Searching For The Center

Brief Overview:

This is a three-lesson unit that discovers and applies points of concurrency of a triangle. The lessons are labs used to introduce the topics of incenter, circumcenter, centroid, circumscribed circles, and inscribed circles. The lesson is intended to provide practice and verification that the incenter must be constructed in order to find a point equidistant from the sides of any triangle, a circumcenter must be constructed in order to find a point equidistant from the vertices of a triangle, and a centroid must be constructed in order to distribute mass evenly. The labs provide a way to link this knowledge so that the students will be able to recall this information a month from now, 3 months from now, and so on. An application is included in each of the three labs in order to demonstrate why, in a real life situation, a person would want to create an incenter, a circumcenter and a centroid.

NCTM Content Standard/National Science Education Standard:

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
- Use visualization, spatial reasoning, and geometric modeling to solve problems.

Grade/Level:

These lessons were created as a linking/remembering device, especially for a co-taught classroom, but can be adapted or used for a regular ed, or even honors level in 9th through 12th Grade. With more modification, these lessons might be appropriate for middle school use as well.

Duration/Length:

Lesson #1	45 minutes
Lesson #2	30 minutes
Lesson #3	30 minutes

Student Outcomes:

Students will:

- Define and differentiate between perpendicular bisector, angle bisector, segment, triangle, circle, radius, point, inscribed circle, circumscribed circle, incenter, circumcenter, and centroid.
- Construct an incenter of a triangle.

- Construct an inscribed circle of a triangle.
- Construct a circumcenter
- Construct a circumscribed circle
- Construct a centroid
- Differentiate when to use an incenter, circumcenter, and centroid.

Materials and Resources:

- Geometer's Sketchpad (Patty Paper or Cabri Junior may be substituted with a modification to the lab sheet)
- Worksheets 1, 2, & 3
- Straight edge

Additional/Optional Resources:

- Patty Paper, pin and dowels, construction paper, and pencils to copy and solve the application problems

Summative Assessment:

There is an assessment consisting of three selected response questions and one BCR and one ECR where students will demonstrate assessments the knowledge gained from the lesson.

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How to Use These Lessons:

These lessons are designed for the teacher with access to a computer with Geometer's Sketchpad or Cabri, or a calculator with Cabri Jr. that can be projected to the class. The lesson can also be implemented using patty paper.

Development/Procedures:

Lesson 1: For The Birds

Preassessment/Procedure

Organize students into cooperative groups. Ask each group to draw a triangle and find the center of that triangle. Give no guidance to them. Have groups share their process and "centers" of triangles. Point out to the students that there is more than one center. A location of the center (i.e., incenter, centroid) can depend on the type of the triangle (acute, obtuse, or right) and what is meant by center. The lab, using ***Worksheet 1*** can now be started. The students must have a working knowledge of Geometer's Sketchpad, Cabri Jr. or use Patty Paper. The students should have already have experience constructing triangles and circles. The students should be familiar with using a compass and a protractor to construct circles and triangles. ***Worksheet 1:*** Students will be guided through the inscribed circle, an incenter, and point of concurrency.

Teacher Facilitation/Student Application

Use Geometer's Sketchpad to demonstrate the step-by-step directions using ***Worksheet 1***, then review what an angle bisectors is. Have students define the terms formally discussed in other lessons to reinforce the terms. Then draw an angle bisector using Geometer's Sketchpad and ask the students to continue with the rest of the steps and locate the point of concurrency. Have a discussion with the class as to what happens to the point of concurrency when the shape of the triangle changes.

Embedded Assessment

To check for understanding and as a follow up to the discussion of point of concurrency, using Geometer's Sketchpad, move on to the next steps of ***Worksheet 1***. Have the students compare the measurements and lead the students in a discussion of what is happening.

Reteaching/Extension

The "Let's Link It" and Memory Technique parts of ***Worksheet 1*** is provided to re-teach and help students remember the concepts covered.

The Application section of ***Worksheet 1*** is provided for a possible homework/reinforcement assignment.
Additional extension exercises are also included.

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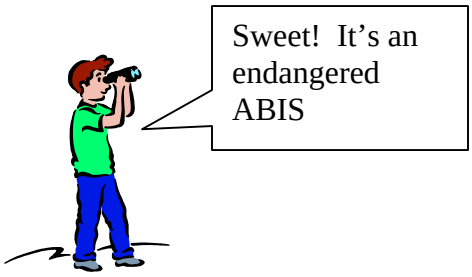


For The Birds

Using Geometer's Sketchpad, Patty Paper, or Cabri Junior:

Step 1: Draw Triangle ΔABC

1. Blast from the Past…..What is an angle bisector?

_____

Step 2: Construct the angle bisector through $\angle A$, $\angle B$, and $\angle C$

A point of concurrency is the point where three or more lines intersect.

Step 3: Label a point O at the point of concurrency of the angle bisectors.

Step 4: Drag a vertex of your triangle so that your triangle looks different (i.e. transforms into either an acute, right, or obtuse triangle). Record the location of the point of concurrency in the chart below.

Type of Triangle	Location of Point of Concurrency
Acute	
Right	
Obtuse	

2. Does the location of the point of concurrence change? Explain.

_____

_____

Step 5: Now hide the angle bisectors.

Step 6: Construct a perpendicular line from point P of ΔABC to $\overline{AB}$ and label a point E where the perpendicular line intersects $\overline{AB}$. Repeat this step with $\overline{BC}$ and $\overline{AC}$.
Use points F and G consecutively.

Step 7: Measure $\overline{PE}$, $\overline{PF}$, and $\overline{PG}$.

Step 8: Drag a vertex of your triangle so that your triangle looks different (i.e. transforms into either an acute, right, or obtuse triangle). Record the measures of the segments after each transformation into the chart below.

Type of Triangle	Measures of Segments $\overline{PE}$	$\overline{PF}$	$\overline{PG}$
Acute			
Right			
Obtuse			

3. How do the measures of these lines compare?

_____

The point of concurrence of the angle bisectors of a triangle is called the incenter. The incenter can then be used to construct an inscribed circle.

An inscribed circle in a triangle has the sides of the triangle tangent to the circle (intersecting at one and only one point) to the circle.

Step 9: Hide the perpendicular lines. Using the incenter as the center of a circle, and $\overline{OE}$ as a radius, construct a circle.

4. **Here's an exploration for a future concept.**

- **If you had to name the inscribed circle, what would you name it? Explain why you would name it this name.**

- **Now identify segments, $\overline{OE}$, $\overline{OF}$, and $\overline{OG}$. Can you make a hypothesis regarding these in any circle?**

Let's Link It!

Fill in the blanks below: Need a hint? All answers are found on this lab sheet.

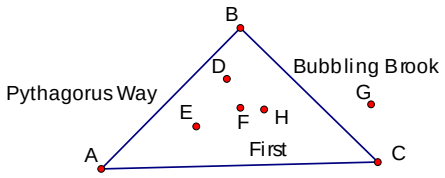
The **A** and **B** in ABIS stand for the _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _.
The **I** in ABIS stands for the _ _ _ _ _ _ _ _, and also for the _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ which can be created using the structure.
The **S** stands for _ _ _ _ _ _ _ _ of the triangle from which we want to be equidistant.

Memory Technique: Sometimes it helps to create a story:

My Story: An A B I S is an endangered tropical bird, which builds its nest equidistant among three streams. It uses angle bisectors to build its nest. The name of its nest is an incenter. All its hatchlings are stay between the three streams.

Now make up your own mnemonic to help you remember that the ANGLE BISECTOR is used to create an INCENTER/INSCRIBED circle which will find a location equidistant from the SEGMENTS of a triangle or create another story about the ABIS.

Application



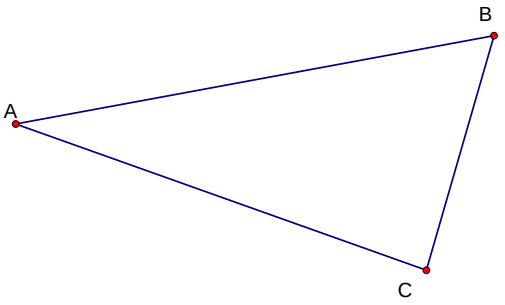
A carnival is coming to Flatland. In order to gain the most business, the carnival will want to be located equidistantly from the streets in Flatland.

a) At which point should the carnival be located? Use what you know about points of concurrency in a triangle to justify your solution.

b) Is there a point in the diagram that a student who knows ABIS should quickly discount as being a correct answer in part (a)? Explain.

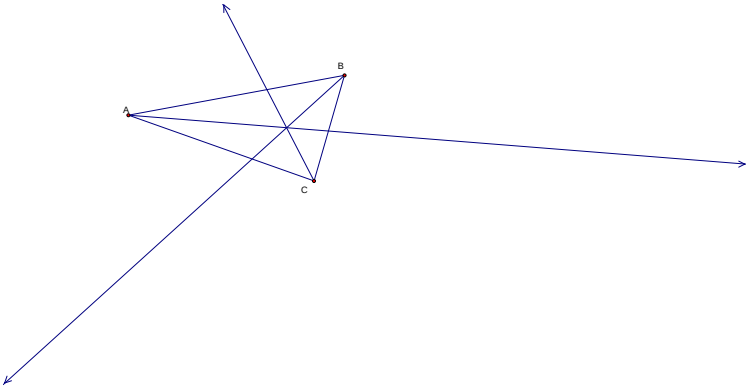
Key – For the Birds

Step 1 Sample Construction

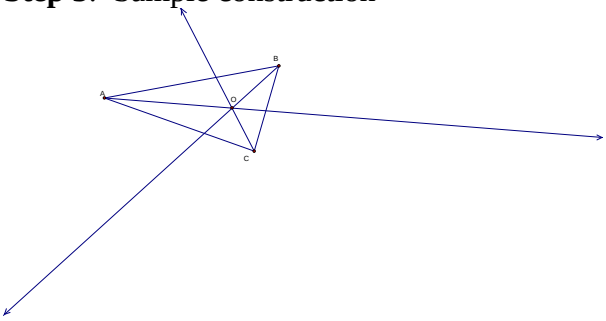


1. An angle bisector is a line that cuts an angle into two congruent parts.

Step 2 Sample Construction



Step 3: Sample construction

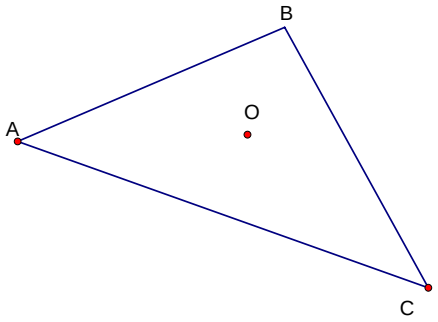


Step 4:

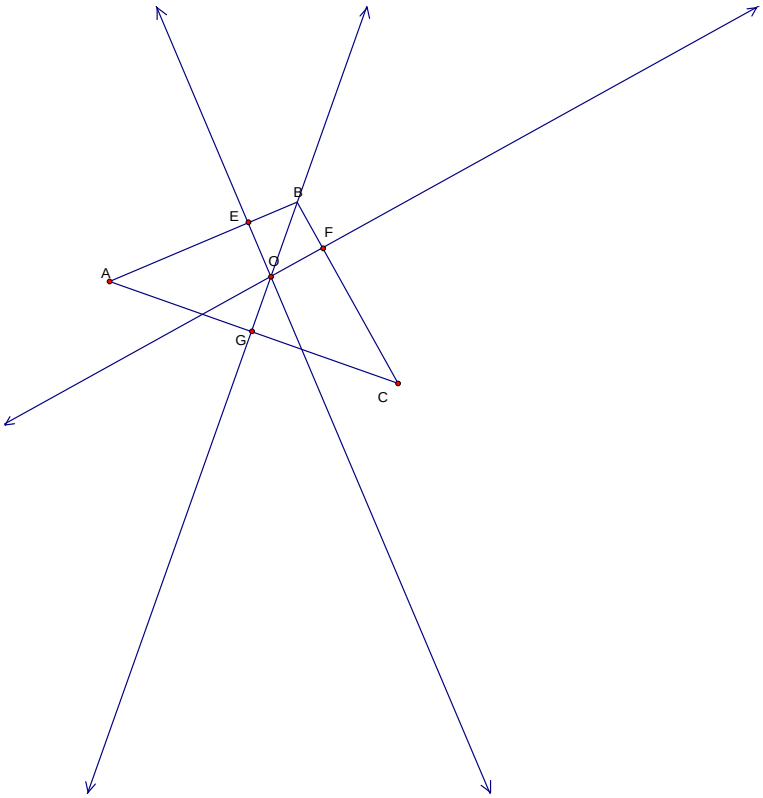
Type of Triangle	Location of Point of Concurrency
Acute	inside
Right	inside
Obtuse	inside

2. The location of the point of concurrency does not change. Since all the angles face into the triangle, all angle bisectors will be inside the triangle. Hence, the point of concurrency must be located inside the triangle.

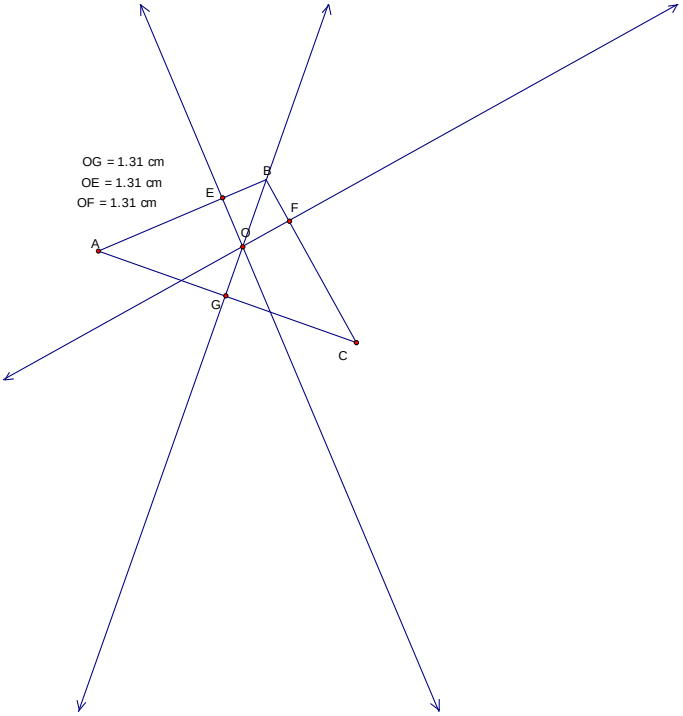
Step 5: Sample Construction



Step 6: Sample Construction



Step 7. Sample measures.

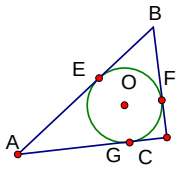


Step 8: Sample values

Type of Triangle	Measures of Segments $\overline{PE}$	$\overline{PF}$	$\overline{PG}$
Acute	.62	.62	.62
Right	.58	.58	.58
Obtuse	.86	.86	.86

3. The measure of the segments remains equal in length.

Step 9:



4.

- Circle O. I used the center to name the circle
- All radii of a circle are congruent

Let's Link It!

Fill in the blanks below: Need a hint? All answers are found on this lab sheet.

The **A** and **B** in ABIS stand for the **ANGLE BISECTOR**.
The **I** in ABIS stands for the **INCENTER**, and also for the **INSCRIBED CIRCLE** that can be created using the structure.
The **S** stands for **SEGMENTS** of the triangle from which we want to be equidistant.

Memory Technique: Sometimes it helps to create a story:

Stories and mnemonic devices will vary.

Application

The students should copy the triangle and create the incenter. The appropriate place to locate the carnival is at Point F.

A student knowing ABIS would discount Point G since Point G is located outside the triangle. All angle bisectors are inside the triangle and thus the point of concurrency must be located inside the triangle. In addition, it's an "incenter" which means it is "inside" the triangle.

Name: _____ **Date:** _____

Other Centers of a Triangle

Use the Internet to find five other triangle centers and describe them. Your description must include how each triangle center is constructed. Use the space below to record your findings.

Name: _____ **Date:** _____

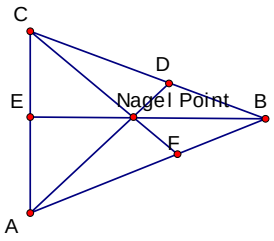
Other Centers of a Triangle

Use the Internet to find five other triangle centers and describe them. Your description must include how each triangle center is constructed. Use the space below to record your findings.

Sample solutions
Information for this solution was found at:
www.imsa.edu/edu/math/journal/volume4/articles/TriangleCenters.pdf

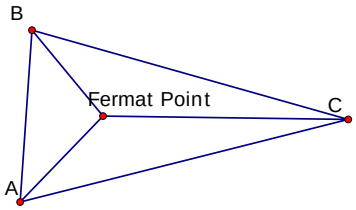
Student should have found Circumcenter and Median. In addition to these, below are others.
Nagel Point

- Point of concurrency of the segments that joins each vertex to the "semi-perimeter" point



Fermat Point

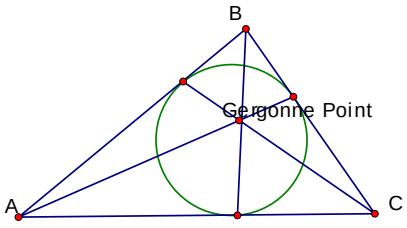
- Point where the sum of the distances to the vertices of a triangle is at a minimum



FA + FB + FC is at a minimum

Gergonne Point

- Point of concurrency of the segments that connect each vertex of a triangle to the point of contact of the inscribed circle of that triangle.



There are other points that centers of triangles that can be found

Development/Procedures:

Lesson 2: Search For the Perfect Peanut Butter Cookie

Preassessment/Procedure

Review lesson 1 with the students. Stress the incenter is just one type of center, and that there are others. Stress that the center used depends upon what the user wants to accomplish. Discuss the results of the extension lesson from Lesson 1. Let students tell you what they found in their research. Let students also tell you why a specific center would be used rather than another. Preview with the students the application project – the broken plate. Have students use their extension from Lesson 1 and class discussion to determine whether one of the centers they've researched would be appropriate for the broken plate application. If so, have them justify their choice of center.

The students must have a working knowledge of Geometer's Sketchpad, Cabri Jr., or use Patty Paper. The students should have already have experience constructing triangles and circles. The students should be familiar with using a compass and a protractor to construct circles and triangles. ***Worksheet 2*** will introduce the students to a circumscribed circle, a circumcenter, and continue to reinforce what was learned through Lesson 1.

Teacher Facilitation/Student Application

Use Geometer's Sketchpad to demonstrate the step by step the creation of a triangle using ***Worksheet 2***, then talk about perpendicular bisectors. Have the students define the terms formally discussed in other lessons to reinforce the terms. Then draw an perpendicular bisector using Geometer's Sketchpad and ask the students to continue with the rest of the steps and locate the circumcenter. Have a discussion with the class as to what happens to the circumcenter when the shape of the triangle changes.

Embedded Assessment

To check for understanding and as a follow up to the discussion of circumcenter, using Geometer's Sketchpad, move on to the next steps of ***Worksheet 2*** compare the measurements of the perpendicular bisectors. Lead the students in a discussion of what is happening.

Reteaching/Extension

The "Let's Link It" and Memory Technique parts of ***Worksheet 2*** is provided to re-teach or help students remember, as a re-teaching exercise.

The Application section of ***Worksheet 2*** is provided for a possible homework/reinforcement assignment.

Additional extension exercises are also included.

Name: _____ **Date:** _____

Search for the **P**erfect **P**eanut **B**utter **C**ookie

Using Geometer's Sketchpad, Patty Paper, or Cabri Junior :

Step 1: Construct ΔABC.

1. Blast from the Past…. What is a perpendicular bisector?

Step 2: Construct the perpendicular bisectors through sides $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$.

Step 3: Label the point of concurrency O .

Remember that a point of concurrency is a point where three or more lines intersect.

Step 4: Drag a vertex of your triangle so that your triangle looks different, (i.e. changes shape into an acute, right, and obtuse triangle).

Type of Triangle	Location of Point of Concurrency
Acute	
Right	
Obtuse	

2. Does the location of the point of concurrency change? Explain.

- **The point of concurrency of perpendicular bisectors of a triangle is called the circumcenter.**

Step 5: Hide the perpendicular bisectors. Construct segments $\overline{OA}$, $\overline{OB}$, and $\overline{OC}$.

Step 6: Measure the lengths of $\overline{OA}$, $\overline{OB}$, and $\overline{OC}$.

3. How do these measures compare?

Step 7: Drag a vertex of your triangle so that your triangle looks different (i.e. changes shape into an acute, right, and obtuse triangle).

Type of Triangle	Segment Measures $\overline{OA}$	$\overline{OB}$	$\overline{OC}$
Acute			
Right			
Obtuse			

4. How are the measures of $\overline{OA}$, $\overline{OB}$, and $\overline{OC}$ related as you drag the triangle?

Step 8: Construct a circle, using point O as the center, and $\overline{OA}$ of ΔABC as a radius.

You have just created a circumscribed circle. A circumscribed circle of a triangle is a circle that has the vertices of the triangle on the circle.

- **< The circumcenter is the center of a circumscribed circle>**

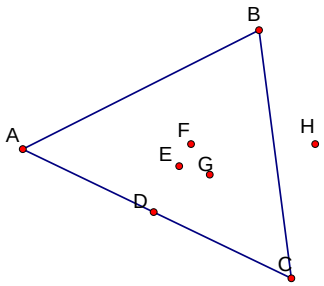
Let's Link This!

How can your search for the **P**erfect **P**eanut **B**utter **C**ookie help you remember how to construct a circumscribed circle and at the same time help you remember when to use this construction?

The **P** in Perfect reminds you that you want to be equidistant from _ _ _ _ _ _.

The **P** in Peanut and **B** in Butter remind you that you want to use _ .
The **C** reminds you that this will create a _ _ _ _ _ _ _ _ _ _ _ _ _ in order to create a _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ circle.

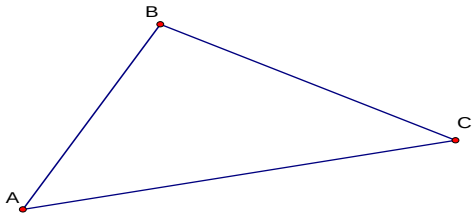
Application
Using Patty paper and a straight edge draw the following:



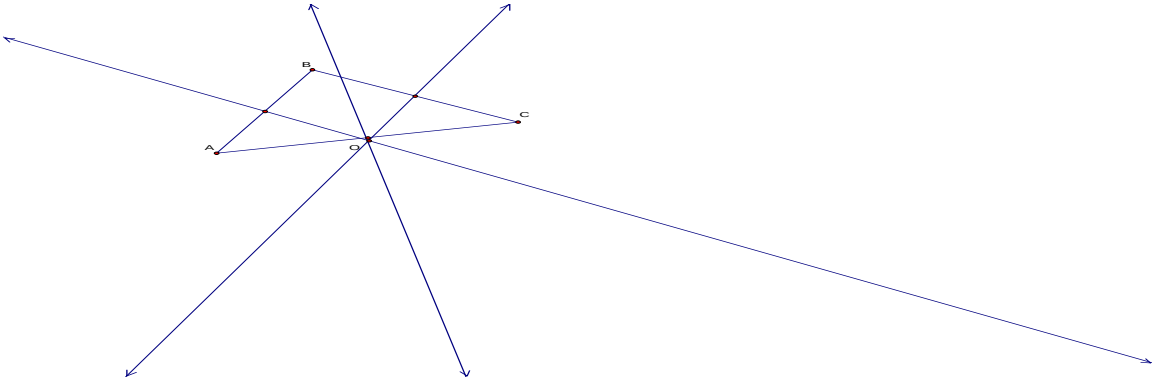
Three families, A, B and C who each reside at the vertices A, B, and C, are planning to meet for a picnic. D, E, F, G, and H represent the towns the families can choose to meet for their picnic. They agree that all families should drive the same distance. In which town should they meet? Justify your solution.

Key Worksheet 2

Step 1 Sample construction



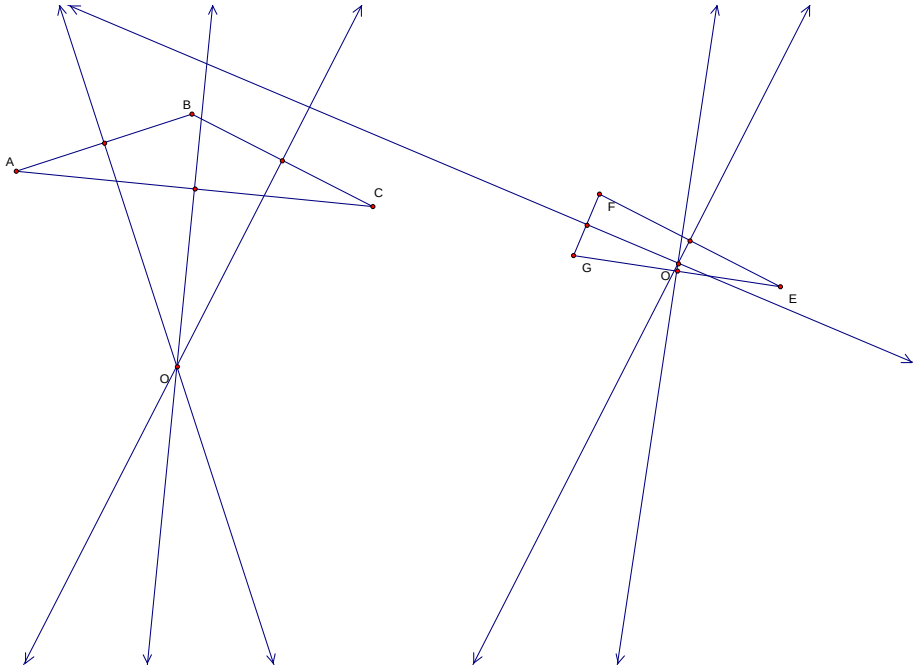
Step 2 Sample construction.



1. A <u>perpendicular bisector</u> is a line that cuts a segment into two congruent parts (passes through the midpoint) and is perpendicular to the segment (intersects to form right angles).

Step 3 See step 2

Step 4. Sample constructions

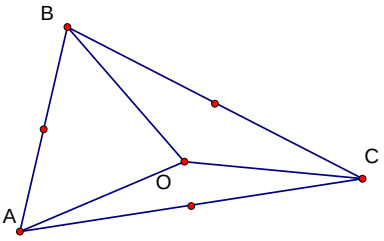


Type of Triangle	Location of Point of Concurrency
Acute	Inside triangle
Right	On the triangle
Obtuse	Outside the triangle

(Answers may vary)

2. The location of the point of concurrency of perpendicular bisectors does change, depending on the angles of the triangle.

Step 5 Sample construction.



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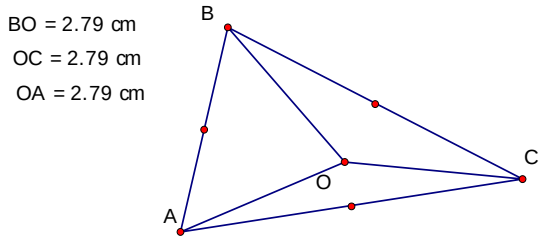
Step 6 Sample construction

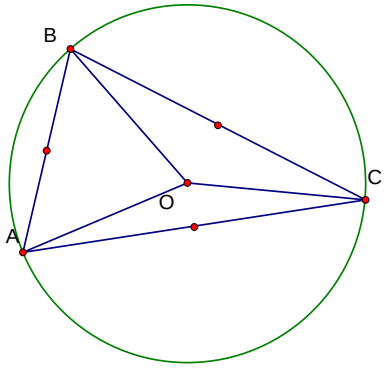


3. The segments are equal in length, meaning point O is equidistant from the vertices of the triangle.

4. The measures of the segments will change, but they will remain equal in length when compared to each other. In other words, the circumcenter remains equidistant from the vertices of the triangle.

Step 7 Sample construction.

Type of Triangle	Segment Measures		
	$\overline{OA}$	$\overline{OB}$	$\overline{OC}$
Acute	2.65	2.65	2.65
Right	1.60	1.60	1.60
Obtuse	4.71	4.71	4.71



Let's Link This!

How can your search for the **P**erfect **P**eanut **B**utter **C**ookie help you remember how to construct a circumscribed circle and at the same time help you remember when to use this construction?

The **P** in Perfect reminds you that you want to be equidistant from **POINTS**.

The **P** in Peanut and **B** in Butter remind you that you want to use **PERPENDICULAR BISECTOR**.

The **C** reminds you that this will create a **CIRCUMCENTER** in order to create a **CIRCUMSCRIBED** circle.

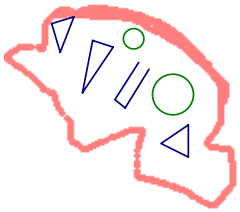
Solution to Application Problem: Point E

Name: _____ **Date:** _____

A Broken Plate

An artist found an interesting antique plate among the rubble found at an excavation site. She would like to duplicate and reproduce the plate as gifts for the students working at the excavation site. There's a problem, though. The plate is broken. The artist, in order to duplicate the plate will need to reconstruct the plate first. One step in the reconstruction is to find the original size of the plate.

Your task is to help the artist re-construct the size of the plate. Plot any three points on the circle and connect them to form a triangle. Then use what you know about circumscribed circles to reconstruct the plate so that the artist will be able to measure the diameter of the plate.



Name: _____ **Date:** _____

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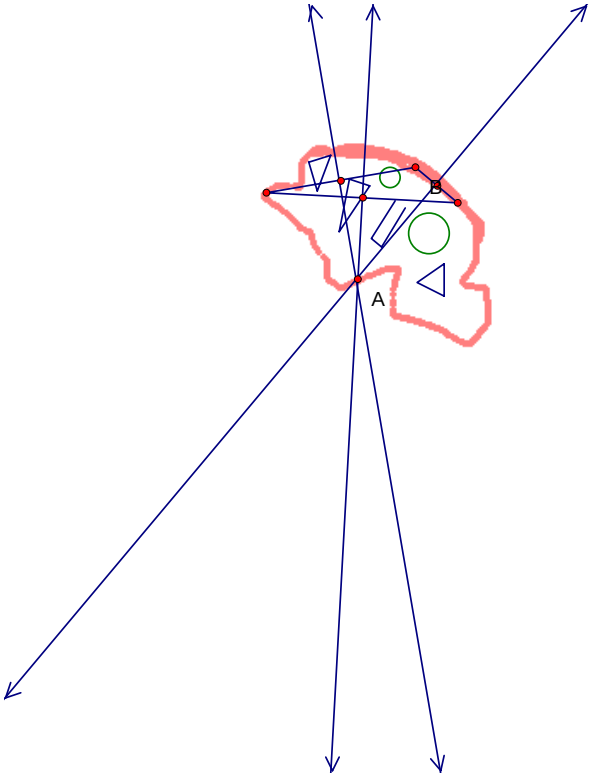
Your task is to help the artist re-construct the size of the plate. Plot any three points on the circle and connect them to form a triangle. Then use what you know about circumscribed circles to reconstruct the plate so that the artist will be able to measure the diameter of the plate.

Solution:



To find the size of the plate, a diameter is needed. A diameter connects two points on the circle, and passes through the center of the circle. A circumscribed circle is needed to find the diameter. To find the circumscribed circle, construct perpendicular bisectors. The artist will need to measure from a point on the edge of the plate to the circumcenter to find the size of the plate.

Development/Procedures:

Lesson 3 More Money Captain!

Preassessment/Procedure

Organize students into cooperative groups of three. Give each group a construction or cardstock triangle with the directions that the group needs to balance the triangle, pick a center from their extension lesson from Lesson 1, and tell why they chose that center. Let one group member be the facilitator. This student will make sure each group member participates in the suggestions of how to balance the triangle. A second group member will be the balancer. A third member will present the group's prediction to the class. Have each group present their prediction of the measure of center needed to distribute that balance of their triangle.

The students must have a working knowledge of Geometer's Sketchpad, Cabri Jr. or use Patty Paper. Students should already been discussing and constructing triangles and circles. The students should be familiar with using a compass and a protractor to construct circles and triangles. ***Worksheet 3:*** is a lesson on medians and centroids.

Teacher Facilitation/Student Application

Use Geometer's Sketchpad to demonstrate the step by step the creation of a triangle using ***Worksheet 3*** then talk about medians. Have the students define the terms formally discussed in other lessons to reinforce the terms. Then draw a median using Geometer's Sketchpad and ask the students to continue with the rest of the steps and locate the centroid. Have a discussion with the class as to what happens to the centroid when the shape of the triangle changes.

Embedded Assessment

To check for understanding and as a follow up to the discussion of centroid, using Geometer's Sketchpad, move on to the next steps of ***Worksheet 3*** compare the measurements of the medians. Lead the students in a discussion of what is happening.

Reteaching/Extension

The "Let's Link It" and Memory Technique parts of ***Worksheet 3*** are provided to re-teach or help students remember, as a re-teaching exercise. The Application section of ***Worksheet 3***is provided for a possible homework/reinforcement assignment. Additional extension exercises are also included.

Name: _____ **Date:** _____

More Money Captain!

Using Geometer's Sketchpad, Patty Paper, Cabri or Cabri Junior :

Step 1: Draw ΔABC.

1. Blast from the Past…..What is a median?

Step 2: Construct the medians of ΔABC.

Step 3: Draw a point O at the point of concurrency of these three medians.

Step 4: Drag a vertex of your triangle so that your triangle looks different (i.e. transforms into an acute, right, or obtuse triangle).

Type of Triangle	Location of Point of Concurrency
Acute	
Right	
Obtuse	

2. Does the location of the point of concurrency change? Explain.

Step 5: Label the midpoints of the sides of ΔABC, and compute the area of each of the six triangles inside ΔABC.

3. What is the relationship between the areas of the smaller triangles?

4. Determine whether the triangles are congruent. Justify your answer.

Aye Mate! The point of concurrency of the medians of a triangle is called the **centroid.** The centroid created 6 triangles of equal area.

5. Blast from the past…. What is area?

A Centroid is used to balance the Mass of an object. Therefore, the centroid of our triangle can be used to suspend a balanced triangle so it balances or remains parallel to a ceiling.

Let's Link It!

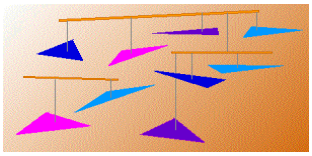
6. **Fill in the blanks below: Need a hint? All answers are found on this lab sheet.**

The **M** in More stands for _ _ _ _ .
The **M** in Money stands for _ _ _ _ _ _.
The **C** in Captain stands for _ _ _ _ _ _ _ _.

Note: To distribute the mass, construct the medians. The point of concurrency of the medians are a centroid.

7. **Memory Technique**: Sometimes it helps to create a story:

Create a story using "**M**ore **M**oney **C**aptain" or devise a mnemonic device of your own that will allow you to remember that when you want to distribute the **MASS** of a triangle, create the **CENTROID**, which is the point of concurrency of the **MEDIANS**.



Assessment Activity: Create a mobile

Supplies needed: Construction paper, three wooden dowels of unequal lengths, string, pin

Step 1: First, cut out a minimum of 6 triangles from construction paper. Each triangle should be a different shape and/or size.
Step 2: Find the centroid of each triangle.
Step 3: Using the pin, puncture the triangle's centroid.
Step 4: Carefully feed the string through the triangle, careful to not tear the triangle's centroid.
Step 5: Tie a knot on one end of the string and tie the other end to a dowel.
Step 6: Tie the dowels together to form a multi-level mobile. Be sure that the dowels balance.

Rubric for Mobile Assessment

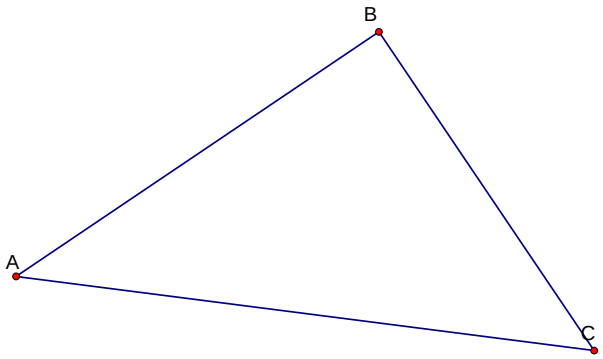
Points Earned	Criteria	(Points possible)
_____	A minimum of six triangles are in the mobile.	(6)
_____	All six triangles are of unequal size or shape.	(6)
_____	One each of acute, right, and obtuse triangles present.	(3)
_____	The mass has been balanced in all six triangles.	(6)
_____	The mobile has a minimum of two levels.	(2)

Cut Scores:

Mobile Score	Grade
23	100
22	95
21	90
20	89
19	85
18	80
17	79
16	75
15	70
14	69
13	65
12	64
11-8	60
7-4	40
below 4	0

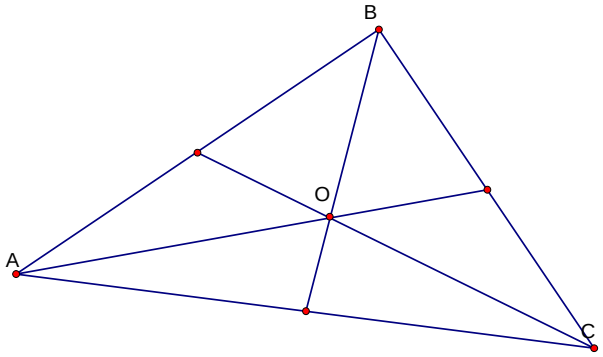
Answer Key – More Money Captain!

Step 1: Sample Sketch



1. A median is a segment that connects a vertex of a triangle to the midpoint of the side opposite of that vertex.

Step 2 and 3: Sample Sketch



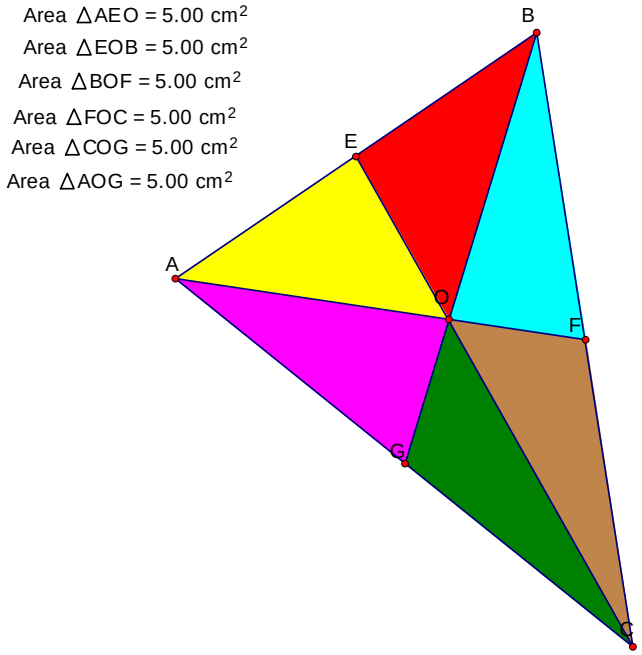
Step 4

Type of Triangle	Location of Point of Concurrency
Acute	Inside
Right	Inside
Obtuse	Inside

2. The Point of concurrency will never be located outside the triangle because by definition, a median must be located inside a triangle (a segment that connects a midpoint of a side to the vertex opposite that side).

Step 5 **Sample construction**. (Remember to construct the interior of the triangles in order to measure the area….also….this triangle is in color on the on-line

version so students can see each smaller triangle.



3. The areas of the smaller triangles are the same.

4. The triangles are not congruent, because congruent means same size and same shape. I can see that not all these triangles have the same shape. (If the triangles happen to appear to be the same shape it might be a good time to preview for example have some discussion on how we can determine if they were congruent (i.e., measure every side and every angle – or use postulates such as SAS, AAS, SSS, ASA)).

5. Area is the measure of the surface of the interior of a triangle….in other words, the amount of material needed to cover the surface of a closed polygon.

Let's Link It!

6. **Fill in the blanks below: Need a hint? All answers are found on this lab sheet.**

The **M** in More stands for **MASS** .
The **M** in Money stands for **MEDIAN**.
The **C** in Captain stands for **CENTROID**.

7. Extension: Answers may vary

Rubric for Mobile Assessment

Points Earned	Criteria	(Points possible)
_____	A minimum of six triangles is in the mobile.	(6)

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below 4	0

Name: _____ Date: _____

Use the Internet find information about the Nine Point Circle and the Euler Line. Describe them. Your description must include how each is constructed and suggest some possible applications of each. Use the space below to record your findings.

Name: _____ **Date:** _____

The Nine Point Circle and the Euler Line

Use the Internet to find information about the Nine Point Circle and the Euler Line. Describe them. Your description must include how each is constructed. Use the space below to record your findings.

Solution:

The Nine Point Circle for any triangle passes through:

- The three mid-points of the sides
- the three feet of the altitudes
- the three midpoints of the segments from the respective vertices to the orthocenter.

Some interesting relationships of the nine point circle are:

- radius of the nine point circle is equal to half the radius of the circumcenter of the triangle
- bisects any line from the orthocenter to a point on the circumcenter

The Euler Line is the straight line that results when the centroid, orthocenter, circumcenter, and the center of the nine-point circle are connected. i.e. the centroid, orthocenter, circumcenter and center of the nine-point circle are collinear.

Other interesting characteristics of the Euler line are:

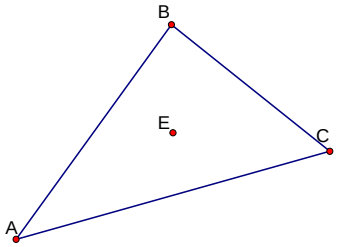
- the center of the nine point circle lies at the midpoint between the orthocenter and the circumcenter,
- the distance between the centroid and the circumcenter is half of the distance between the centroid and the orthocenter.

Information for these solutions was found at
http://encyclopedia.worldvillage.com/s/b/Nine_point_circle
and
http://encyclopedia.worldvillage.com/s/b/Euler%27_line

Name _____ Date _____

Assessment for Points of Concurrency of a Triangle

Selected Response. (1pt) Select the best choice for each problem.



1. A martial arts expert, E, spars in a triangular ring with three of his equally talented students, A, B, and C. The worst place he could stand is where the three students could deliver a chop or leg kick to the expert at the same time. Which point of concurrency would represent this worst place he could stand?

A) Centroid B) Circumcenter
C) Incenter D) Altitude

2. A contractor is building a gazebo in a triangular garden. He needs to determine the incenter. Which of the following will he need to determine the incenter?

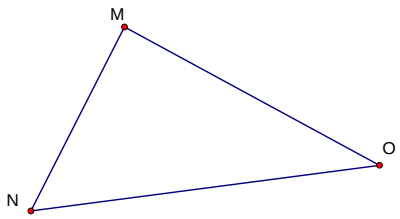
A) Angle Bisector B) Apothem
C) Median D) Perpendicular Bisector

3. You are a sculptor and have just completed a large metal mobile. You want to hang the mobile, made of flat triangular metal plates, in the State Capitol Building. Each triangular piece will hang so that it will be suspended with the triangular surface parallel to the ground. Which of the following would you use as a center of each triangular plate?

A) Centroid B) Circumcenter
C) Incenter D) Altitude

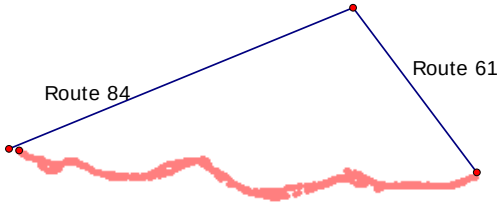
BCR. (3pts)

4. a) Construct the inscribed circle of ΔNMO. For full credit, show all your work.



ECR. (4pts)

5.

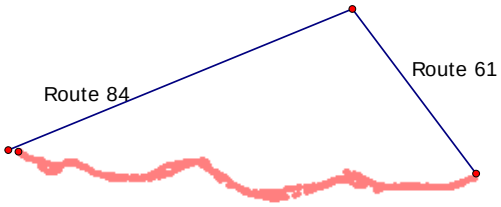


Raccoon River

The northernmost county of Tantonia is nestled between Route 84, Route 61, and the Raccoon River. County officials are looking for a place to locate the county "fair" .The criteria for the location of the county fair is that the location must be where households in the boundaries of the county will not have a larger drive than any other household.

A) County Official McBride ordered the county planners to make sure the location is equidistant from all three county borders. Use the drawing above to find the location for the county fair. Use what you know about points of concurrency of a triangle to explain your solution.

B) County Official Henderson protests the location of the fair as ordered by County Official McBride. He asserts that most of the population of the county lives in three towns located at the intersections of Routes 84, 61, and the Raccoon River. A "fair" location for the county fair would be one that served the majority of the population. Use the drawing below to find the new location of the county fair according to County Official Henderson's requirement. Use what you know about points of concurrency of a triangle to explain your solution.



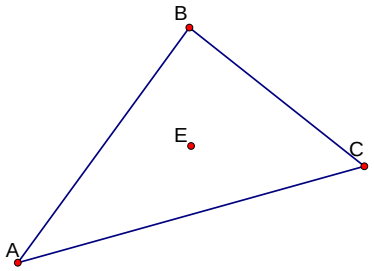
Raccoon River

C) Which location do you think is the better choice? Explain your opinion.

Answer Key

Assessment for Points of Concurrency of a Triangle

Selected Response. (1pt) Select the best choice for each problem.



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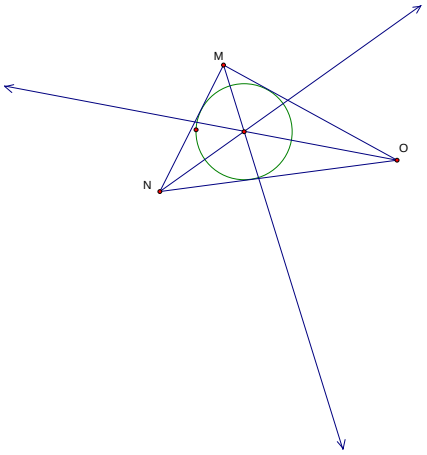
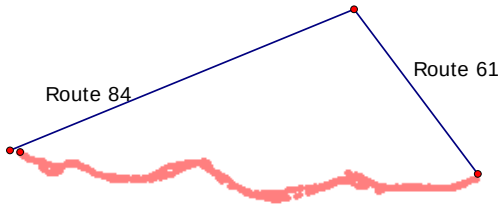


ECR. (4pts)

5.



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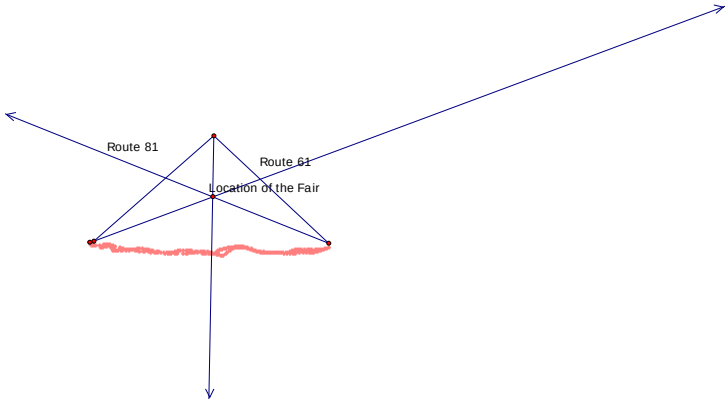
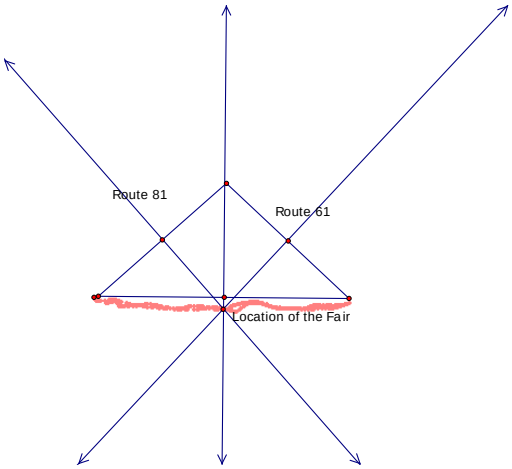


I had to be equidistant from boundaries of county lines. I determined the incenter by constructing angle bisectors.

D) County Official Henderson protests the location of the fair as ordered by County Official McBride. He asserts that most of the population of the county lives in three towns located at the intersections of Routes 84, 61, and the Raccoon River. A "fair" location for the county fair would be one that served the majority of the population. Use the drawing below to find the new location of the county fair according to County Official Henderson's requirement. Use what you know about points of concurrency of a triangle to explain your solution.



Raccoon River

I had to be equidistant from three towns. I determined the circumcenter by constructing perpendicular bisectors of boundaries of the county.

E) Which location do you think is the better choice? Explain your opinion.

County Official McBride's idea is the better choice because the county fair will be located in the county and equally accessible to all county residents. County Official Henderson's choice would locate the County fair outside of the county.

Summative Assessment:

There is an assessment consisting of three selected response questions and one BCR and one ECR where students will demonstrate assessments the knowledge gained from the lesson.

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