Title: The Zit Theorem: Changing the Complexion of Trapezoids

Brief Overview:

In this unit students will use computers to: 1) investigate and explain relationships involving lengths of bases and the median of a trapezoid, 2) use angle measures to determine whether or not the median is parallel to the base, and 3) discover the connection between a trapezoid median and a triangle midline.

Link to Standards:

• **Problem Solving**	Students will use critical thinking and *The Geometer's Sketchpad*™ to discover the relationship between the bases and the median of a trapezoid.
• **Communication**	Students will explain in writing the relationships and theorems discovered in their trapezoid investigations including animation.
• **Reasoning**	Students will use trapezoid facts to determine logically the relationship between the median and the two bases of a trapezoid and extend this relationship to a triangle's midline and sides.
• **Patterns and Relationships**	Students will use data collected in tables to discover relationships between the median of a trapezoid and the two bases. A relationship between a trapezoid and a triangle will be observed through the use of animation.
• **Geometry**	Students will demonstrate their ability to construct and measure segments and angles using *The Geometer's Sketchpad™*.

Grade/Level:

Grades 9-12, Geometry

Duration/Length:

This lesson will take 2-3 days

Prerequisite Knowledge:

- Definition of a trapezoid and facts about parallel lines and related angles
- Definition of a median and a base of a trapezoid
- Basic knowledge of *The Geometer's Sketchpad*™
- Definition of a midpoint

Objectives:

Students will be able to:

- inductively prove the size of the median of a trapezoid.
- discover relationships between trapezoid bases and the median.
- define a midline of a triangle.
- discover the length of a midline of a triangle.

Materials/Resources/Printed Materials:

- Computer
- *The Geometer's Sketchpad*™ Software
- Calculators
- Student Worksheets

Development/Procedures:

Throughout this unit students will work cooperatively in groups. Students will complete a worksheet and an extended activity. The worksheet involves an observation of the characteristics of the median of a trapezoid. From these observations using *The Geometer's Sketchpad*™, the student is to discern and classify properties of trapezoids.

Once students are split into heterogeneous groups of 3 (if the teacher desires), they are assigned to computer lab stations such that one is a reader, one is the computer operator, and the other is the facilitator and liaison between the first two. Student groups should progress step by step, following the included worksheet in order to discover the relationships through which the activity will lead them. If students complete the worksheet, they can progress to the optional animation activities; these activities allow faster-moving groups to challenge themselves while the other groups complete their work. Note that the second of the two animations is challenging due to the limitations on the paths of *Sketchpad*'s animate command.

Evaluation:

Students should follow up this activity by writing a short, coherent essay summarizing their procedures and findings the day following the completion of the assignment. The discovery nature of this activity lends itself perfectly to performance assessment of any kind. If the teacher desires to construct an assessment based on this activity, there are a number of possible essay questions included in the text of the worksheet. Simply choose one of those and describe specifically what should be included in a correct answer. For example, one question might be "Why does it conceptually make sense for the median's length to be the average of the bases' length? Include in your answer how the definition of median plays a role."

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Name: ______________________________

Procedure: You will use this lab in conjunction with *The Geometer's Sketchpad*™ in order to discern and classify properties of trapezoids. In particular, you will be observing the behavior of the median of a trapezoid. Answer the questions completely as you progress rather than waiting until the end. So, grab your Oxy™ and your pen, and let's begin.

1. This investigation will center around trapezoids. To begin, we ask ourselves a fundamental question: what are trapezoids? What differentiates them from other quadrilaterals? from parallelograms?

2. Construct a line segment AB using your computer--this will be a base of your trapezoid. Knowing what a trapezoid is (from question one above), you know that the other base of the trapezoid will be _______________ to the first. Construct a point outside your original base and through it, construct another base (its length is unimportant). In order to make your drawing neat (if you are a *Sketchpad* pro) you may want to construct a segment on that new line and hide the line as well.

3. In order to form a trapezoid now, name the second base CD and connect the vertices so that the trapezoid's name is ABCD. Note that this doesn't happen automatically! Points B and C will have to be consecutive and adjacent. The legs of the trapezoid should now be constructed (they are segments AD and BC. What special classification would this trapezoid have if the legs were congruent at this stage? (Note that they do not have to be.)

4. At this point, you should be looking at your trapezoid and it should be looking at you. Smile at it. Make pleasant conversation. (Make sure that any and all conversation is two-dimensional, or it may not understand!)

5. It is now time to bring the median into the picture. What is the median of a trapezoid and how is it formed? Note that the median of a trapezoid is really <u>really</u> different from the median of a triangle. Why in the world would geometers choose to give these two different concepts the same name? Do they have anything in common? Highlight the stark differences as well.

6. In order to construct the median, we are going to have to find particular points on the legs (not the bases) of the trapezoids, and your answer to number five should identify those points. Construct them and the segment that connects them--Viva Median! Name the point on segment AD "M" and name the other endpoint N.

7. What observations can you make immediately about the diagram based on the nature of a trapezoid and the nature of a median? Be complete and thorough--rack your brain and list as many as possible. Hint: think about the interior angles of the trapezoid.

8. Did you figure anything interesting out about the angles in the trapezoid? Don't read on unless you have. If you have, it was probably that angle D and angle A must be supplementary. Check this using *Sketchpad*. More importantly, why is it true? Hint: consider the properties of parallel lines (bases) being cut by a transversal (AD).

9. Now, make the quantum leap and draw some conclusions about segment MN using the same reasoning techniques (measuring with *Sketchpad* and justifying geometrically). This should give you the first major property of trapezoidal medians, regarding their relationship to the bases of the trapezoid.

Conclusion #1: The median of a trapezoid is ___________________________.

10. Time to head for the other major property of medians. Are you ready? Of course! You can't stop us--you can only hope to contain us. Anyway...use *Sketchpad*'s Star Trek-like technology to determine the length of AB using the measure menu. Do the same for CD and position both bits of information on the screen so you can see them and your diagram at the same time. You may want to set your units to tenths; in fact, that's probably a good idea, so do it already for goodness' sakes!

11. Now, use the measure command to determine the length of MN. Can you divine a relationship between the lengths of the bases and the length of the median? Perhaps putting this on pencil and paper for a second will help--let's make a chart with the tabulate command. Highlight the lengths of AB, CD, and MN on the screen holding the <shift> key and choose tabulate. Copy those values into the first row of your table on the next page.

AB	CD	MN

12. By clicking and dragging one of the trapezoids vertices, change the length of base AB, noting that the other measurements on the screen will change also. Double-click on the table in order to get the new, updated values; copy these into your table. Try changing the length of CD and MN; record those values in the table as well. Now, using the table, try to discern the relationship between the lengths of the bases of the trapezoid and the length of the median.

13. Record your observation below.

Conclusion #2: The length of the median is equal to ________________________

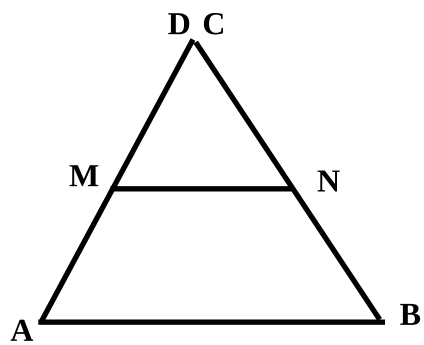
____.

14. Enter the formula you derived for number 13 in the *Sketchpad*'s calculate option and add it to your table. It should equal MN whenever you grab vertices and change the size of the trapezoid. If you can't figure it out, here's a big hint: arithmetic mean. What is another way of saying arithmetic mean? Average. Why does this word make sense from a conceptual view? In other words, why would that seem to make sense from looking at the picture?

15. Now we know a lot about trapezoid's medians, but you may be puzzled about the title of the lesson...the Zit Theorem.... Imagine, if you will, a trapezoid oriented in such a way that the smaller of the two bases appears "above" the other. Imagine this as a skin blemish whose top base is also the surface of the skin; all have experienced this "blind pimple" (a pimple with no head) and few remember it with pleasure.

Any teenager with complexion issues can tell you the best, though not always most hygenically-friendly, way to eliminate a blemish is the direct attack...the squeeze play. Imagine now your trapezoid being squeezed by imaginary fingers on each of its legs. "To what end?" you may be wondering, perhaps a bit sick to your stomach because of the imagery. To this end--the top base of the trapezoid becomes the head of the zit...a point. In other words, the top base of the trapezoid is no longer really a base at all, as it is no longer a line segment; the endpoints of the "segment" now overlap (coincide) and the top base becomes a single solitary point (sure to appear on school picture day).

Adjust your picture in *Sketchpad* so that the endpoints of the smaller base overlap and coincide. You should have a picture that resembles the drawing below.



What we have now is no longer really a trapezoid at all...we have collapsed it into a triangle, but note that our two conclusions about the median of a trapezoid hold true. One other important observation is that we cannot call this segment MN the median in the triangle ADB (or ACB for that matter). Why not?

16. "Median" is already a term we use in conjunction with triangles, as mentioned before, so we will use a different term--the "midline." In the above diagram, MN is the midline of triangle ADB. The Zit Theorem, then, is a generalization of our conclusions about the median of a trapezoid to the midline of a triangle. Minor modifications are necessary in the wording, so make these below. Conclusion three is the generalization of conclusion one and similarly for conclusion four and conclusion two.

The Zit Theorem:

Conclusion #3: The midline of a triangle is ______________________________.

Conclusion #4: The length of the midline of a triangle is __________________

____.

17. Congratulations, Jedi. You have done well, and your geometric training will be complete the day you can take this compass from my hand ....

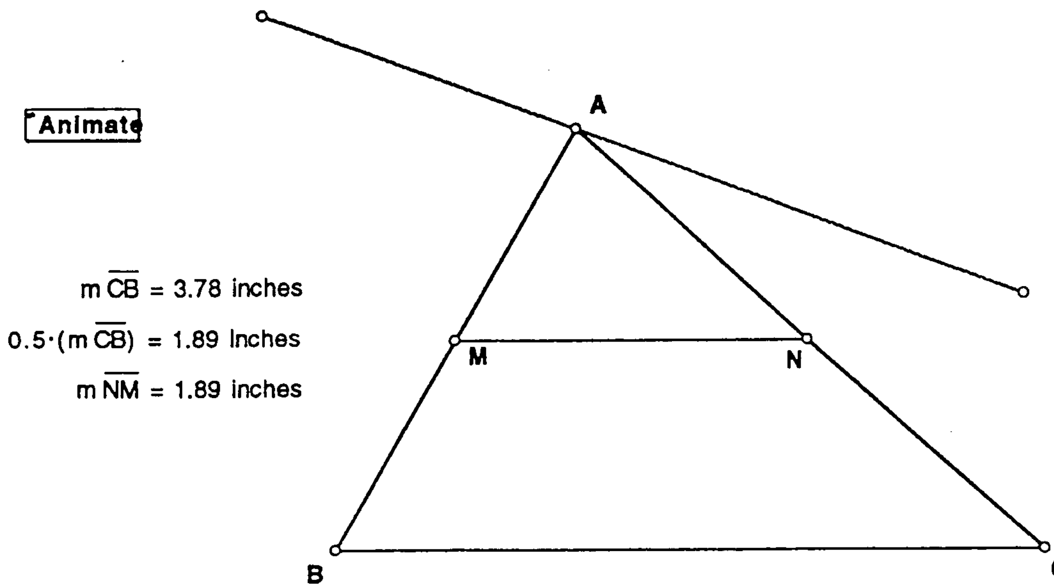
Challenge Animations for the Zit Theorem Activity

The following animation challenges will enhance your understanding of the midline of a triangle, should you choose to undertake them. A basic understanding of the animation capabilities of *The Geometer's Sketchpad*™ is required.

1. Construct a triangle ABC such that BC forms the base parallel to an included midline MN. (Clearly, angle A would be opposite both the midline and BC). Construct a segment near point A and instruct A to travel back and forth along it. Show, with on-screen calculations, that the midline remains constant as the triangle shifts. Try varying slopes of segments upon which A travels. Explain geometrically why this is true.

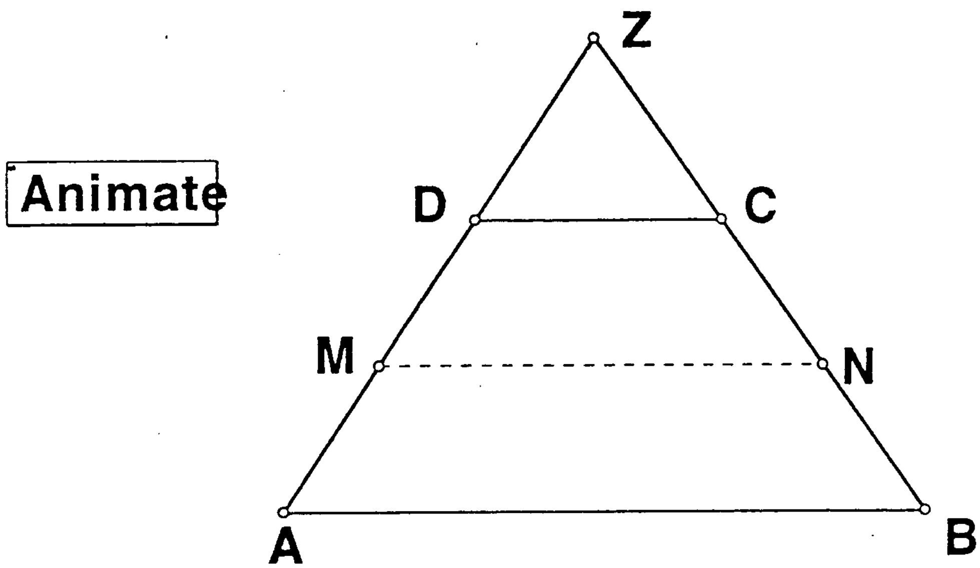
2. Construct a trapezoid ABCD but extend the legs above the trapezoid so that they intersect at a point Z (for Zit Theorem). Animate your diagram so that the top base of the trapezoid rises, shrinking as it approaches Z; at this point the trapezoid has collapsed into a triangle and the Zit Theorem can be applied. For additional points, construct the median MN of trapezoid ABCD and draw conclusions about your work.

Solution to Challenge Animation Number One



The first order of business is to construct triangle ABC. Construct the midpoints of sides AB and AC, labeling them M and N respectively. Finally, construct a segment that appears to overlap point A as you construct it. Instruct point A to animate along that segment (by first clicking on A and then holding <shift> as you click on the unlabeled segment; then choose action button --> animation from the Edit menu).

Solution to Challenge Animation Number Two



In order to construct the above, create triangle ABZ as shown. Then choose a point D on segment AZ. Through point D, construct a parallel line to segment AB. Find the intersection of that parallel line and BZ and label it C. Hide the newly-found parallel line and construct segment CD. Instruct point D to animate along segment AZ--not segment DZ or an error will result. For additional grins, construct segments AD and BC when all else is done and find their midpoints; label them M and N respectively. Constructing segment MN and displaying it as a dotted line visually exhibits the median of trapezoid ABCD which approaches the midline of triangle ABZ as segment DC approaches point Z.