Title: Can Cruddy Cool As Fast As Clean?

Brief Overview:

Students will investigate variations in cooling times for ordinary tap water and salt water. Data will be collected using the CBL unit and the TI-82 calculator to create and interpret data plots. Students will become familiar with Newton's Law of Cooling and be able to verbalize and write conclusions made from the collected data.

Links to Standards:

- **Mathematics as Problem Solving**
  Students will investigate cooling data of two solutions and demonstrate the ability to collect and make inferences from data.

- **Mathematics as Communication**
  Students will express mathematical ideas orally and in writing.

- **Mathematics as Reasoning**
  Students will be able to make predictions, collect data, and determine the appropriate algebraic model for a clear solution and a salty solution.

- **Mathematical Connections**
  Students will utilize algebraic concepts in confirming Newton's Model of Cooling throughout the activity.

- **Algebra**
  Students will use the graphs of data to determine an equation.

- **Functions**
  Students will represent and analyze relationships using tables, equations, and graphs. Students will analyze the effects of parameter changes on the graphs of functions.

- **Conceptual Underpinnings of Calculus**
  Students will understand the conceptual foundations of limit, rate of change, and their application to other disciplines.

Links to Maryland Core Learning Goals:

- **1.1.4** Students will describe the graph of a nonlinear function in terms of the basic concepts of limits and rate of change.
- **3.1** Students will collect, organize, analyze, and present data.
- **3.2.1** Students will make informed decisions and predictions based on the results of simulations.
- **3.2.2** Students will make predictions by finding and using a curve of best fit.

Grade/Level:

Grades 9 - 12, Pre-Calculus, Calculus

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Using a TI-82 with a CBL to collect and analyze data
- Given a data set, use the TI-82 to construct and interpret scatter plots and determine a regression equation

Objectives:

Students will:

- describe the graph of a nonlinear function in terms of the basic concepts of maxima and minima, roots, limits, rate of change, and continuity.
- use CBL and TI-82 calculator to collect, organize, analyze, and present data.
- make informed decisions and predictions based on the results of simulations.
- make predictions by finding and using a curve of best fit.

Materials/Resources/Printed Materials:

- CBL unit
- TI-82 calculator with link cable
- TI Temperature Probe (provided with the CBL)
- 5 - 10ml of hot water
- 5 - 10ml of hot cruddy water (Cruddy water can be made by adding a teaspoon or more of salt, corn syrup, or soil to tap water.)
- Worksheets (attached)

Development/Procedures:

Students will collect data and answer directed questions on the worksheet provided. The data will be used to confirm student's predictions and analyze Newton's Law of Cooling.

Performance Assessment:

Student worksheets can be graded based on the information collected. A rubric can be developed which will help teachers evaluate student performance.

Extension/Follow Up:

- Students may investigate the colligative property determining freezing temperatures and boiling point for each solution.
- Students may study variations in the capacity(volume) of solutions and how it affects cooling times.
- Students may examine the effects contaminants may have on cooling time of water and form conjectures on how this may affect climate or other environmental factors.

Authors:

Del Deligianis
Quince Orchard High School
Montgomery County, MD

George Reidy
School Without Walls
Washington, DC

Introduction to Activity

Newton's Law of Cooling says that the rate at which an object loses or gains heat is proportional to the difference between its temperature and the temperature of the surrounding air. Mathematically this law can be expressed by the equation

$$\frac{\Delta T}{\Delta t} = k(T - A)$$

where **T** represents the Temperature of the object
k is a constant, **A** is the Temperature of the air
and **t** is the time

Assuming the air temperature remains constant, the solution of the differential equation is

$$T = A + Ce^{kt}$$

Note here that only t represents a variable.
A, k and C are constants which we must identify.
C is the difference between initial air and water temperatures.

In this activity, you will use a CBL system with a temperature probe to investigate the relationship between time and temperature in cooling two solutions, one with ordinary water and the second with "cruddy" water.

Predict what you think will happen to the temperature of the two solutions before beginning the activity. ____ (2)

Will the solutions cool faster when hot or when simply warm? ____________________ (1)

How long will it take for the water to cool? ________________________________ (1)

How long will it take for the cruddy water to cool? _____________________________ (1)

Activity Worksheet

In this activity you will collect data using a graphing calculator and a CBL. You will be asked to draw conclusions from the data. It is important to start with very hot water in order to best observe the graph of the data. Follow the steps below to determine the appropriate algebraic model for the experiment. Think about how your model confirms/rejects Newton's Law of Cooling.

PROCEDURE

Insert the Temperature Probe in CH 1 on the CBL.
Find and record the temperature of the air. The calculator records direct data in Celsius.

____
air temperature (1)

Obtain a container of very hot water. Put the temperature probe in the water for about one minute. Then record the temperature.

____
water temperature (1)

Link the calculator to the CBL.

Turn on your calculator. Clear your memory of all lists to more easily follow the instructions (2nd, MEM, DELETE, LIST, ENTER as many times necessary).

Press PRGM on the calculator, scroll down to highlight the HEAT program , ENTER to choose the program, then ENTER again to activate the program.

Now follow the displayed directions.

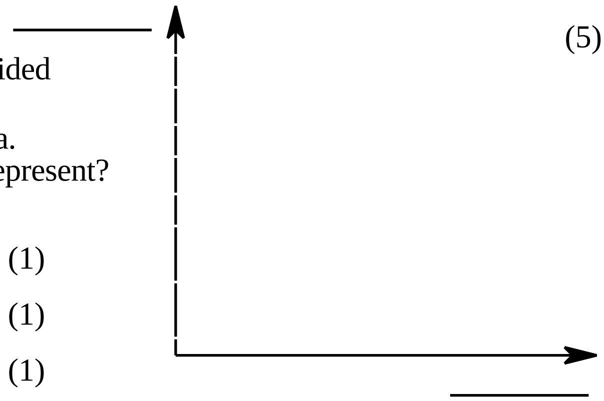
The first question is "HOW MUCH TIME BETWEEN POINTS IN SECONDS?" Enter 60.

Make sure the probe does not leave the water.

Move data in L4 to L6 in order to collect additional data.

Repeat the process for the cruddy water, trying to record data with the same air temperature and the same initial hot water temperature.

Analyzing the results



(5)

Sketch the scatter plots in the space provided on the right. (L_3, L_4) and (L_3, L_6)
Label the sketch to help visualize the data.
What do the numbers in L_3, L_4 and L_6 represent?

L_3 ________________________________ (1)

L_4 ________________________________ (1)

L_6 ________________________________ (1)

Use the TI-82 calculator to view the graph of exponential regressions. Let Y_1 contain data relating L_3 and L_6 and Y_2 the data relating L_3 and L_4. Report your equations to three significant figures. How do the graphs of the models compare to the scatter plots?

Y_1 = ______________________ ____ (1)

Y_2 = ______________________ ____ (1)

Identify the y intercept for each of the equations. Explain the meaning of the y intercept.

Y_1 ______________________ ____ (1)

Y_2 ______________________ ____ (1)

The calculator uses the following window according to the program. Identify a window which better reflects the collected data.

x min	-120	x min __________
x max	2160	x max __________
x scl	60	x scl __________
y min	-10	y min __________
y max	90	y max __________
y scl	10	y scl __________

Use the revised graph to verbally describe how the water cooled. Include your interpretation of the different cooling rates of the two solutions.

____

____ (2)

Explain how your model differs from the equation formulated by Newton's model.

____

____

____ (2)

Activity Scoring

Points for the activity items are shown in parentheses after each item. An additional 5 points can be used for participation. The total is 30 points; we recommend the following scale for grading:

27-30	A
24-26	B
21-23	C
18-20	D
0-17	E

Culminating Activity

Choose one of the following culminating activities:

Write a letter to Newton explaining how his work has helped you see the variations in which various solutions change their temperature. You might reach him at his Cambridge address.

Make a poster showing the describe the change in temperature for the two solutions.

Do a web search of at least five Internet sites explaining Newton's Model of Cooling. Summarize the three best sites.

Recreate the activity starting with ice water. Explain the relationship of the new graphs to the graphs produced by the original experiment.

Make a cartoon showing Newton's investigation how quickly an object cools over a period of time.

Write a program which will allow both temperatures to be entered in the calculator simultaneously.

Create a quiz with 2 true false questions, 2 multiple choice questions, and 2 free-response questions which demonstrate your understanding of Newton's Model of Cooling and the graphs generated by this activity.

Write a script for a radio commercial for a new product which maintains a stable temperature for an object. Include information about how this product defies Newton's Law of Cooling.

Explain how the graphs of the functions would be modified if the readings were recorded in Fahrenheit instead of Celsius. Share how the rate of change would be affected over a given time period. What temperature scale did Newton use?

Create your own culminating activity which demonstrates your knowledge of Newton's Model of Cooling. You will need prior teacher approval for this option.

HEAT • Program

```
:PlotsOff
:Func
:FnOff
:AxesOn
:ClrDraw
:-10ÍYmin
:90ÍYmax
:10ÍYscl
:ClrHome
:{1,0}ÍLÏ
:Send(LÏ)
:{1,1,1}ÍLÏ
:Send(LÏ)
:36Ídim LÓ
:36Ídim LÔ
:Disp "HOW MUCH TIME"
:Disp "BETWEEN POINTS"
:Disp "IN SECONDS?"
:Input T
:-2*TÍXmin
:36*TÍXmax
:TÍXscl
:seq(I,I,T,36*T,T)ÍLÓ
:ClrHome
:Disp "HIT ENTER"
:Disp "TO START"
:Pause
:ClrHome
:{3,T,-1,0}ÍLÏ
:Send(LÏ)
:For(I,1,36,1)
:Get(LÔ(I))
:Pt-On(LÓ(I),LÔ(I))
:End
:ClrHome
:Plot1(Scatter,LÓ,LÔ,Å)
:DispGraph
:Stop
```