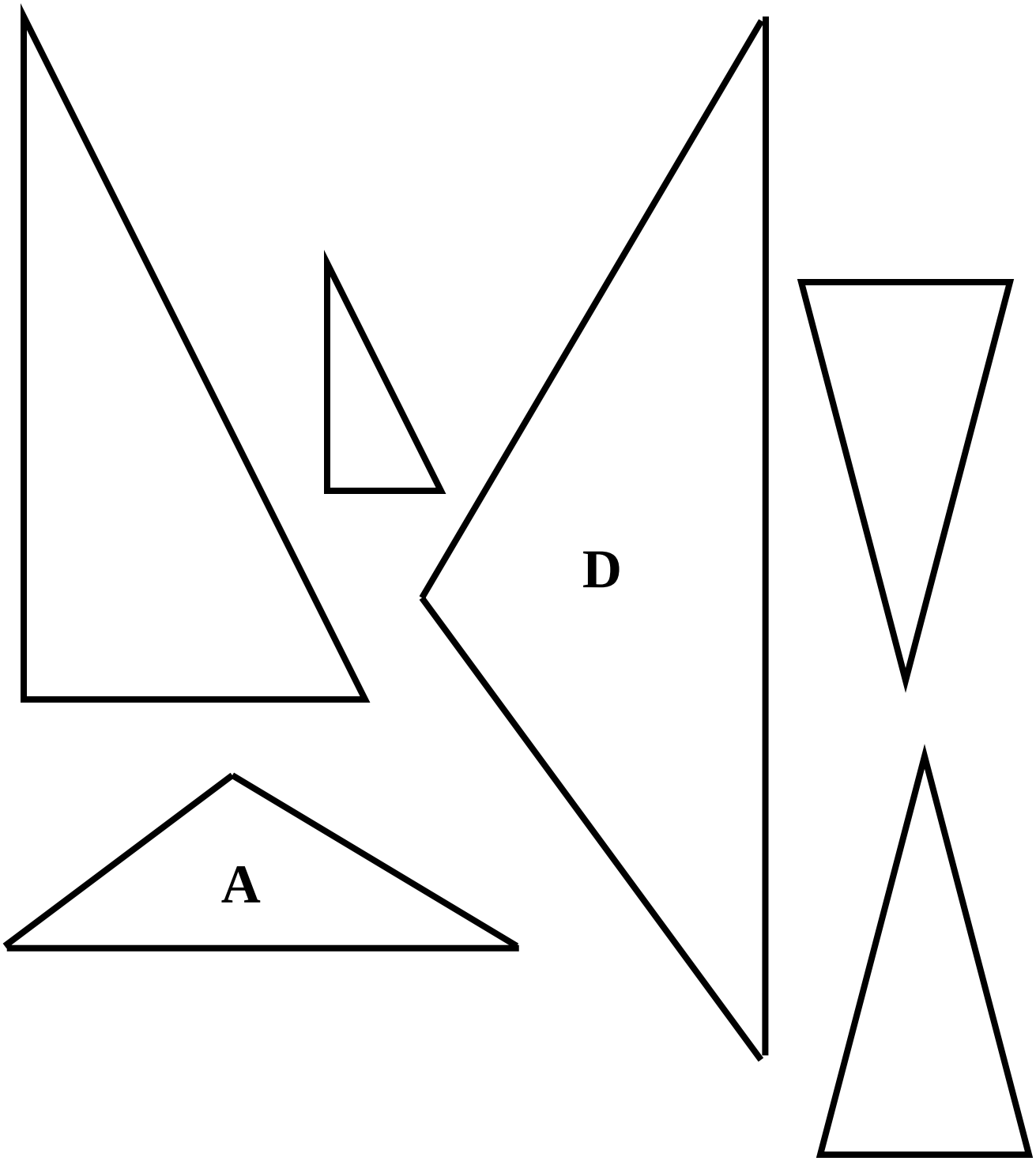
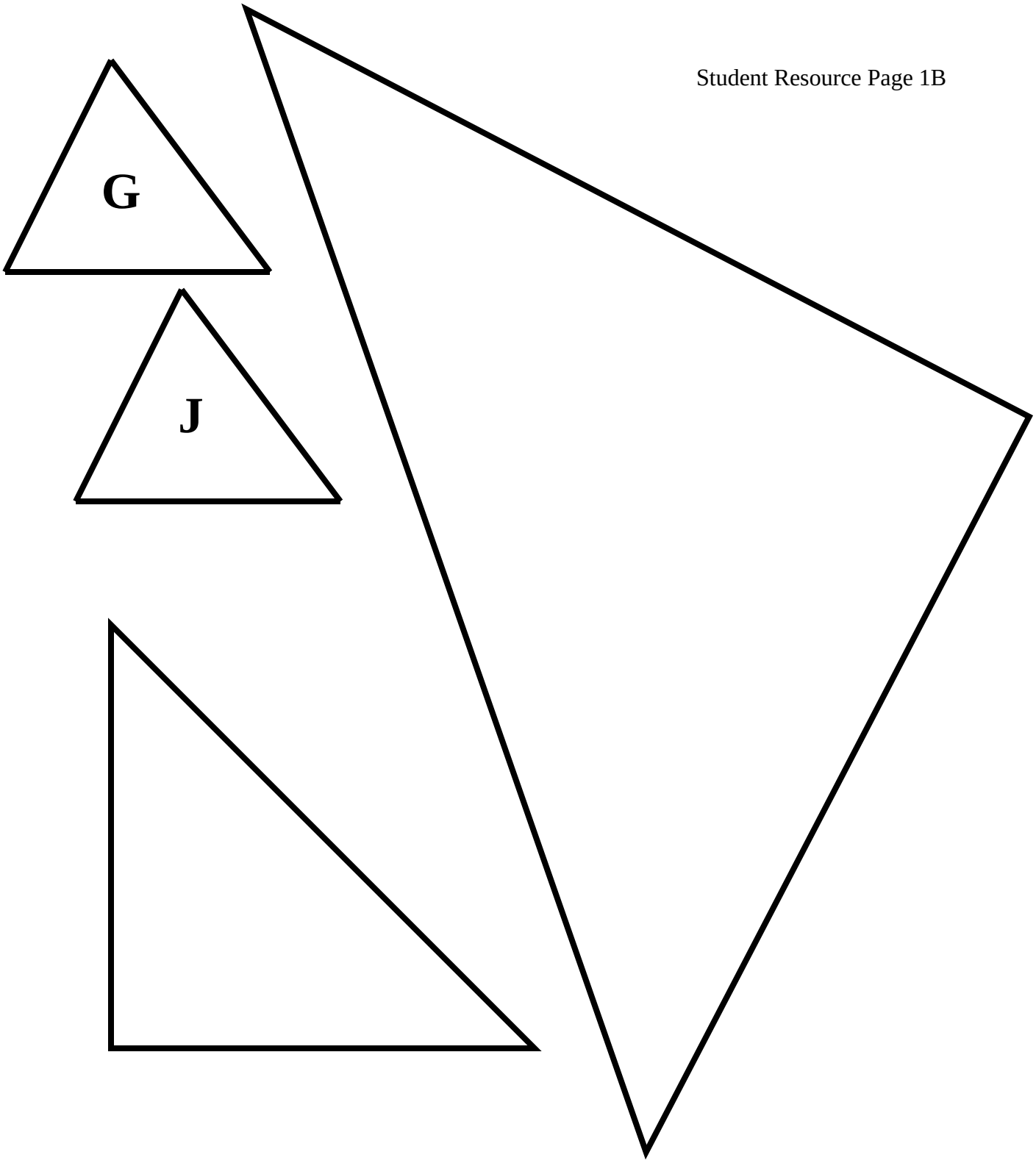
Student Resource Page 1A

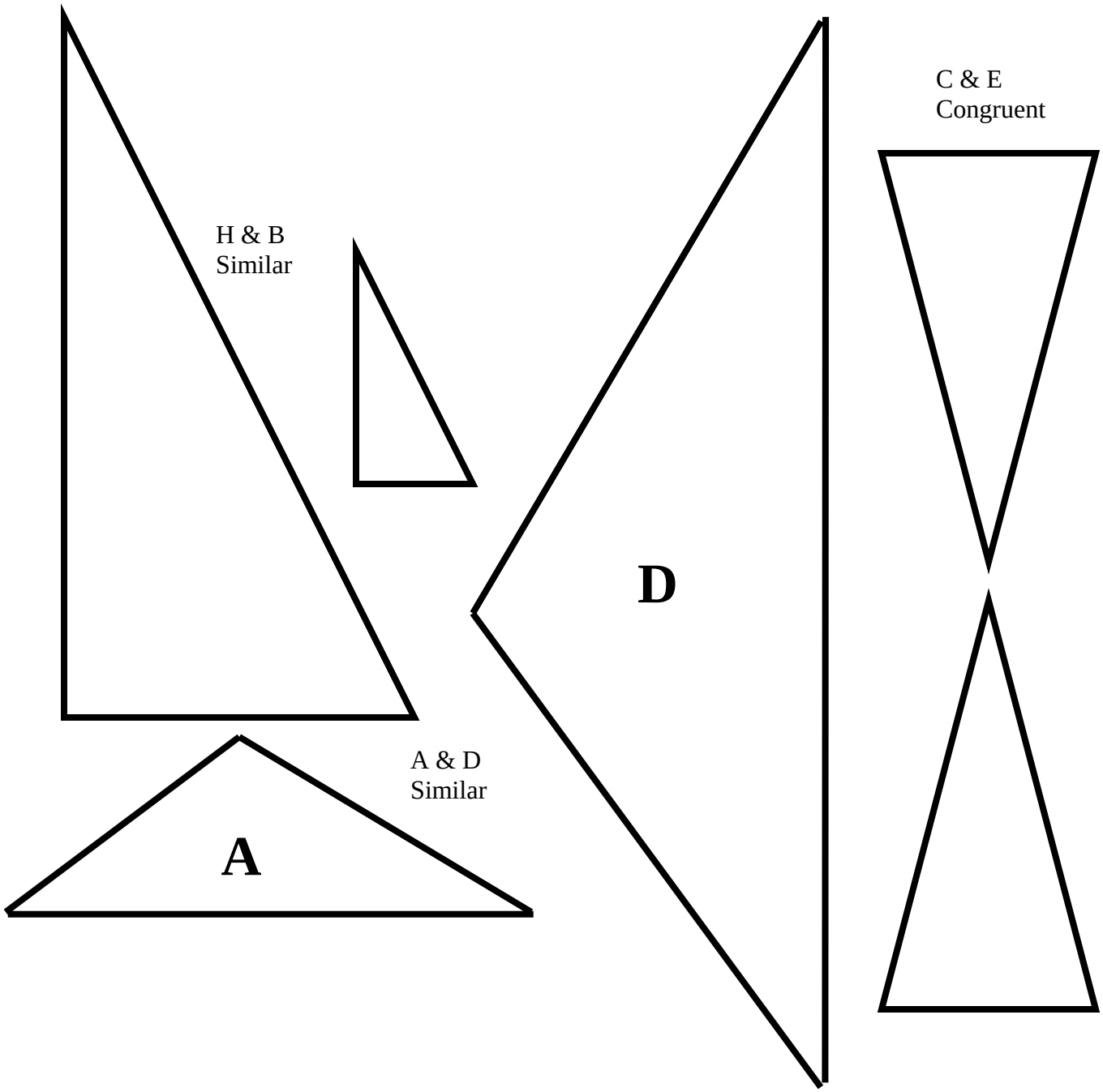
Teacher cuts triangles and places in bags for student pairs to complete the Day 1 Launch.

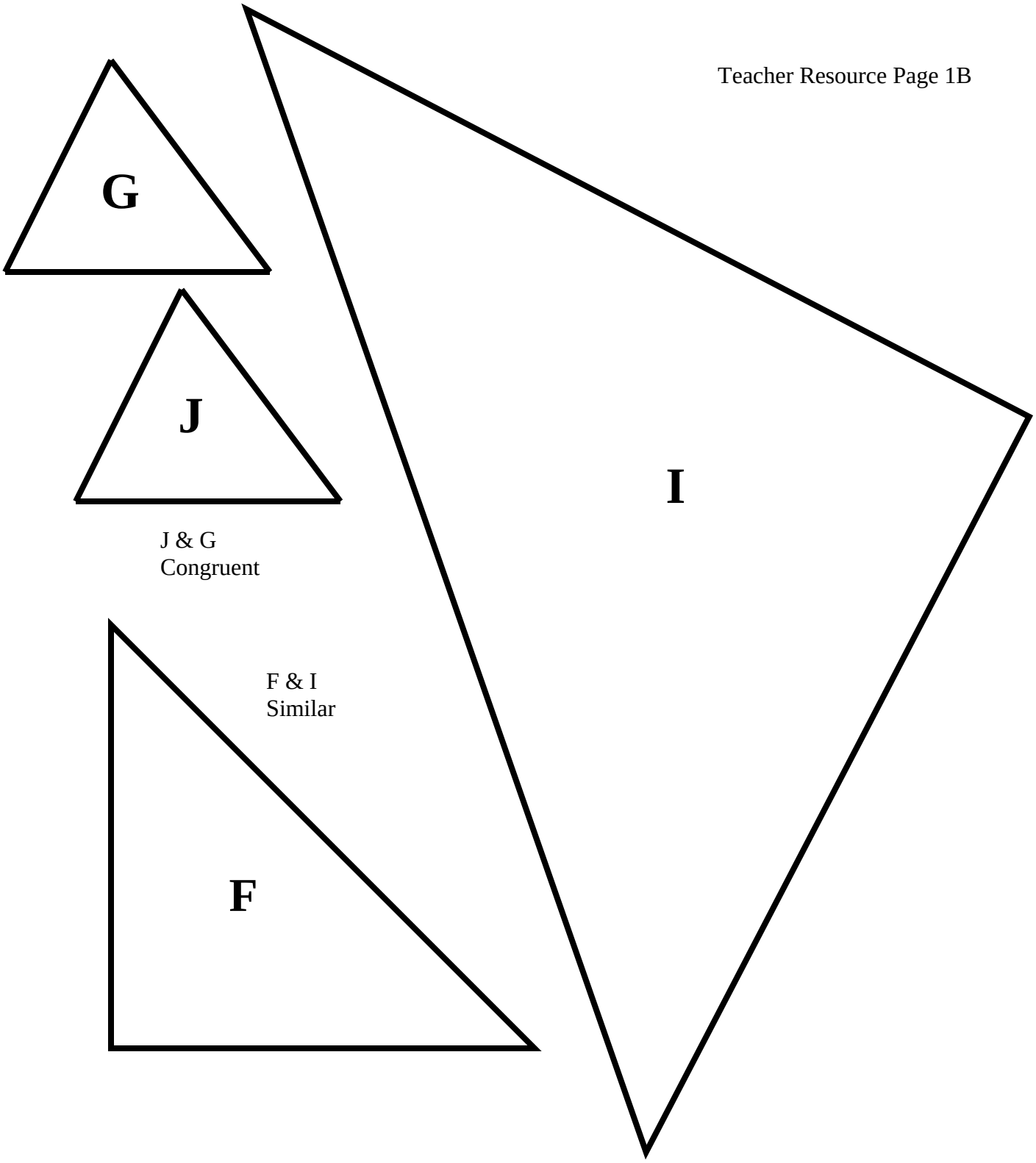



G
J

Teacher Resource Page 1A

These are the desired pairings of the triangles for the Day 1 Launch.

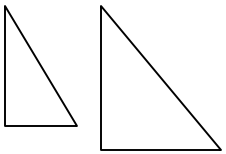



G
J
I
J & G
Congruent
F & I
Similar
F

Student Resource Page 1C

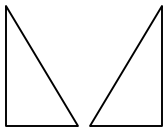
Note Sheet – Similar vs. Congruent

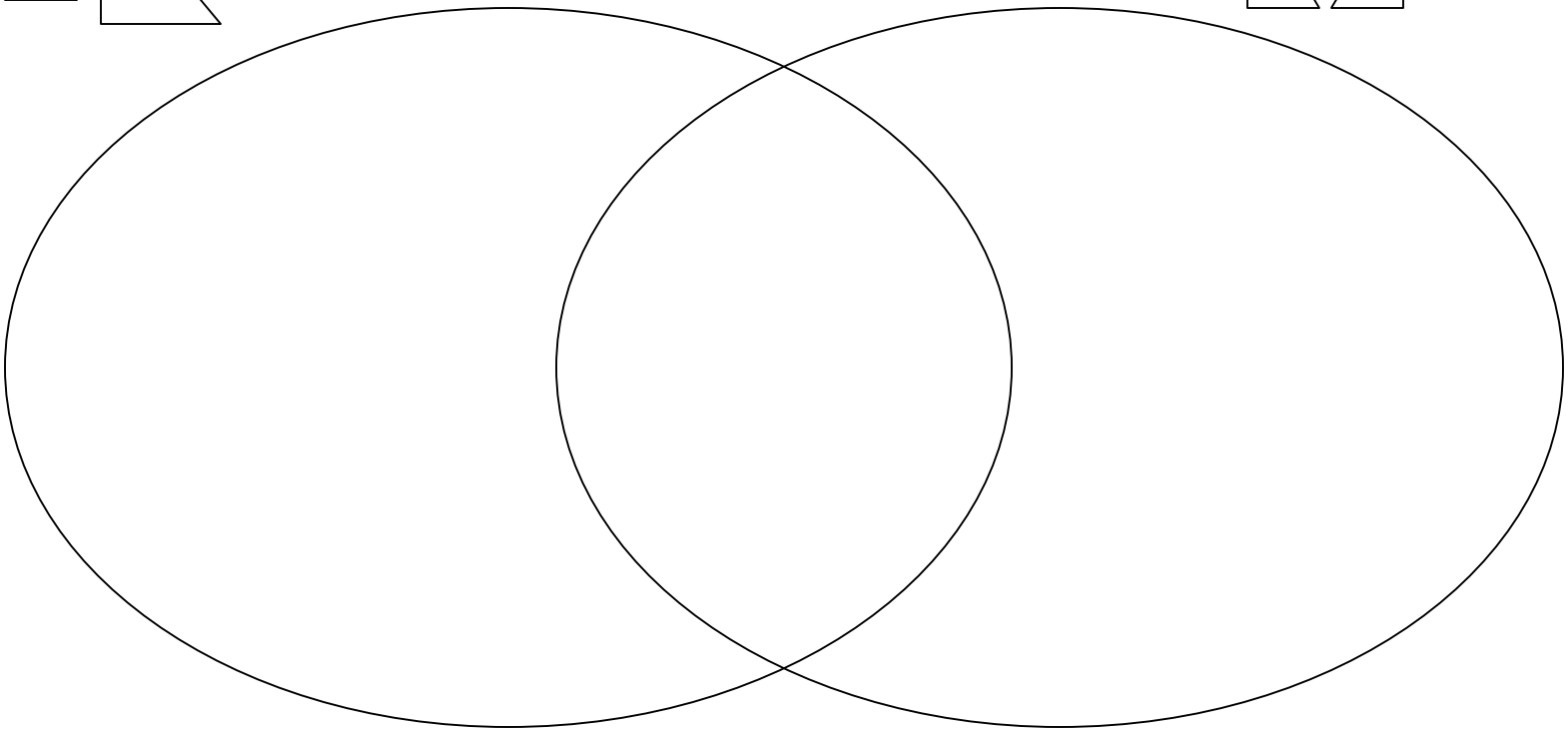
Name: Class: Date:



Similar Figures

Congruent Figures





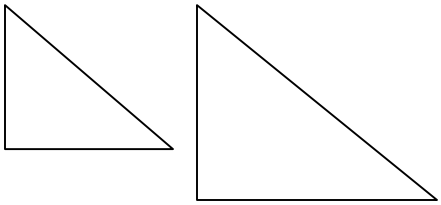
Student Resource Page 1D - Applying the Learning

Name: Class: Date:

<u>Directions:</u> Use your knowledge of similar and congruent figures to identify the relationship between the two triangles.

- Label each pair as **congruent, similar,** or **neither.**
- Justify your response using mathematical definitions.

1.

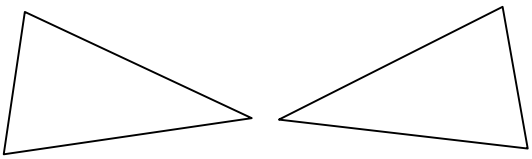


These two triangles are _____ because

_____

_____

2.

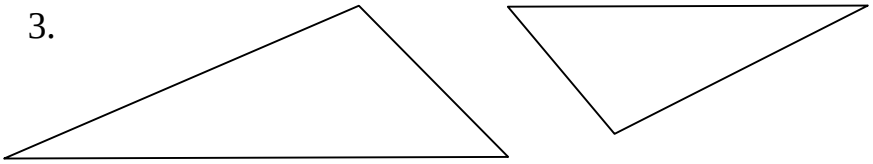


These two triangles are _____ because

_____

_____

3.

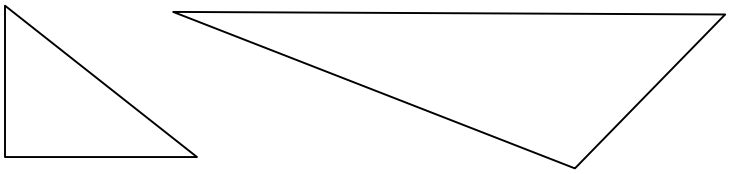


These two triangles are _____ because

_____

_____

4.



These two triangles are _____ because

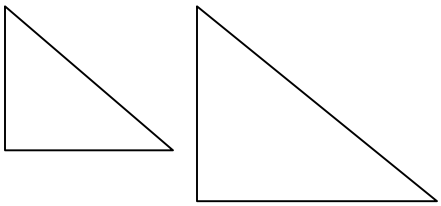
_____

_____

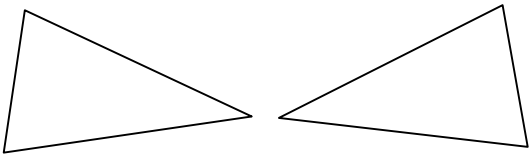
Teacher Resource Page 1D - Applying the Learning

Directions: Use your knowledge of similar and congruent figures to identify the relationship between the two triangles.

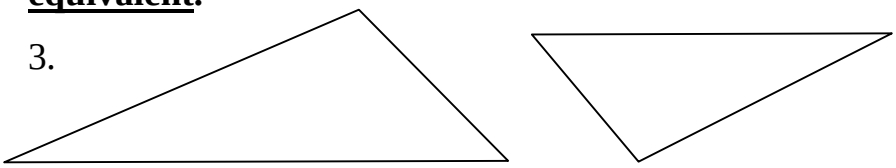
- Label each pair as **congruent, similar,** or **neither.**
- Justify your response using mathematical definitions.

1. 

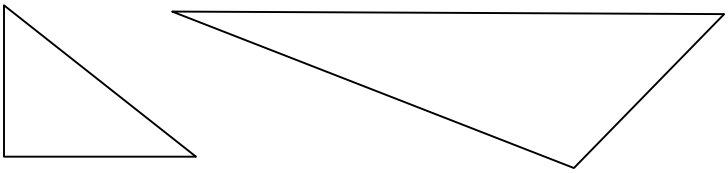
These two triangles are **similar** because **corresponding angles are equivalent (equal) and corresponding sides are proportional, (or any explanation demonstrating understanding of similar.)**

2. 

These two triangles are **congruent** because **they are the same triangle just turned another direction or corresponding sides and angles are equivalent.**

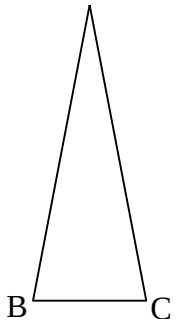
3. 

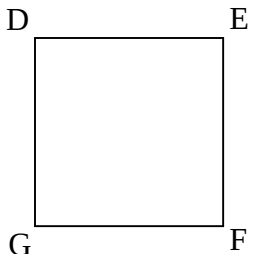
These two triangles are **similar** because **corresponding angles are equivalent (equal) and corresponding sides are proportional, (or any explanation demonstrating understanding of similar.)**

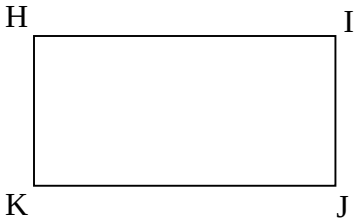
4. 

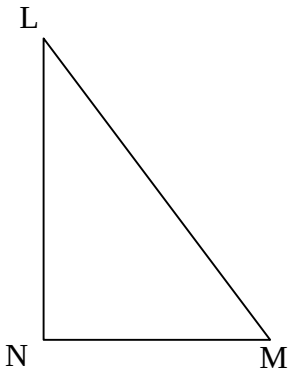
These two triangles are **neither** because **there is no proportional relationship among the sides and the angles of the two triangles.**

Teacher Resource Page 2A









Student Resource Page 2A – Recording Sheet

Name: Class: Date:

Directions: Record the measurements from the transparency and the overhead for each figure.

Triangle ABC

Side	Actual	Position 1	Position 2	Position 3
AB				
BC				
CA				

Rectangle DEFG

Side	Actual	Position 1	Position 2	Position 3
DE				
EF				
FG				
GD				

Square HIJK

Side	Actual	Position 1	Position 2	Position 3
any				

Triangle LMN

Side	Actual	Position 1	Position 2	Position 3
LM				
MN				
NL				

How many times bigger????

Student Resource Page 2B

Name: Class: Date:

Directions: Imagine that your teacher has projected a shape on the overhead three times. You have recorded some of the measurements. Fill in the missing values in each chart.

Triangle STU

Side	Actual	Position 1	Position 2	Position 3
ST	3 in.	18 in.		
TU	5 in.		2.5 in.	
US	6.5 in.			13 in.

Choose one position (one column) and describe how you determined the values of the two missing sides. _____

_____

_____

_____

Student Resource Page 2B

Name: Class: Date:

Imagine that your teacher has projected a shape on the overhead three times. You have recorded some of the measurements. Fill in the missing values in each chart.

Triangle STU

Side	Actual	Position 1	Position 2	Position 3
ST	3 in.	18 in.		
TU	5 in.		2.5 in.	
US	6.5 in.			13 in.

Choose one position (one column) and describe how you determined the values of the two missing sides. _____

_____

_____

_____

Teacher Resource Page 2B

NOTE: This is the closure activity for Day 2. Each student receives just 1/2 a sheet.

Name: Class: Date:

Directions: Imagine that your teacher has projected a shape on the overhead three times. You have recorded some of the measurements. Fill in the missing values in each chart.

Triangle STU

Side	Actual	Position 1	Position 2	Position 3
ST	3 in.	18 in.	**1.5 in.**	**6 in.**
TU	5 in.	**30 in.**	2.5 in.	**10 in.**
US	6.5 in.	**39 in.**	**3.25 in.**	13 in.

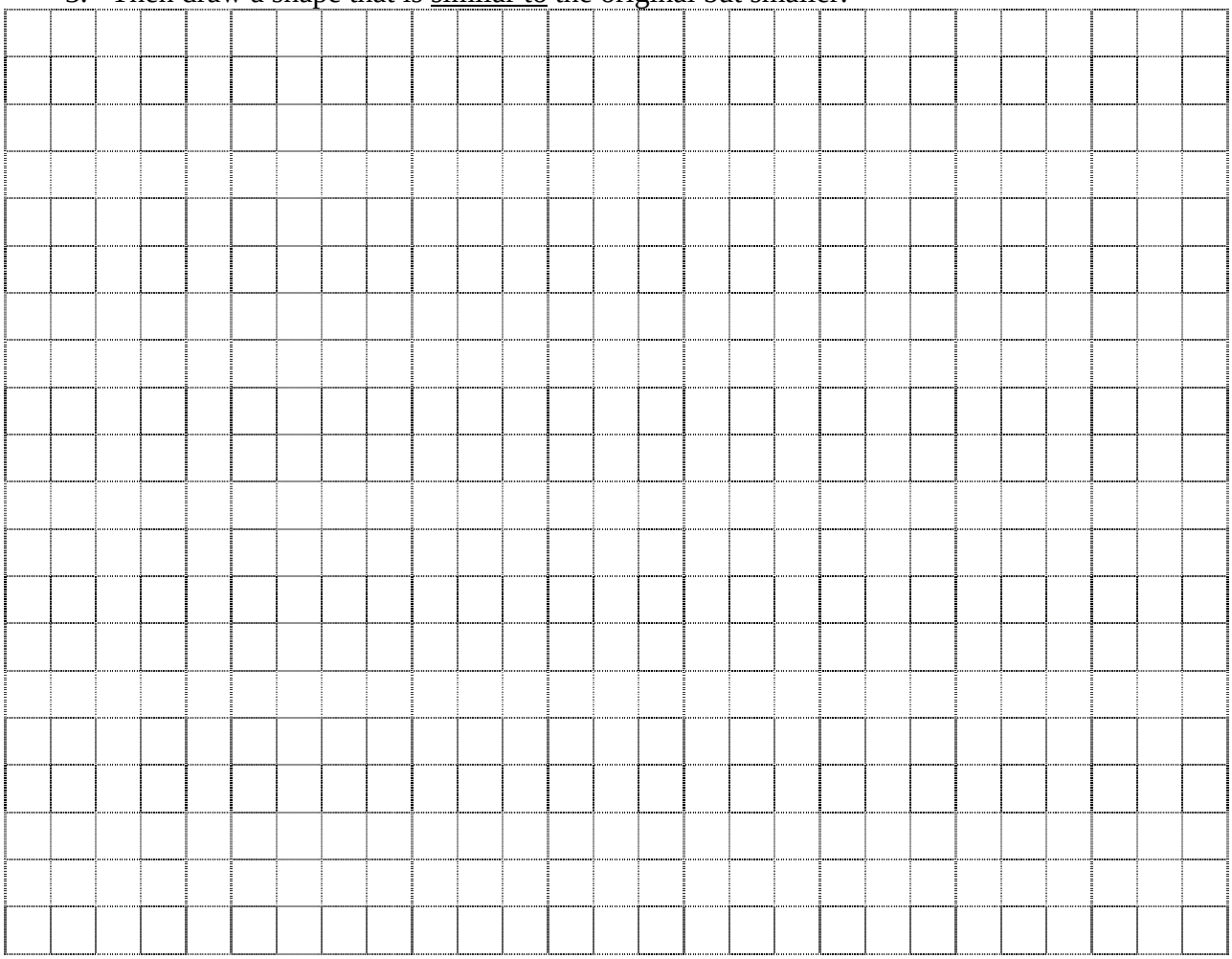
Choose one position (one column) and describe how you determined the values of the two missing sides. **Answers will vary based on which column is chosen, but students should say something about using the actual measurement and position and dividing or multiplying to find the relationship between them. Then use that factor to find the missing measurements.**

Student Resource Page 3A

Name: Class: Date:

Directions:

1. On the grid below, draw a simple shape.
2. Draw a shape that is similar to the original but larger.
3. Then draw a shape that is similar to the original but smaller.



4. Are your shapes similar? _____ How do you know?

5. How much bigger is the larger shape?

6. How much smaller is the smaller shape?

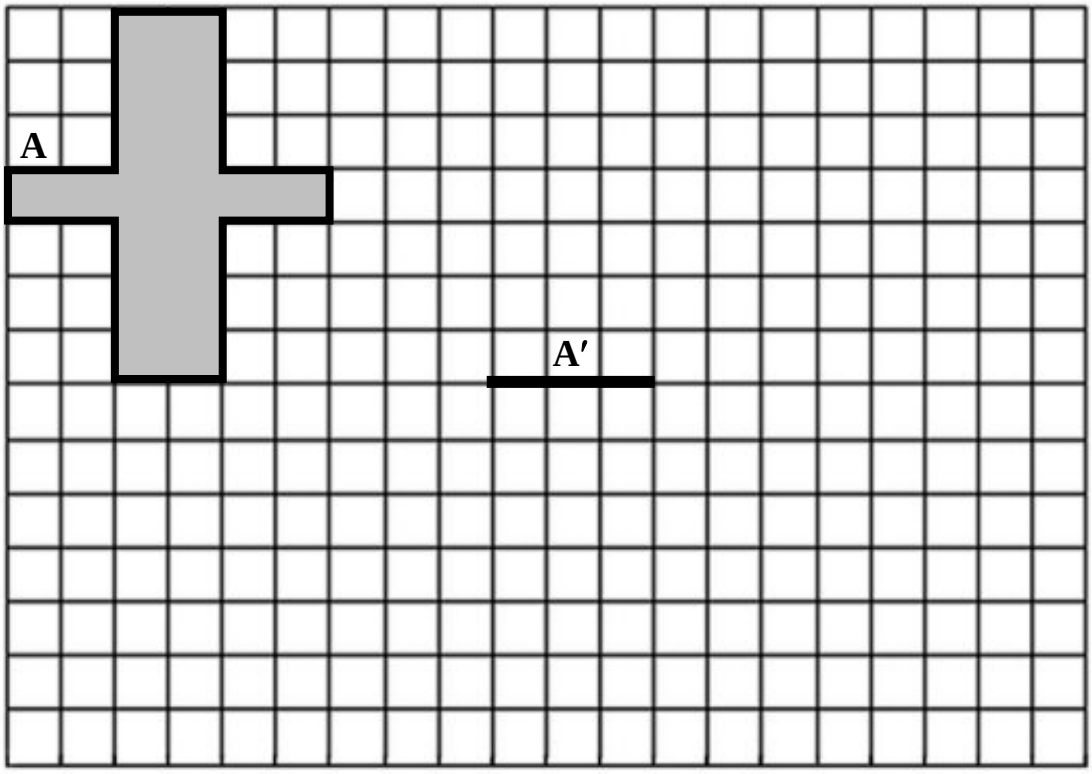
Student Resource Page 3B

Student Resource Page 3C

Name: Class: Date:

<u>Directions</u>: Use the grid paper to draw a shape similar to the given figure. Use proportions to verify that the measurements make the shapes similar.

Note: The given side **A′** corresponds to side **A** in the given figure. Use **A′** as the given measurement in your similar figure.



1. Show the proportions you used to find/verify the missing measurements.

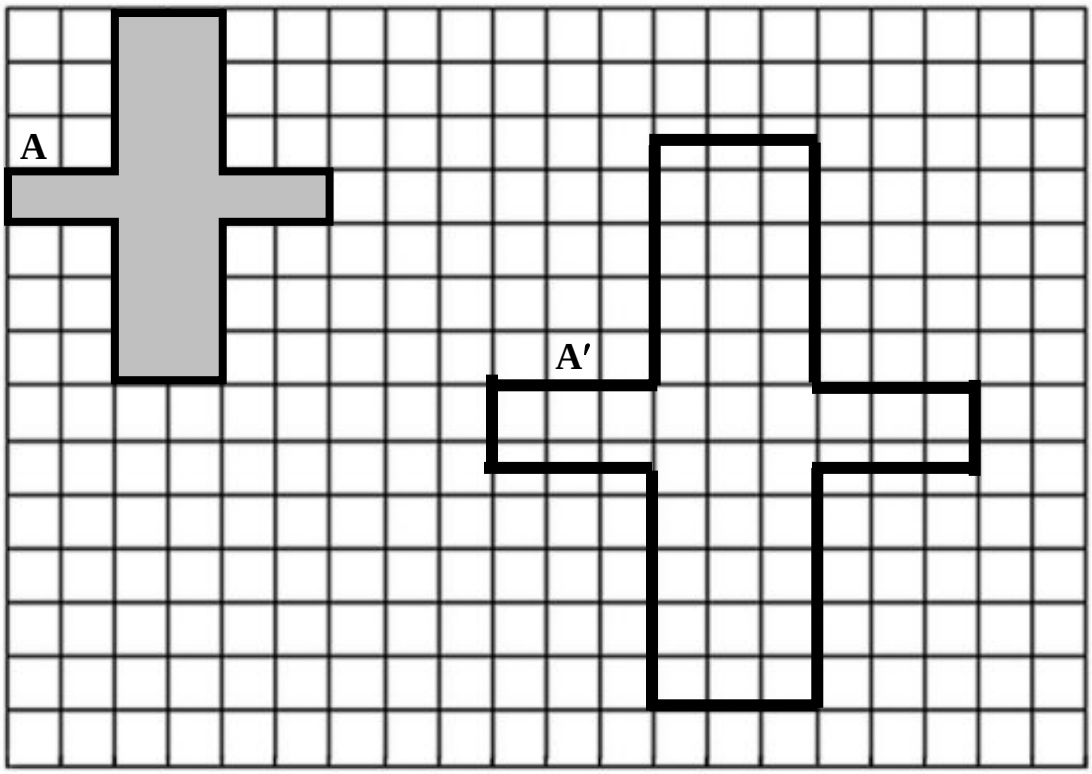
2. How much larger is the new shape?

Teacher Resource Page 3C

Name: Class: Date:

Directions: Use the grid paper to draw a shape similar to the given figure. Use proportions to verify that the measurements make the shapes similar.

Note: The given side **A′** corresponds to side **A** in the given figure. Use **A′** as the given measurement in your similar figure.



3. Show the proportions you used to find/verify the missing measurements.

Student should have proportions that determine or verify corresponding measurements. The given are samples.

$$\frac{1}{2} = \frac{x}{3}$$

$$\frac{2}{3} = \frac{3}{x}$$

4. How much larger is the new shape? **1.5 times bigger**

Student Resource Page 3D

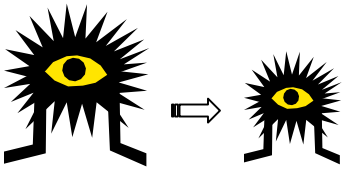
Name: Class: Date:

Whose spoon is this???

Directions: Use the materials provided by your teacher, along with what you know about proportions and similar figures to make a prediction about the size of a person who would use this as a cereal spoon.

Student Resource Page 3E

Name: Class: Date:



Honey, Math Shrunk the Kids!
You have been asked by the Toys 'N' You company to make an action figure of yourself. The only thing you know about this action figure is that it is to be 25 cm high.

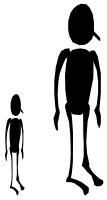
Your action figure will need some clothes and accessories. These items need to be in proportion to your figure. Your assignment is to use what you have learned about proportions and similar figures to determine the size of these items. Make sure you show all your work.

1. What would the length of a shoe be for your figure?

2. How big would a pencil for your figure be?

3. What would be the dimensions of a math textbook for your action figure?

4. Choose one of the above measurements and explain in words in detail how you came up with your answer.



Giants 'R' Us

Giants Department Store would like to attract customers to their store so they have decided to build a giant-sized statue of you to put outside. The giant version of you is to be 300 cm tall. The items need to be in proportion to your figure. Use what you have learned about proportions and similar figures to determine the size of these items for the statue. Make sure you show all your work.

5. What would the length of the giant's arm be, from shoulder to fingertip?

6. What would the length of the giant's fingernail?

7. What would be the circumference of the giant's wrist?

8. Choose one of the above measurements and explain in words in detail how you came up with your answer.

Honey, Math Shrunk the Kids!

Students need to use their height in centimeters in proportion to a 25 cm high figure. Students should measure the real objects listed in each problem and set up a proportion with their height, the figure's height, and the object size to solve for the missing measurement. Students should then be able to explain how they got one of their answers.

Score as you see fit.

Giants 'R' Us

Students need to use their height again in proportion to a 300 cm high figure. Students should measure the real body parts listed in each problem and set up a proportion with their height, the figure's height, and the body part to solve for the missing measurement. Students should then be able to explain how they got one of their answers.

Score as you see fit.