

A Balancing Act II

Brief Overview:

In this lesson students will learn the definition of “median” and will investigate the characteristic of the centroid as the center of gravity. It is assumed that students can name and classify triangles by sides and angles, and will know basic terms, such as vertices and midpoint. The lesson will culminate with students constructing a mobile using the properties of the centroid.

NCTM Content Standard/National Science Education Standard:

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
- Use visualization, spatial reasoning, and geometric modeling to solve problems.

Grade/Level:

9-12 Geometry or Honors Geometry Class

Duration/Length:

At least 2 classes of 45-50 minutes

Student Outcomes:

Students will:

- Be able to draw medians of a triangle by at least three of the following different methods: paper folding, drawing in Geometer’s Sketchpad software, or constructing using a MIRA or compass and straight edge.
- Locate the centroid and demonstrate the function of a centroid as the center of gravity in real-life situations, such as balancing a Geometric component of a mobile so that it is parallel to the floor.
- Discover by measurement the ratio of the two segments of the median formed at the centroid.

Required Materials and Resources:

- Cardboard, card stock, or poster board for students to make polygons.
- Strong thin string, such as carpet or upholstery thread, and 5 lb. weight fish line
- Push pin, scissors and coat hanger per group

Optional Supplies:

- Geometer’s Sketchpad Software, Compass and Straightedge per student, MIRAs

Development/Procedures:

Lesson 1

Preassessment – The students are expected to be able to identify the different types of triangles, and given a triangle, to draw a segment from the vertex of a triangle to the midpoint of the opposite side.

Launch – The students will draw their own triangle on card stock paper (like a manila folder), then cut out the triangle. They will balance the triangle their finger, an eraser, or a pointed object. They will use a pin or push pin to mark this point.

Teacher Facilitation – After the triangle balance investigation, the teacher will introduce the concepts of medians, the intersection of the three medians, which is called the centroid, and the center of gravity, or balance point. The teacher will also review the different types of triangles according to their angles and lengths of sides. Each teacher will then have the option of using Sketchpad, MIRA, paper folding, or compass and straightedge construction to investigate the segments of the median formed at the centroid.

Student Application – Each student in the group will then be responsible for drawing one of five different types of triangles (scalene, obtuse, right, isosceles, and equilateral), constructing the three medians of the triangle using the tools mentioned above, and labeling the intersection as the centroid. They will also write any observations or conclusions, such as the special cases of the isosceles and equilateral triangles.

Embedded Assessment – The teacher will check to see if the students' three medians intersect at the centroid, and the correct ratio of 2 to 1 has been found for the segments of the median formed by the centroid.

Reteaching/Extension –

- For those who have not completely understood the lesson, review the terms needed.
- For those who have understood the lesson, the student will write the steps used to find the centroid of any triangle and will describe in words or diagrams how a median differs from an altitude, angle bisector and perpendicular bisector.

Lesson 2

Preassessment – The students will be given a triangle to find the centroid. (Warm-up Lesson 2)

Launch – The students will use the same triangle from the previous lesson to find the centroid by construction or Sketchpad software to compare it to their hypothetical centroid obtained by balancing on the tip of an eraser.

Teacher Facilitation – Check for understanding of the vocabulary (median and Centroid). Discuss the function of the centroid as the center of gravity, the balance point. Have the students read the “Making a Mobile” assignment and discuss the ideas. Explain your expectation that each group will use the idea of centroid to make group mobiles using triangles. These triangles will hang from the fish wire as decorations in the mathematics laboratory classroom, and will be parallel to the floor if balanced at the centroid. Time permitting, the teacher might possibly review and compare the incenter, circumcenter, and orthocenter to the centroid.

Student Application – Each student in the group will draw a different type of triangle (scalene, isosceles, equilateral, right, or obtuse) on cardstock, construct the medians to find the centroid, attach a string or fish wire at the centroid, and suspend it from the group coat hanger.

Embedded Assessment – Each student will complete the mobile worksheet. S/he will explain the type of triangle used, the steps needed to find the centroid, and the group’s results (success or failure).

Reteaching/Extension -

- For those who have not completely understood the lesson, review the definitions of median and centroid. Have student draw another triangle to find the centroid using an alternative method to find the midpoint (i.e. paper folding or MIRA or Sketchpad instead of construction with compass and straight edge).
- For those who have understood the lesson, they will be encouraged to try the same method or devise a new method to find the centroid of various quadrilaterals.

Lesson 3

Preassessment - The students are expected to be able to identify the different types of quadrilaterals (square, rectangle, parallelogram, rhombus, trapezoid and kite) and draw the diagonals and the midpoints of segments.

Launch – Each group will be assigned one of the types of quadrilaterals. They will create a transparency that names the quadrilateral and its characteristics, and illustrates the diagonals and midpoints.

Teacher Facilitation – The teacher will lead the following discussion before distributing the “Making A Mobile Part 2” worksheet. Use the transparency created by the students to discuss the various types of quadrilaterals and their parts and definitions. Discuss using quadrilaterals as shapes for the mobile. Ask the students what possible methods could be used to find the centroid (center of gravity, center of mass) of a quadrilateral. As with the first lesson, have students discover (play with) the centroid of various quadrilaterals by balancing them on a finger, eraser or pencil point, and marking their hypothetical centroid with a push pin. In a parallelogram, the intersection of the line joining the midpoints of the opposite sides will give the centroid. In any non-specific quadrilateral, the line joining the midpoints of two opposite sides is called a **bi-median**. The centroid is the intersection of the bi-medians of a quadrilateral. Another property of a quadrilateral’s centroid is that it is also the midpoint of the segment joining the midpoints of the diagonals.
(<http://www.pballew.net/centroid.html>)

Student Application – Students will make several quadrilaterals to test the methods presented by the teacher to find the centroid. Then they will check to see if the quadrilateral is balanced.

Embedded Assessment – Check to see if students are able to draw diagonals and find midpoints. Discuss the progress with each group concerning their quadrilaterals.

Reteaching/Extension -

- For those who have not completely understood the lesson, review the terms needed.
- For those who have understood the lesson, the student will try to devise methods to find the centroids of other polygons such as quadrilaterals, pentagons, hexagons, etc. Students might also explore a variety of problems based on the $\frac{2}{3}$ - $\frac{1}{3}$ division of the median by the centroid.

Name _____ Date _____ Class _____

CENTROID LESSON I PRE-ASSESSMENT

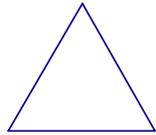
Sketch an illustration of each of the following:

1.) An Equilateral Triangle	2.) An Isosceles Triangle
3.) A Scalene Triangle	4.) A Right Triangle
5.) An Obtuse Triangle	6.) An Acute Triangle
7.) Midpoints of Sides of a Triangle	8.) A Line Segment from a Vertex to the Midpoint of the Opposite Side of a Triangle

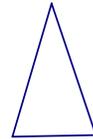
CENTROID LESSON I PRE-ASSESSMENT ANSWER SHEET

Sketch an illustration of each of the following: *(Possible Answers)*

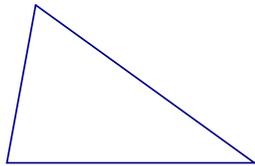
1.) An Equilateral Triangle



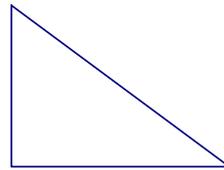
2.) An Isosceles Triangle



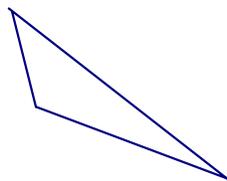
3.) A Scalene Triangle



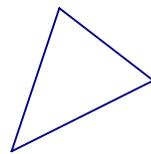
4.) A Right Triangle



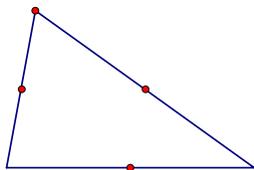
5.) An Obtuse Triangle



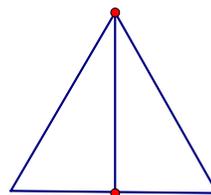
6.) An Acute Triangle



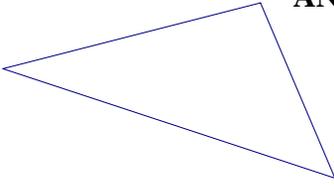
7.) Midpoints of the Sides of a Triangle



8.) A Line Segment from a Vertex to the Midpoint of the Opposite Side of a Triangle



AN INVESTIGATION: Medians of a Triangle



As a group, draw five different types of triangles (scalene, obtuse, right, isosceles, and equilateral). You should draw your triangle below. Then construct the three medians. Label the vertices of the triangle A, B, and C and midpoints of the opposite sides L, M, and N. Label the point of intersection (the centroid) X. Measure the two segments of the median formed by the centroid.

DRAWING

OBSERVATIONS

1. Did your medians intersect at exactly the same point? If not, can you explain why?

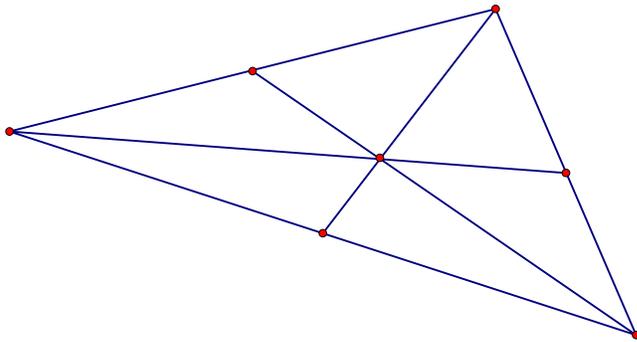
2. Of the two segments of a median formed by the centroid, what the ratio of the larger segment to the smaller segment?

3. Write the observations you have noticed about the three medians in special triangles such as the isosceles, equilateral, or right triangle?

AN INVESTIGATION: Medians of a Triangle
ANSWERS

As a group, draw five different types of triangles (scalene, obtuse, right, isosceles, and equilateral). You should draw your triangle below. Then draw or construct the three medians. Label the vertices of the triangle A, B, and C and midpoints of the opposite sides L, M, and N. Label the point of intersection (the centroid) X.

DRAWING



Answers may vary. Constructed with Sketchpad.

OBSERVATIONS

1. Did your medians intersect at exactly the same point? If not, can you explain why?

If the three medians did not intersect exactly, the construction may not have been exactly perfect. Sometimes this can be improved by keeping the pencil very sharp. A small triangle is often formed when the medians do not intersect at the centroid. This might be a “margin of error.”

2. Of the two segments of a median formed by the centroid, what the ratio of the larger segment to the smaller segment?

The ratio of the longer segment to the shorter segment is 2 to 1, or, you might say that the longer is 2 times the shorter, or the shorter is one half of the longer, or that the centroid is $\frac{2}{3}$ of the distance from the vertex to the midpoint of the opposite side.

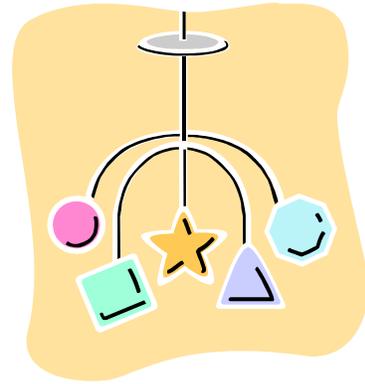
3. Write the observations you have noticed about the three medians in special triangles such as the isosceles, equilateral, or right triangle?

The medians of a triangle always meet at a point inside the triangle, unlike the altitudes and perpendicular bisectors, which can meet outside the triangle. In an isosceles triangle, the median from the vertex angle is the same segment as the altitude and the perpendicular bisector, and the centroid will always fall on this same segment. In an equilateral triangle, the centroid, incenter, circumcenter, and orthocenter are all exactly the same point.

Lesson 2

AN INVESTIGATION: MAKING A MOBILE

Our mathematics class has been asked to decorate the mathematics lab. The principal decided mobiles hanging from the ceiling would add to the room. Each mobile will consist of properly balanced geometric shapes hanging parallel to the floor. It is our job to determine what geometric shapes to use and how to make the decorations.



What geometric shape do you think we should consider making? Explain your response.

What do you need to do to make a well balanced shape? Explain your response.

MAKING TRIANGLES FOR YOUR MOBILE

Follow the directions below.

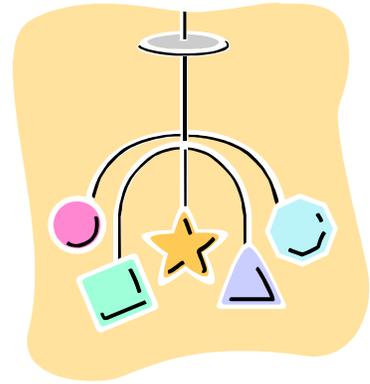
1. With your group, make a mobile. Each one should choose a triangle to make (scalene, isosceles, equilateral, right, or obtuse).
2. Collect materials to make your triangle (cardboard, fish line, coat hanger, etc.).
3. Use the method of your choice to draw or construct the medians in order to find the centroid.
4. Carefully cut out your triangle.
5. Attach string or fish line at the centroid then attach to a coat hanger.
6. Balance your shapes to make a good mobile.

Reflections

Write a summary about making your group's mobile include what your group did and how you did it.

AN INVESTIGATION: MAKING A MOBILE

Our mathematics class has been asked to decorate the new mathematics lab. The principal decided mobiles hanging from the ceiling would add to the room. Each mobile will consist of properly balanced geometric shapes hanging parallel to the floor. It is our job to determine what geometric shapes to use and how to make the decorations.



What geometric shape do you think we should consider making?

Triangles (Answers may vary.)

What do you need to do to make a well balanced shape?

To find the centroid of the triangle.

MAKING TRIANGLES FOR YOUR MOBILE

Follow the directions below.

7. With your group, make a mobile. Each one should choose a triangle to make (scalene, isosceles, equilateral, right, or obtuse).
8. Collect materials to make your triangle (cardboard, fish line, coat hanger, etc.).
9. Use the method of your choice to draw or construct the medians in order to find the centroid.
10. Carefully cut out your triangle.
11. Attach string or fish line at the centroid then attach to a coat hanger.
12. Balance your shapes to make a good mobile.

Reflections

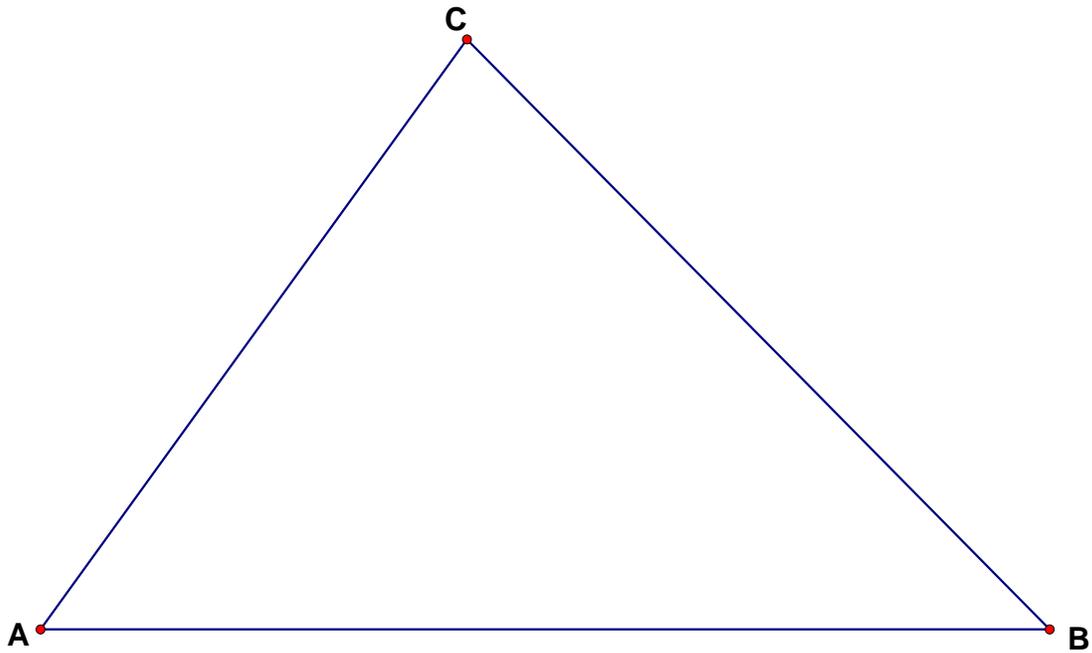
Write a summary about making your group's mobile include what your group did and how you did it.

(Answers may vary.)

Name _____ Date _____ Class _____

CENTROID LESSON II PRE-ASSESSMENT

In the triangle ABC given below, do a free-hand sketch and list the steps you would do to locate the centroid.

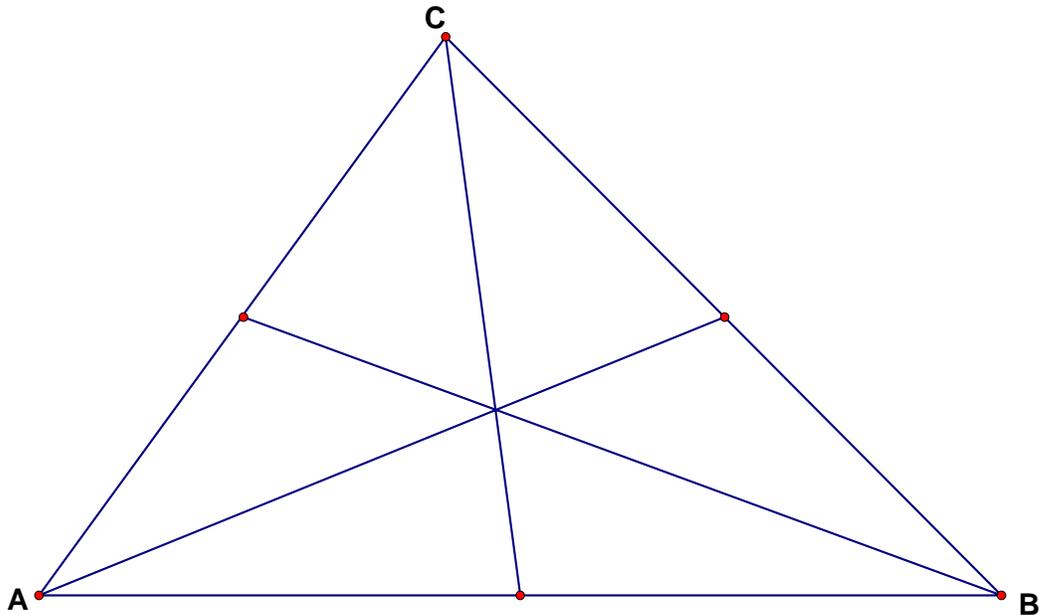


Write the steps to find the centroid:

Name _____ Date _____ Class _____

CENTROID LESSON II PRE-ASSESSMENT **ANSWER SHEET**

In the triangle ABC given below, do a free-hand sketch and list the steps you would do to locate the centroid.



Write the steps to find the centroid:

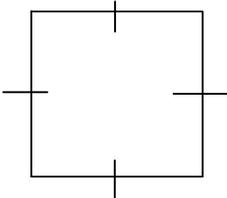
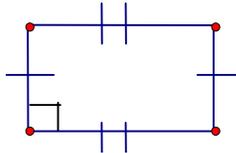
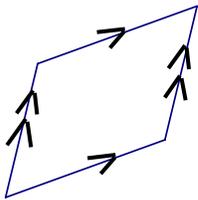
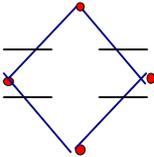
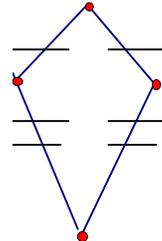
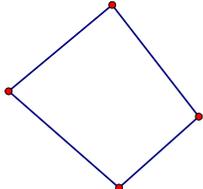
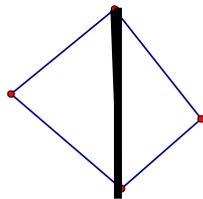
Answers may vary

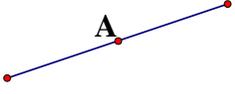
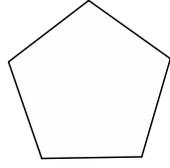
1. Find the midpoint of each side of the triangle.
2. Draw a segment joining the vertex of the triangle to the midpoint of the opposite side.
3. The point of intersection of the three segments is the centroid.

Name _____ Date _____ Class _____

CENTROID LESSON III PRE-ASSESSMENT

In each of the following, name the figure or the concept that is illustrated.

1.  _____	2.  _____
3.  _____	4.  _____
5.  _____	6.  _____
7.  _____	8.  _____
9.	10.

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Name _____ Date _____ Class _____

CENTROID LESSON III PRE-ASSESSMENT ANSWER SHEET

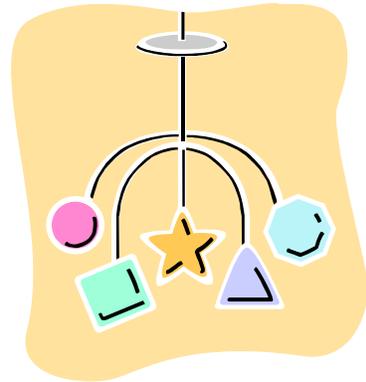
In each of the following, name the figure or the concept that is illustrated.

1. SQUARE	2. RECTANGLE
3. TRAPEZOID	4. PARALLELOGRAM
5. RHOMBUS	6. KITE
7. QUADRILATERAL	8. DIAGONAL
9. MIDPOINT	10. PENTAGON

Lesson 3

AN INVESTIGATION: MAKING A MOBILE PART 2

The mobiles our mathematics class created with various triangles are a big hit. The principal would like us to add other geometric shapes such as quadrilaterals to our mobiles. Remember each mobile has properly balanced geometric shapes hanging parallel to the floor. Find the centroid of various quadrilaterals to add to the mobile.



Name at least four quadrilaterals that you would like to add to the mobiles.

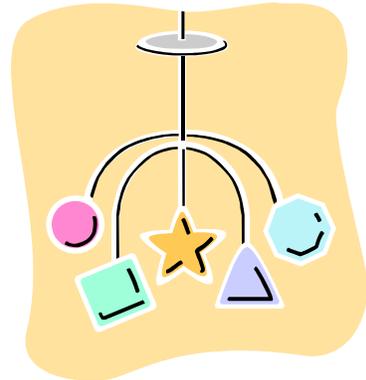
Draw one of the quadrilaterals in the list above on cardboard. Cut it out.
Use one of the following methods to find the centroid of a quadrilateral.

- In a parallelogram, the intersection of the line joining the midpoints of the opposite sides will give the centroid.
- In any non-specific quadrilateral, the line joining the midpoints of two opposite sides is called a **bi-median**. The centroid is the intersection of the bi-medians of a quadrilateral. Another property of a quadrilateral's centroid is that it is also the midpoint of the segment joining the midpoints of the diagonals. (<http://www.pballew.net/centroid.html>)

Extension: Try other polygons to find their balance points experimentally or theoretically.

AN INVESTIGATION: MAKING A MOBILE PART 2

The mobiles our mathematics class created with various triangles are a big hit. The principal would like us to add other geometric shapes such as quadrilaterals to our mobiles. Remember each mobile has properly balanced geometric shapes hanging parallel to the floor. Find the centroid of various quadrilaterals to add to the mobile.

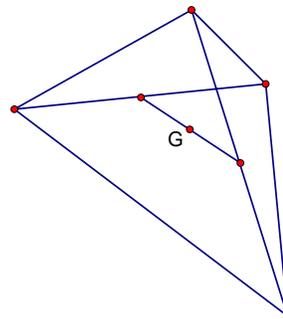
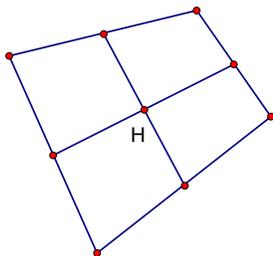


Name at least four quadrilaterals that you would like to add to the mobiles.

_____ square, rectangle, parallelogram, kite, rhombus, etc. _____

Draw one of the quadrilaterals in the list above on cardboard. Cut it out.
Try using one of the following methods to find the centroid of a parallelogram (quadrilateral). You may devise your own method.

- In a parallelogram, the intersection of the line joining the midpoints of the opposite sides will give the centroid.
- In any non-specific quadrilateral, the line joining the midpoints of two opposite sides is called a **bi-median**. The centroid is the intersection of the bi-medians of a quadrilateral. Another property of a quadrilateral's centroid is that it is also the midpoint of the segment joining the midpoints of the diagonals. (<http://www.pballew.net/centroid.html>)

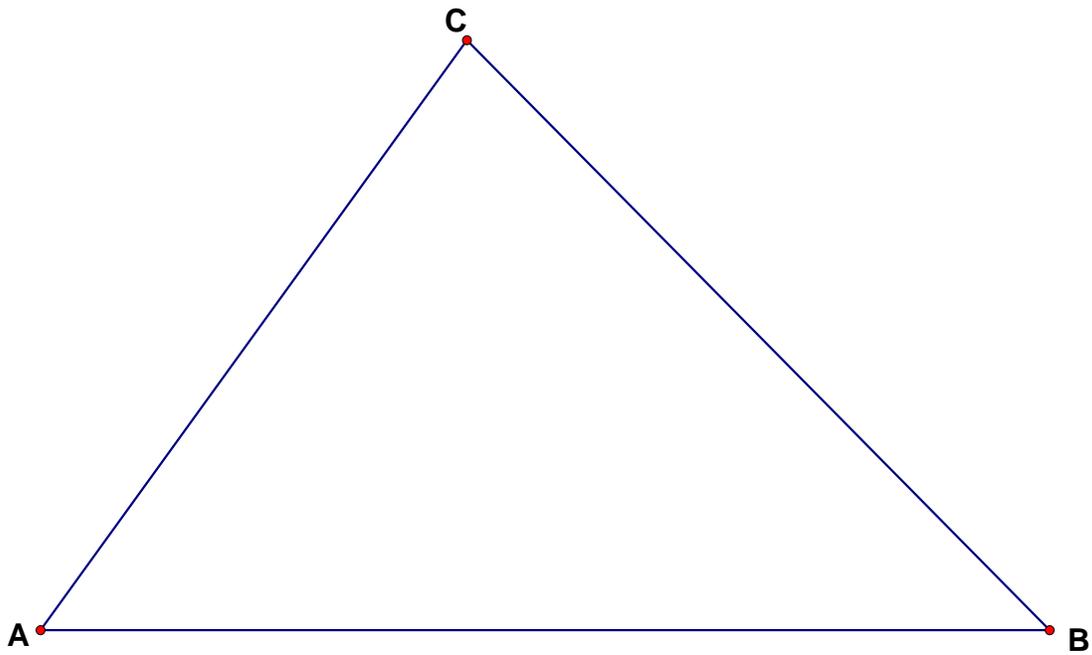


Extension: Try other polygons to find their balance points experimentally or theoretically.

Name _____ Date _____ Class _____

SUMMATIVE ASSESSMENT

In the triangle ABC given below, find the centroid and list the steps you would do to locate the centroid.

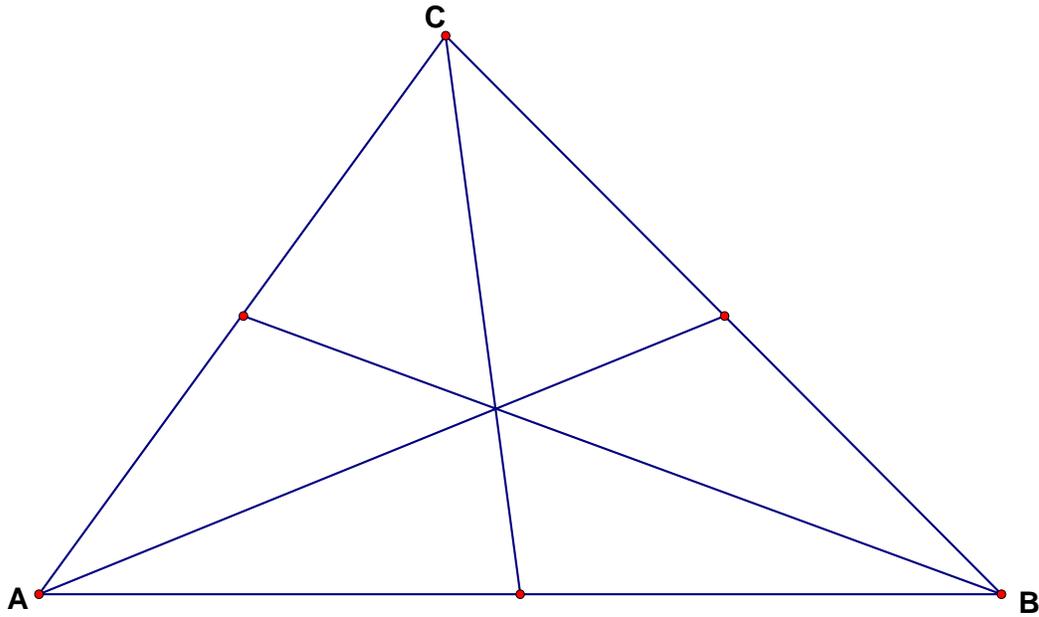


Write the steps to find the centroid:

Name _____ Date _____ Class _____

SUMMATIVE ASSESSMENT

In the triangle ABC given below, find the centroid and list the steps you would do to locate the centroid.



Write the steps to find the centroid:

Answers may vary

- 1. Find the midpoint of each side of the triangle.*
- 2. Draw a segment joining the vertex of the triangle to the midpoint of the opposite side.*
- 3. The point of intersection of the three segments is the centroid.*

Authors:

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