Title: Quadrilaterals Still Aren't Just Squares! – Part II – The Sequel

Brief Overview:

This is Part II in a seven lesson series studying quadrilaterals, including trapezoids, parallelograms, rhombi, rectangles, squares, kites and mid-point quadrilaterals. Students will investigate the characteristics of each group of quadrilaterals using compass and straight edge, Geometer's Sketchpad software, and other optional hand-held learning tools.

NCTM Content Standards/National Science Education Standards:

Geometry:

Students should analyze properties of and determine attributes of two-and three-dimensional geometric shapes

Students should explore and develop mathematical arguments about geometric relationships (including congruence) among classes of two-dimensional geometric objects, make conjectures about them, and solve problems involving them.

Students should establish the validity of geometric conjectures using deduction, prove theorems, and critique arguments made by others.

Students should draw and construct representations of two-and three- dimensional geometric objects using a variety of tools.

Students should specify locations and describe spatial relationships using coordinate geometry and other representational systems.

Algebra:

All students should be able to represent and analyze mathematical situations and structures using algebraic symbols.

Grade/Level:

Grades 8-12 Geometry

Duration/Length:

3-4 class periods of 45-60 minutes in length

Student Outcomes:

Students will:

Create coordinate proofs to identify types of quadrilateral and to prove particular properties of special quadrilaterals using graph paper and the graph functions of Geometer's Sketchpad software.

Compare and contrast groups of quadrilaterals possessing special characteristics.

Create and manipulate drawings of quadrilaterals using Geometers Sketchpad software to discover special relationships in specific quadrilaterals.

Materials and Resources:

- Geometer's Sketchpad software
- Student worksheets (activity sheets, homework sheets and assessments)
- Graph paper
- Teacher notes and lesson plans for using GSP software demonstrations
- Textbook

Summative Assessments:

A short pre-assessment will be included in the plan for day one.

A partial chapter assessment will be included after lesson three and a comprehensive final assessment after lesson seven.

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Lesson 4: Using Coordinate Geometry with Parallelograms

Brief Overview:

In this lesson, students will use Geometer's Sketchpad to prove that a quadrilateral is a parallelogram using coordinate geometry.

Teacher Facilitation:

Present the theorems for proving a quadrilateral is a parallelogram. Use Student Resource Sheet 4.1 to make an overhead. Do activity in Preassessment/Launch. Use Sketchpad to complete Student Resource Sheet 4.2 on how to make a parallelogram on a coordinate plane.

Preassessment/Launch:

Put students into groups. Students will plot the points A (-1, -2), B (6,0), C (0,2), and D (7,4) on graph paper. Connect the points together to make a parallelogram. Have students discuss various methods to prove that the figure is a parallelogram. Students should list what measurements would be necessary to prove the figure is a parallelogram and how to find those measurements. Review slope, midpoint, and distance formulas.

Student Application:

Students will complete Student Resource Sheet 4.3

Embedded Assessment:

Monitor progress of the students by reviewing their responses on Student Resource Sheets 4.2 and 4.3.

Reteaching/Extension:

Use examples from Student Resource Sheet 4.4 to review methods for proving a quadrilateral is a parallelogram when coordinates for vertices are not given. Diagrams have congruency marks and measurements given instead.

Proving Quadrilaterals are Parallelograms

Show that both pairs of opposite sides are parallel.

Show that both pairs of opposite sides are congruent.

Show that one pair of opposite sides are congruent and parallel.

Show that both pairs of opposite angles are congruent.

Show that one angle is supplementary to both consecutive angles.

Show that the diagonals bisect each other.

Name _____ Date _____

Coordinate Geometry

Students will use Geometer's Sketchpad to draw a parallelogram on a coordinate plane and prove that it is a parallelogram.

Part A: Graphing a Parallelogram.

1. Under the Graph menu, select Show Grid
2. Graph the following points by using Plot Points option in the Graph menu: A (-1, -2), B (6,0), C (0,2), and D (7,4)
3. Draw $\overline{AC}$, $\overline{CD}$, $\overline{DB}$, and $\overline{AB}$
4. Draw the diagonals $\overline{AD}$ and $\overline{BC}$. Draw a point at the intersection of the diagonals and label it E.

Part B: Proving Parallelograms Using Coordinate Geometry

1. Use the slope formula to prove the figure above is a parallelogram by showing its opposite sides are parallel. Show your work to calculate the slopes of $\overline{AC}$, $\overline{CD}$, $\overline{DB}$, and $\overline{AB}$. Verify your answers by measuring the slopes using Sketchpad. Remember that your slopes are in fraction form and the Sketchpad slopes are in decimal form.

2. Use the distance formula to prove the figure is a parallelogram by showing opposite sides are congruent. Show your work to calculate the lengths of $\overline{AC}$, $\overline{CD}$, $\overline{DB}$, and $\overline{AB}$. Verify your answers by measuring the sides using Sketchpad.

3. Use the midpoint formula to prove the figure is parallelogram by showing that the diagonals bisect each other. Show your work to find the midpoints of $\overline{AD}$ and $\overline{BC}$. The midpoint of each diagonal should have the same coordinates. Verify that E is the midpoint of both diagonals by using Sketchpad.

4. Prove the figure is parallelogram by showing that the opposite angles are congruent. Use Sketchpad to measure pairs of opposite angles and list your results.

5. Using your results from #1-4 explain two other methods to prove that the figure is a parallelogram.

Name _____Answer Key_____ Date _____

Coordinate Geometry

Students will use Geometer's Sketchpad to draw a parallelogram on a coordinate plane and prove that it is a parallelogram.

Part A: Graphing a Parallelogram.

1. Under the Graph menu, select Show Grid
2. Graph the following points by using Plot Points option in the Graph menu: A(-1, -2), B(6,0), C(0,2), and D(7,4)
3. Draw $\overline{AC}$, $\overline{CD}$, $\overline{DB}$, and $\overline{AB}$
4. Draw the diagonals $\overline{AD}$ and $\overline{BC}$. Place a point at the intersection of the diagonals and label it E.

Part B: Proving Parallelograms Using Coordinate Geometry

1. Use the slope formula to prove the figure above is a parallelogram by showing its opposite sides are parallel. Show your work to calculate the slopes of $\overline{AC}$, $\overline{CD}$, $\overline{DB}$, and $\overline{AB}$. Verify your answers by measuring the slopes using Sketchpad. Remember that your slopes are in fraction form and the Sketchpad slopes are in decimal form.

ANSWERS:

Slope of $\overline{AB}$: $\frac{0-{}^{-}2}{6-{}^{-}1} = \frac{2}{7}$ ($\approx$0.29)

Slope of $\overline{CD}$: $\frac{4-2}{7-0} = \frac{2}{7}$ ($\approx$0.29)

Slope of $\overline{AC}$: $\frac{2-{}^{-}2}{0-{}^{-}1} = \frac{4}{1} = 4$

Slope of $\overline{BD}$: $\frac{4-0}{7-6} = \frac{4}{1} = 4$

2. Use the distance formula to prove the figure is a parallelogram by showing opposite sides are congruent. Show your work to calculate the lengths of $\overline{AC}$, $\overline{CD}$, $\overline{DB}$, and $\overline{AB}$. Verify your answers by measuring the sides using Sketchpad.

ANSWERS:

AB = $\sqrt{(6-{}^{-}1)^2+(0-{}^{-}2)^2} = \sqrt{7^2+2^2} = \sqrt{53}$ ($\approx$7.28)

CD = $\sqrt{7-0)^2+(4-2)^2} = \sqrt{7^2+2^2} = \sqrt{53}$

AC = $\sqrt{(0-{}^{-}1)^2+(2-{}^{-}2)^2} = \sqrt{1^2+4^2} = \sqrt{17}$ ($\approx$4.12)

BD = $\sqrt{(7-6)^2+(4-1)^2} = \sqrt{1^2+4^2} = \sqrt{17}$

3. Use the midpoint formula to prove the figure is parallelogram by showing that the diagonals bisect each other. Show your work to find the midpoints of $\overline{AD}$ and $\overline{BC}$. The midpoint of each diagonal should have the same coordinates. Verify that E is the midpoint of both diagonals by using Sketchpad.

ANSWERS:

midpoint of $\overline{AD}$: $\left(\frac{{}^{-}1+7}{2}, \frac{{}^{-}2+4}{2}\right) = \left(\frac{6}{2}, \frac{2}{2}\right) = (3,1)$

midpoint of $\overline{BC}$: $\left(\frac{0+6}{2}, \frac{0+2}{2}\right) = \left(\frac{6}{2}, \frac{2}{2}\right) = (3,1)$

Note: Students could also verify that the diagonals are bisected by finding the length of each of the 4 halves of the diagonals.

4. Use Sketchpad to prove the figure is parallelogram by showing that the opposite angles are congruent. Measure the pairs of opposite angles and list your results.

ANSWERS:

m∠CAB = 60°, m∠CDB = 60°, m∠ACD = 120°, and m∠ABD = 120°

Students will measure the interior angles of the parallelogram, and should notice they have two pairs of opposite angles congruent.

5. Using your results from #1-4 to explain two other methods to prove that the figure is a parallelogram.

Students can prove that a quadrilateral is a parallelogram by proving consecutive angles are supplementary, OR by proving that one pair of sides is both parallel and congruent.

(See copy of Sketchpad graph on next page.)

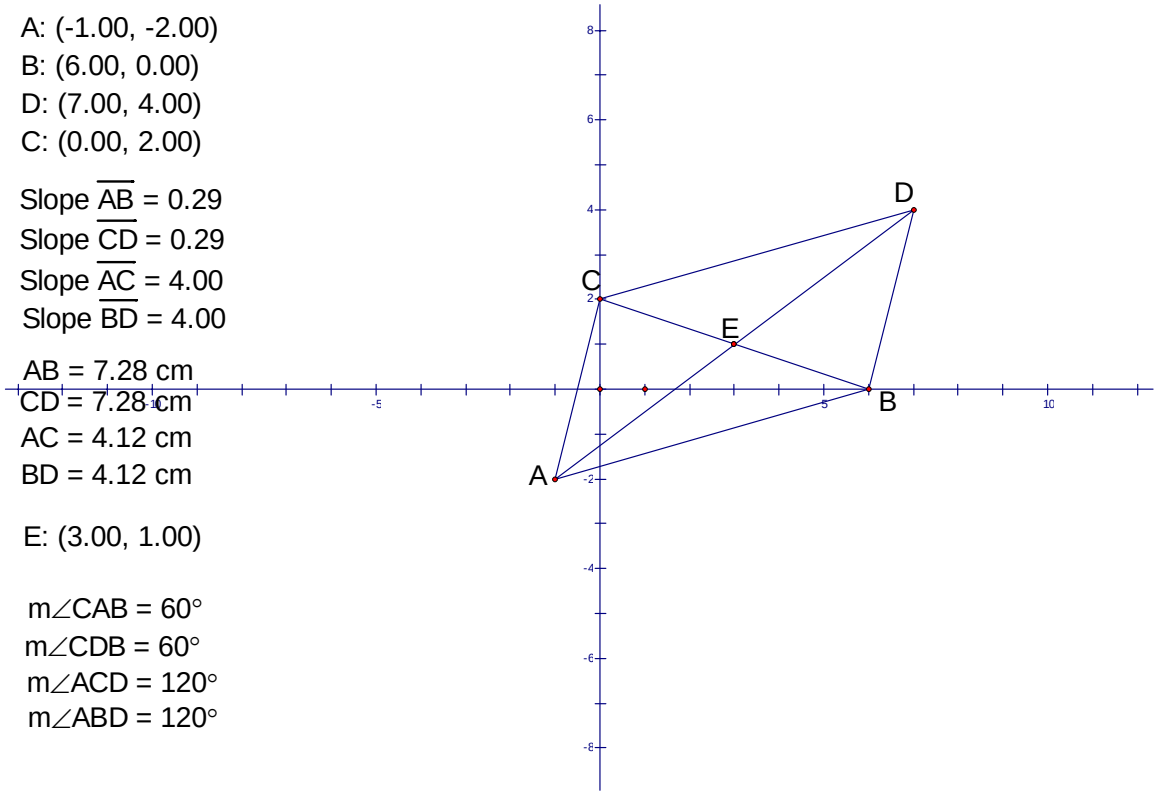
A: (-1.00, -2.00)
B: (6.00, 0.00)
D: (7.00, 4.00)
C: (0.00, 2.00)

Slope $\overline{AB}$ = 0.29
Slope $\overline{CD}$ = 0.29
Slope $\overline{AC}$ = 4.00
Slope $\overline{BD}$ = 4.00

AB = 7.28 cm
CD = 7.28 cm
AC = 4.12 cm
BD = 4.12 cm

E: (3.00, 1.00)

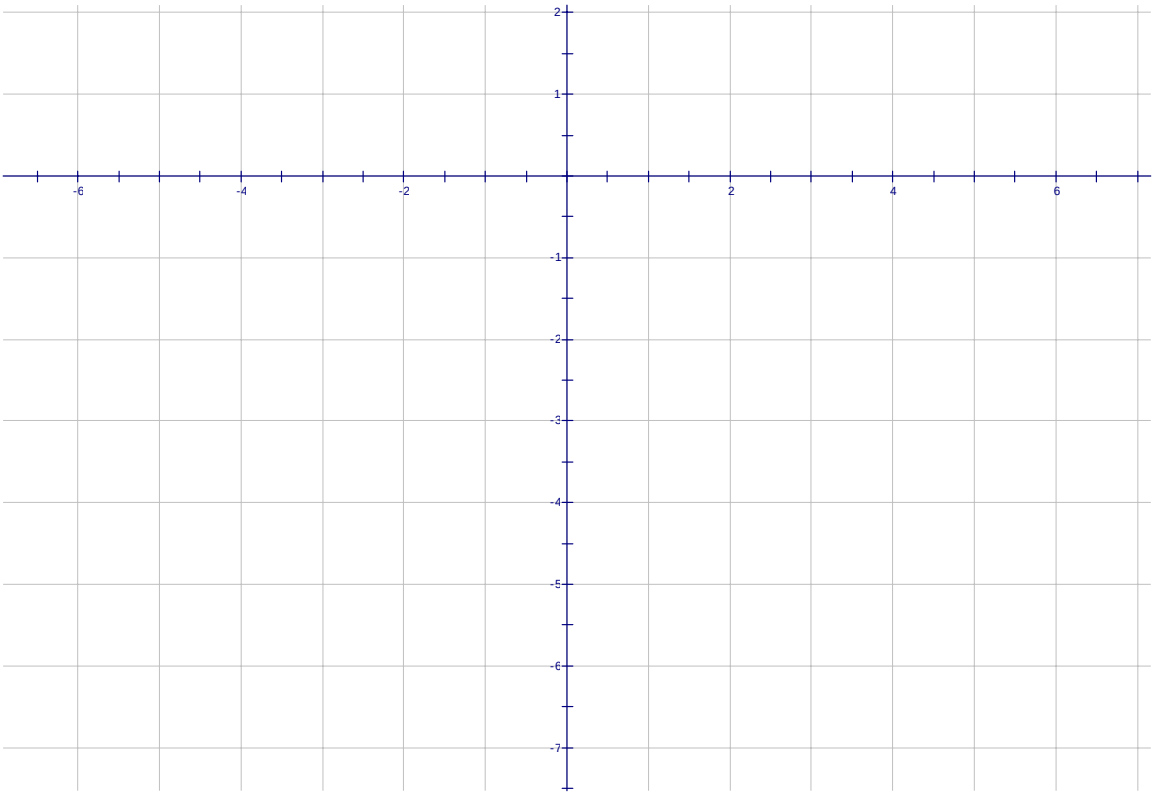
m∠CAB = 60°
m∠CDB = 60°
m∠ACD = 120°
m∠ABD = 120°



Name _____ Date _____

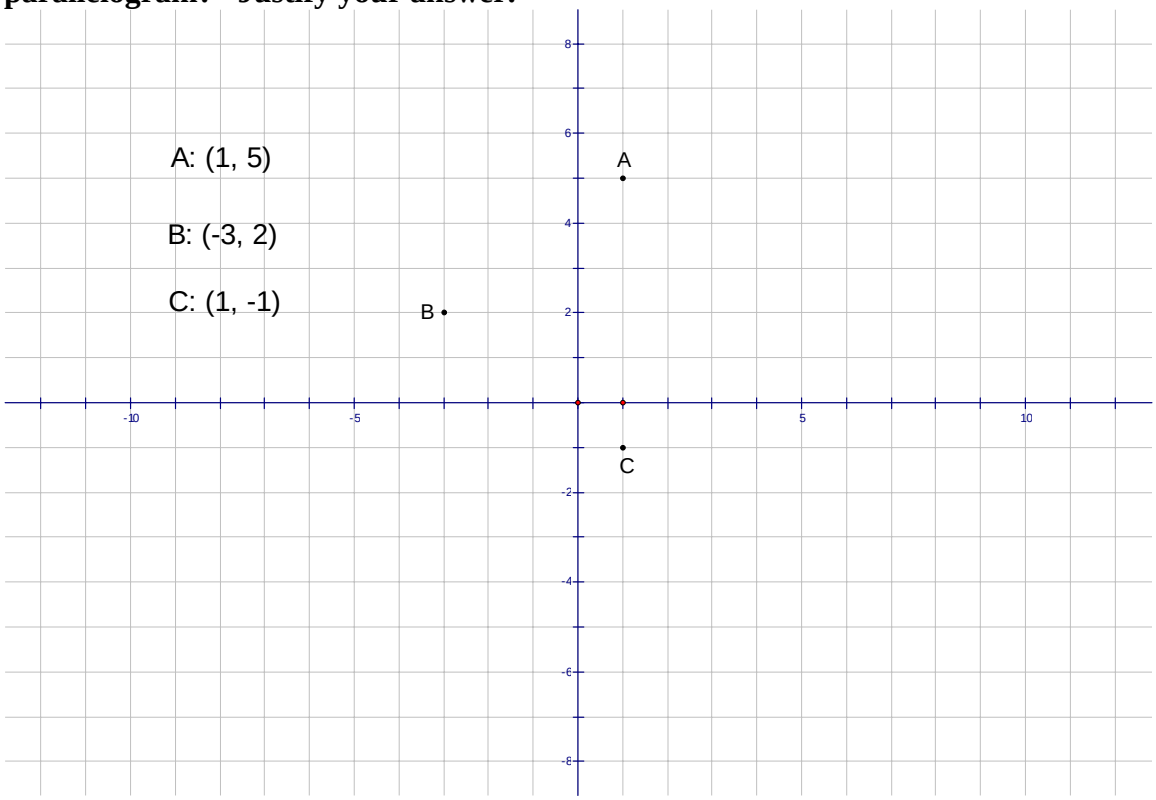
Coordinate Geometry and Parallelograms

1. a. Plot the points: A(-4, 1), B(3, 0), C(5, -7), and D(-2, -6). Connect the points to make a quadrilateral.



b. Use two methods to prove that the figure above is a parallelogram. Show all work.

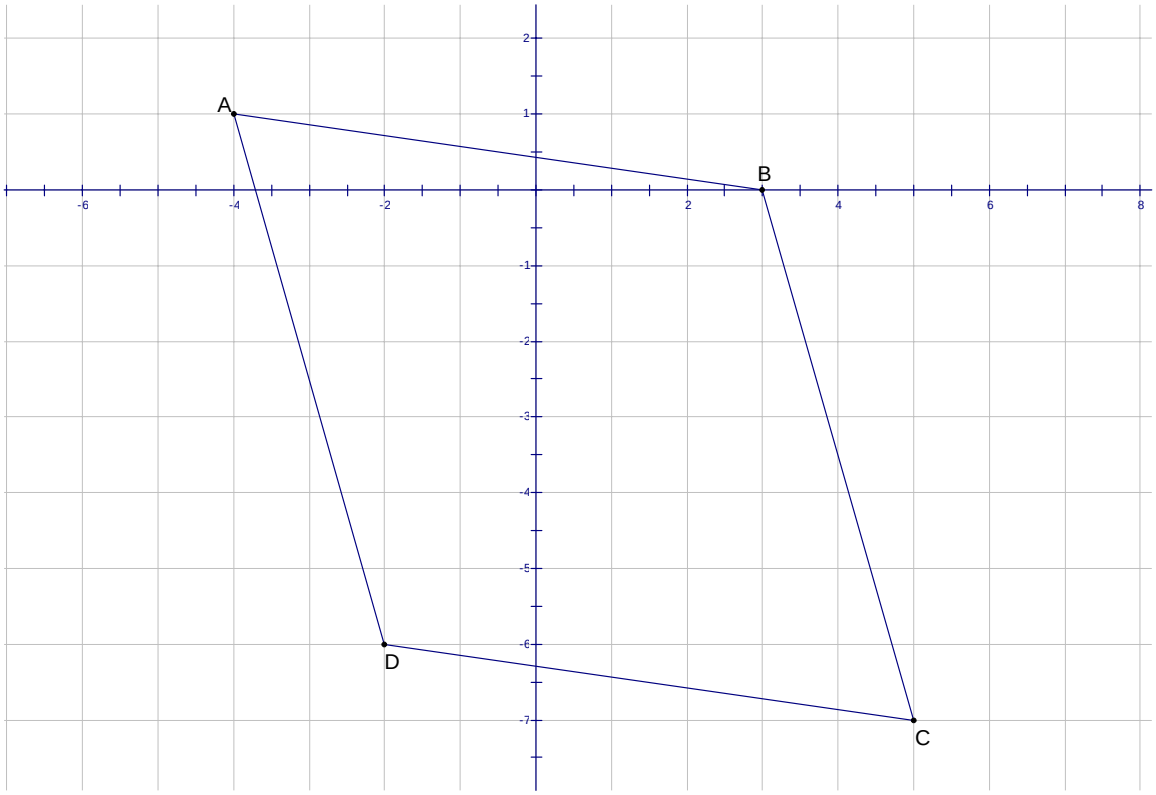
2. Where would you plot point D so that the four points would form a parallelogram? Justify your answer.



Name___Answer Key_____ Date _____

Coordinate Geometry and Parallelograms

1. a. Plot the points: A(-4, 1), B(3, 0), C(5, -7), and D(-2, -6). Connect the points to make a quadrilateral.



c. Use two methods to prove that the figure above is a parallelogram. Show all work.

1. Show both pairs of opposite sides parallel.

Slope $\overline{AB} = -\frac{1}{7}$ **Slope** $\overline{DC} = -\frac{1}{7}$

Slope $\overline{AD} = -\frac{7}{2}$ **Slope** $\overline{CB} = -\frac{7}{2}$

2. Show both pairs of opposite sides congruent.

AB = $\sqrt{50} = 5\sqrt{2} = 7.07$ **DC =** $\sqrt{50} = 5\sqrt{2} = 7.07$

AD = $\sqrt{53} = 7.28$ **BC =** $\sqrt{53} = 7.28$

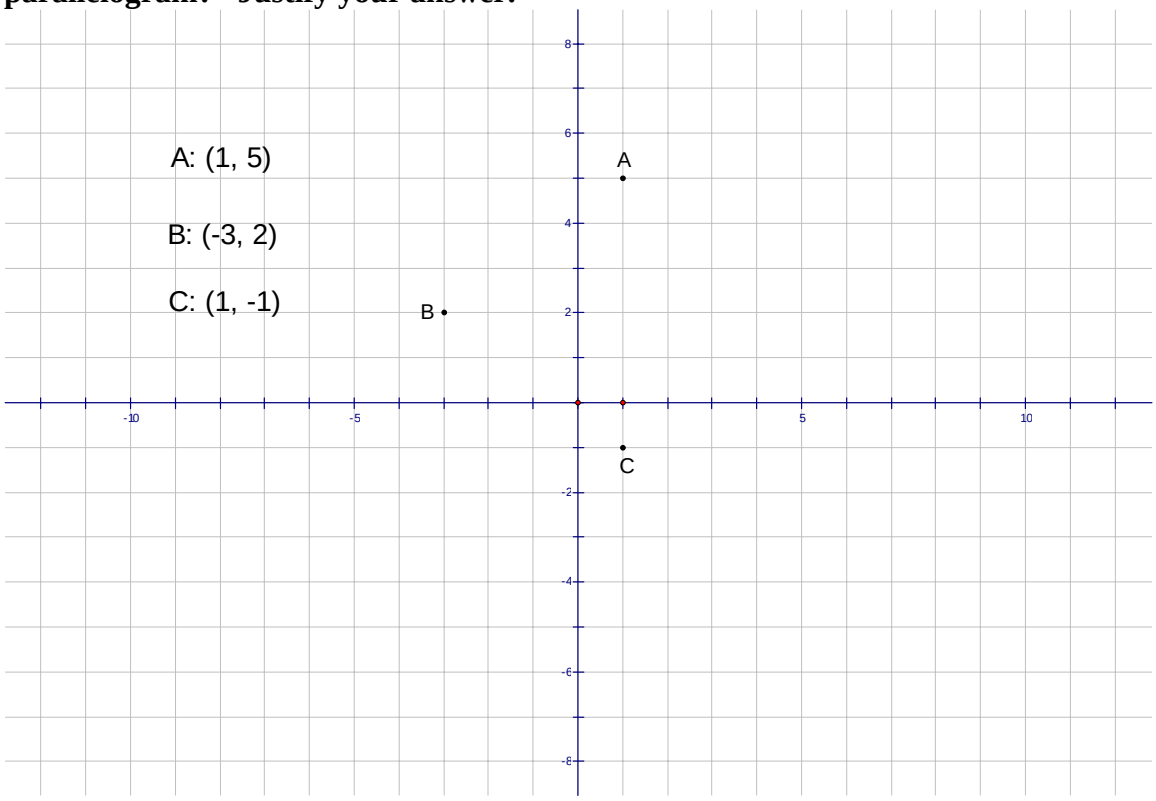
3. Show that the diagonals bisect each other.

Midpoint of $\overline{AC} = \left(\frac{1}{2}, -3\right)$

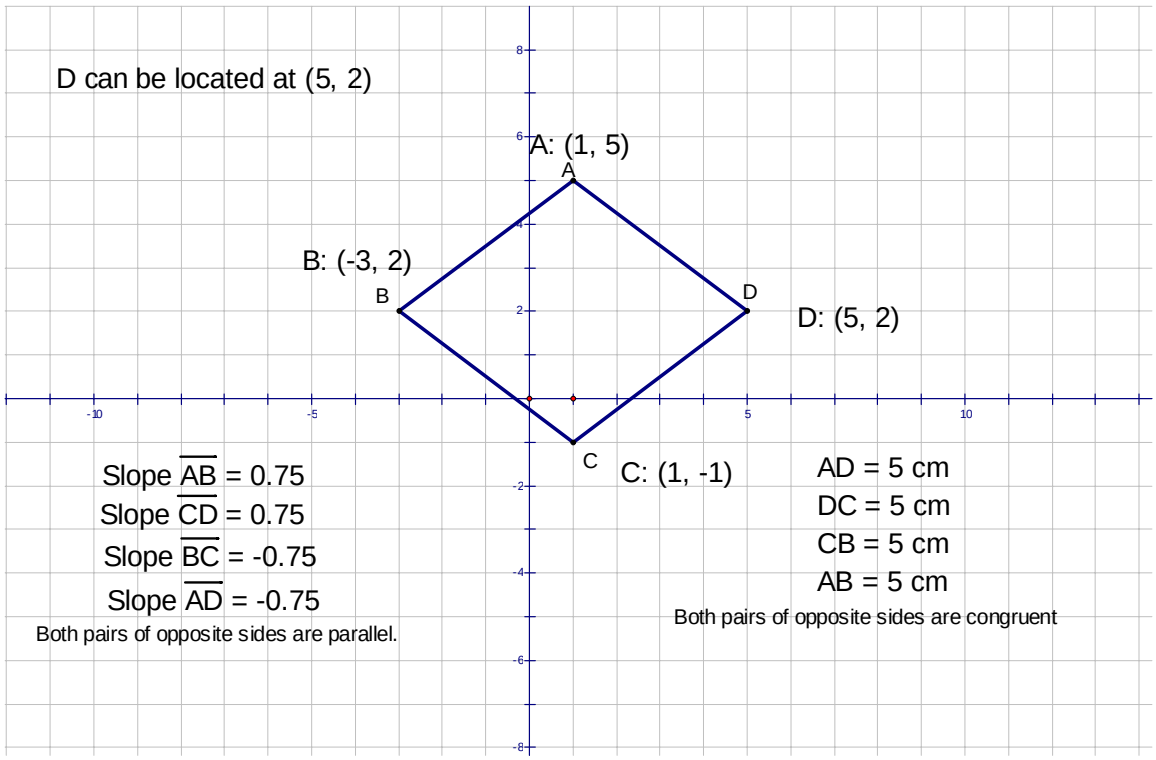
Midpoint of $\overline{BD} = \left(\frac{1}{2}, -3\right)$

If two segments have the same midpoint then they must be bisecting each other.

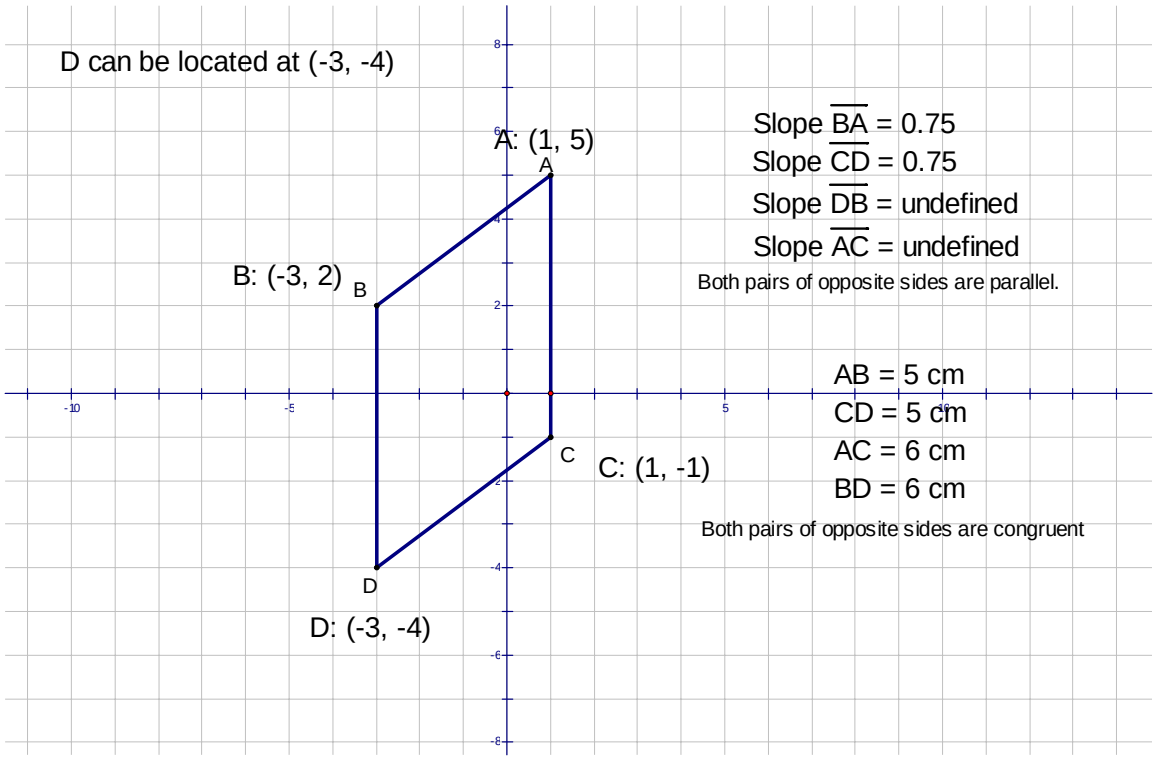
2. Where would you plot point D so that the four points would form a parallelogram? Justify your answer.



Solution I:



Solution II:

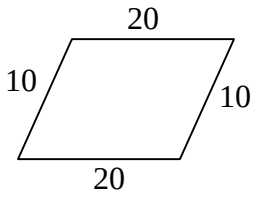

D can be located at (-3, -4)
A: (1, 5)
A
B: (-3, 2)
B
C
C: (1, -1)
D
D: (-3, -4)
Slope $\overline{BA}$ = 0.75
Slope $\overline{CD}$ = 0.75
Slope $\overline{DB}$ = undefined
Slope $\overline{AC}$ = undefined
Both pairs of opposite sides are parallel.
AB = 5 cm
CD = 5 cm
AC = 6 cm
BD = 6 cm
Both pairs of opposite sides are congruent

Name _____ **Date** _____

Reteaching/Extension

Are you given enough information to determine whether the quadrilateral is a parallelogram? Explain.

1.



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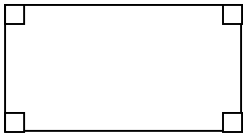

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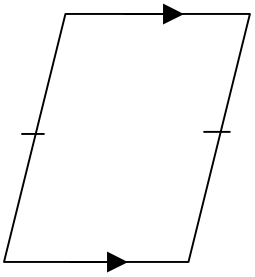
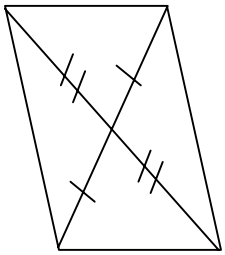
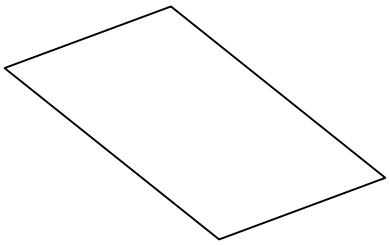


4.



5.

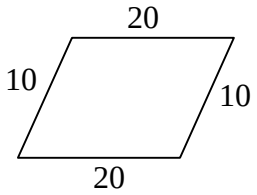


Name ____**Answer Key**_____ **Date** _____

Reteaching/Extension

Are you given enough information to determine whether the quadrilateral is a parallelogram? Explain.

1.

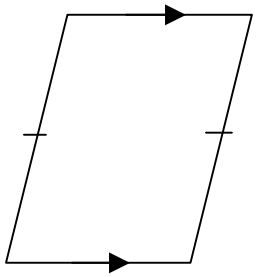


Yes; Both pairs of opposite sides are congruent.

2.

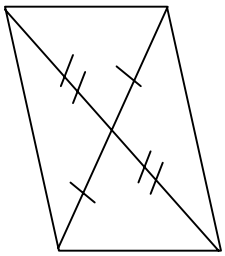
Yes; Both pairs of opposite angles are congruent. Also, one angle is supplementary to both consecutive angles.

3.



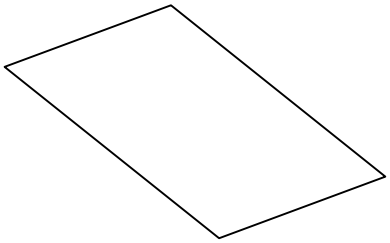
No; One pair of opposite sides is NOT parallel and congruent.

4.



Yes; The diagonals bisect each other.

5.



No; One angle is not supplementary to both consecutive angles.

Lesson 5: <u>Trapezoids</u>

Brief Overview:

In this lesson, students will use Geometer's Sketchpad to investigate properties of trapezoids. They will discover properties of isosceles trapezoids including the trapezoid base angles theorem and the trapezoid diagonals theorem. Students will also discover the midsegment theorem for trapezoids and the property of supplementary angles in trapezoids.

Pre-assessment:

Students will warm-up by evaluating a proposed definition of a trapezoid (see Student Resource Sheet 5.1). Students will also be presented with the names of the parts of trapezoids.

Launch:

Students will investigate properties of trapezoids using Geometer's Sketchpad. Students will create non-isosceles and isosceles trapezoids and explore the relationships of their angles, side-lengths, diagonals, and midsegments.

Teacher Facilitation:

Teachers will model the lesson, the constructions, and the key learning points using Geometer's Sketchpad on the overhead as students follow along on the worksheets. It is desirable that each student is able to work on Geometer's Sketchpad, though teacher modeling can suffice.

Student Application:

Students will complete Student Resource Sheets 5.1, 5.2, and 5.3.

Embedded Assessment:

The teacher will assess students by a running discussion and by monitoring each student's progress through the worksheets.

Reteaching/Extension:

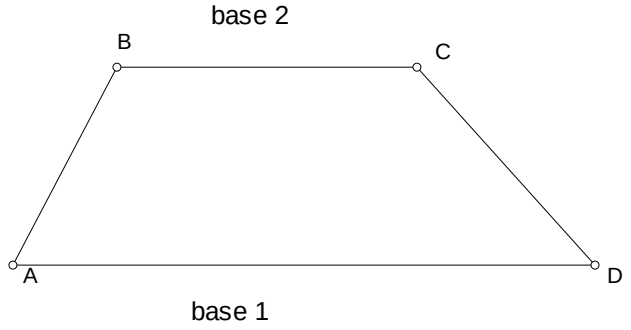
Students can use Geometer's Sketchpad to verify angle, segment length, and area relationships. Students will write out discovered theorems. Students will complete assigned problems.

Lesson 5: Trapezoids **Name:_____**

I. Warm-up.

A trapezoid is a quadrilateral with one pair of parallel sides.

Is the definition above a good one? Why or why not?

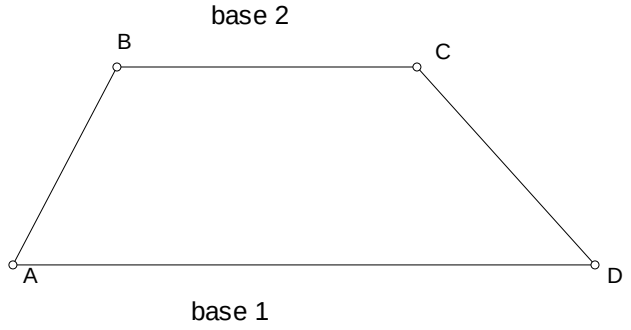


Lesson 5: Trapezoids **Name:_____**

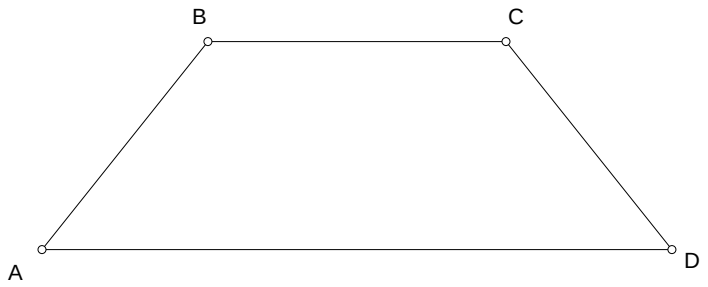
I. Warm-up.

A trapezoid is a quadrilateral with one pair of parallel sides.

Is the definition above a good one? Why or why not?



Lesson 5: Trapezoids **Name: _____**



I. <u>Trapezoid</u>

- Sketch a non-isosceles trapezoid using Geometer's Sketchpad.
- Explore the measures of the base angles. Which angles are supplementary? Why?
- Construct the midpoints of legs of the trapezoid. Construct the mid-segment by connecting these midpoints.
- Measure the bases and measure the mid-segment. Highlight these measures and select **Graph – Tabulate**. Manipulate the figure by dragging a vertex and look for a relationship of the lengths of the bases and that of the midsegment.

II. <u>The Isosceles Trapezoid</u>: Now, we will construct an isosceles trapezoid and discover its properties.

- Start by constructing an isosceles triangle.
  - Construct a segment.
  - Highlight the segment and select **Construct – Midpoint**.
  - Highlight the midpoint and the segment and select **Construct – Perpendicular**.
  - Create a point on the perpendicular, and then hide the perpendicular.
  - Construct the segments that connect the endpoints of the segment and the point on the perpendicular.
- Construct a point on one side of the triangle. Highlight the point and the base of the triangle and select **Construct – Parallel**.
- Hide the top of the triangle. (Be creative.)

- Explore the properties of the Isosceles Trapezoid. Consider the angle measures and the lengths of the diagonals. What conjectures can you make?

III. **<u>Exploration</u>**. Finally, we will learn that any trapezoid can be doubled to create a parallelogram.

- Using the same trapezoid that we constructed above, construct the midpoint of one of the legs. Highlight this point by double-clicking on it. (It will expand to a bull's eye for a moment.)
- Now, highlight the bases and the opposite leg of the figure.

- Select **Transform – Rotate**. Rotate the figure 180 degrees. What figure resulted? Why?

Trapezoid Theorems **Name: _____**

Below are listed theorems relevant to trapezoids. Please use the knowledge you have gained from exploring in Geometer's Sketchpad to write the theorems. Have you made any other discoveries?

Supplementary Angles in a Trapezoid

_____
_____
_____.

The Mid-segment Theorem for Trapezoids

_____
_____
_____.

Theorems specific to Isosceles Trapezoids

Trapezoid Base Angles Theorem

_____
_____
_____.

Trapezoid Diagonals Theorem

_____
_____
_____.

Others?

_____
_____
_____
_____
_____

Trapezoid Theorems – For Teacher

Supplementary Angles in a Trapezoid
In any trapezoid, adjacent angles formed by the two bases and each leg are supplementary.

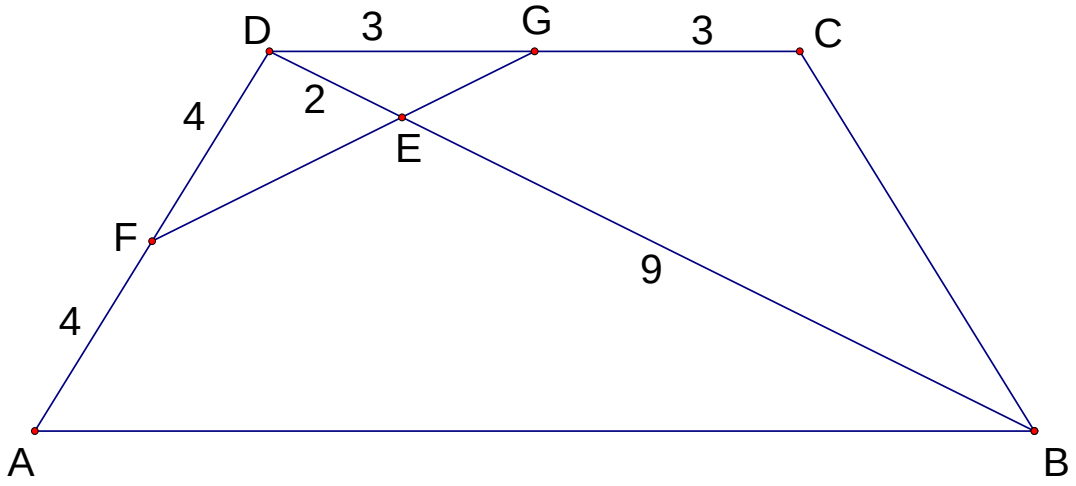
Mid-segment Theorem for Trapezoids
The mid-segment of a trapezoid is parallel to the bases and its measure is one-half the sum of the lengths of the bases.

Trapezoid Base Angles Theorem
The pairs of base angles of an isosceles trapezoid are congruent.

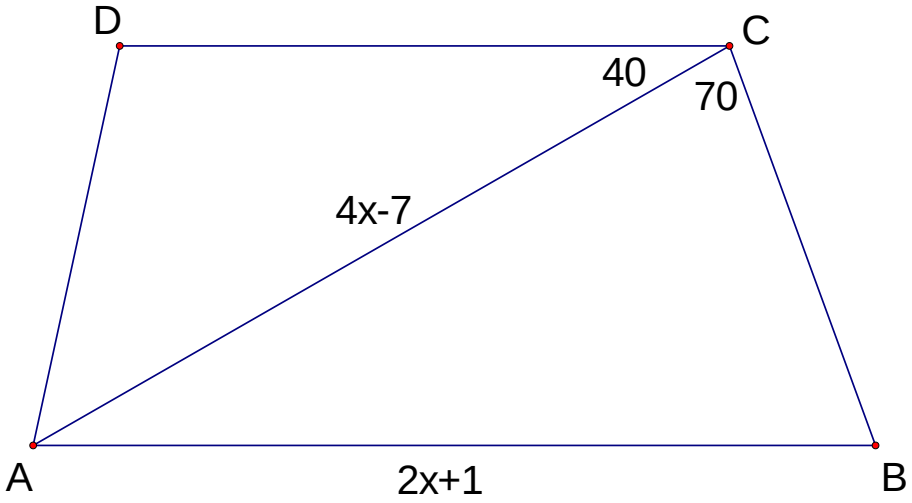
Trapezoid Diagonals Theorem
The diagonals of an isosceles trapezoid are congruent.

Trapezoid Problems

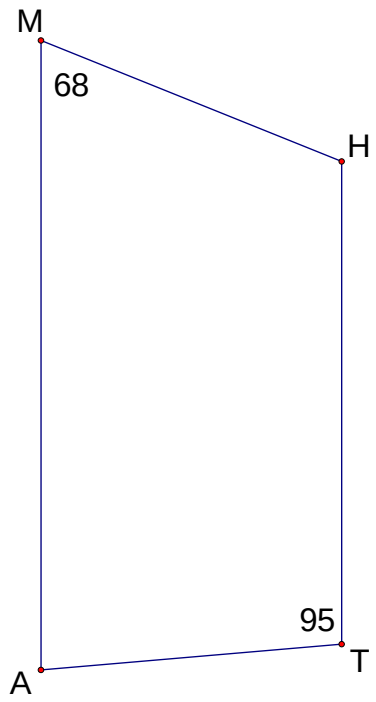
1. Given isosceles trapezoid ABCD, find FG.



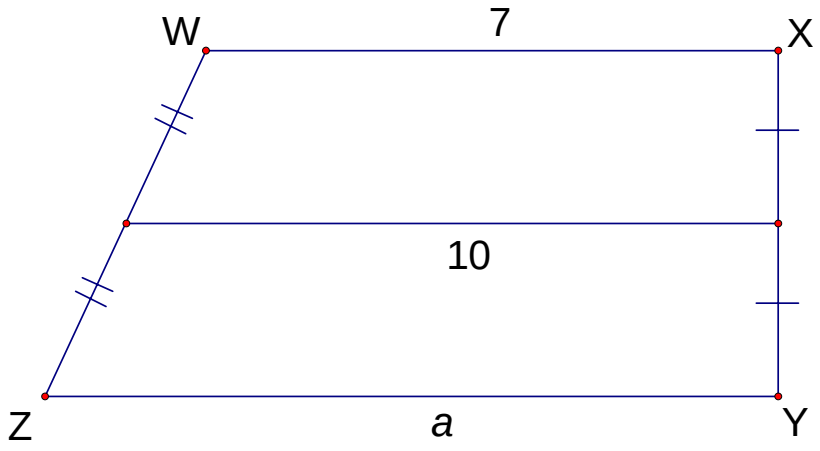
2. Given trapezoid ABCD, solve for *x*.



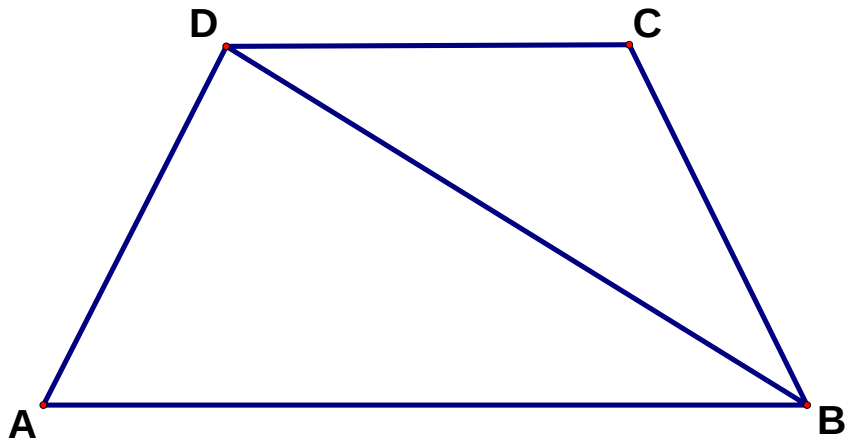
3. Given trapezoid MATH…
   a. Name the bases.
   b. Find m∠A and m∠H.



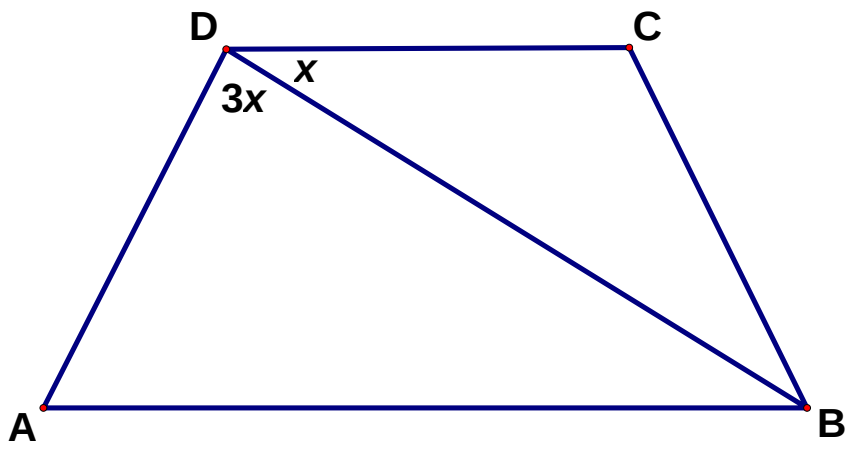
4. Solve for *a* in trapezoid WXYZ.



5. Given trapezoid ABCD below, AD = DC = CB, and m∠ABD = 32°. Find m∠A.



6. Given trapezoid ABCD below, AD = DC = CB. Find m∠A



Lesson 6: Midpoint Quadrilaterals

Brief Overview:

In this lesson, students will use Geometer's Sketchpad to discover properties of the quadrilaterals formed by constructing the midpoints of quadrilaterals and connecting their segments.

Pre-assessment:

We will warm-up with a challenging problem using Geometer's Sketchpad. Students will discover how to bisect a parallelogram to form two congruent trapezoids.

Launch:

Students will investigate properties of midpoint quadrilaterals using Geometer's Sketchpad.

Teacher Facilitation:

Lead the investigation of the properties of midpoint quadrilaterals as students follow along on the worksheet. Model the lesson, the constructions, and the key learning points using Geometer's Sketchpad on the overhead.

Student Application:

Students will follow directions as they explore constructions using Geometer's Sketchpad.

Embedded Assessment:

Students will participate in informal discussion about the lesson and they will complete worksheets.

Reteaching/Extension:

Students will complete assigned problems.

Lesson 6: <u>Midpoint Quadrilaterals</u> Name: _____
Warm-up

I. <u>Sketch and Discover</u>: For a warm-up, we will relate parallelograms to trapezoids by construction and we will learn how to determine the area of a polygon using Geometer's Sketchpad.

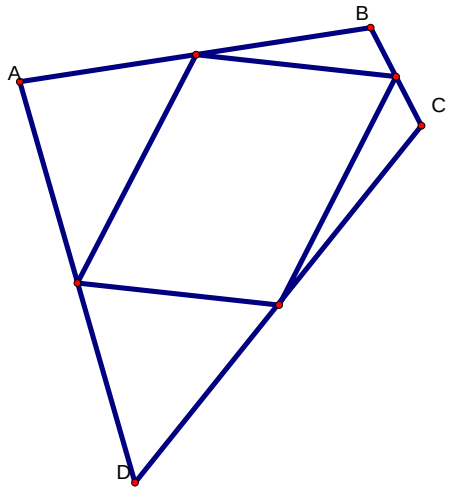
- Construct a parallelogram (see lesson 2). Draw one segment connecting the bases that divides the parallelogram into two congruent trapezoids. Think carefully about this construction. Take some chances and verify your work by using the Calculate Area function of Geometer's Sketchpad.

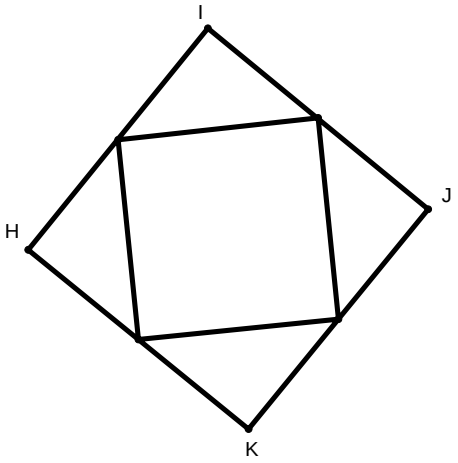
- To determine the area of your original parallelogram:

  - First, highlight each vertex in consecutive order.
  - Select **Construct – Quadrilateral Interior**.
  - Select **Measure – Area**. (The area measure will be shown in the upper left corner of the screen.)

Midpoint Quadrilaterals **Lesson 6** Name: _____

In this lesson, you will discover an interesting truth about the quadrilateral formed by connecting the midpoints of another quadrilateral.





<u>Sketch and Investigate</u>

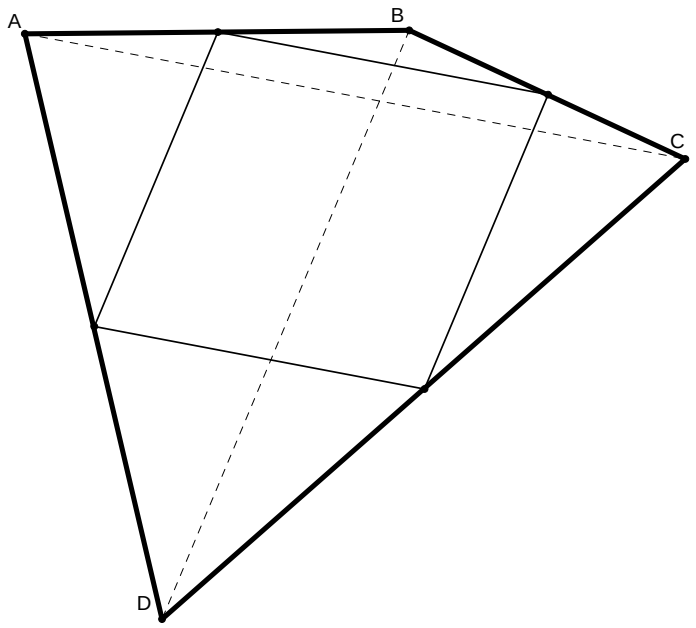
- Construct any quadrilateral ABCD.
- Construct the midpoints of the sides.
- Connect the midpoints to construct another quadrilateral EFGH.
- Drag vertices of your original quadrilateral and observe the midpoint quadrilateral.
- Measure the four side lengths of this midpoint quadrilateral.

<u>Question 1</u>. Measure the slopes of the four sides of the midpoint quadrilateral. What kind of quadrilateral does the midpoint quadrilateral appear to be? How do the measurements support that conjecture?

<u>Sketch and Investigate</u>

- Construct a diagonal.
- Measure the length and the slope of the diagonal.
- Drag the vertices of the original quadrilateral and observe how the length and slope of the diagonal are related to the lengths and slopes of the sides of the midpoint quadrilateral.

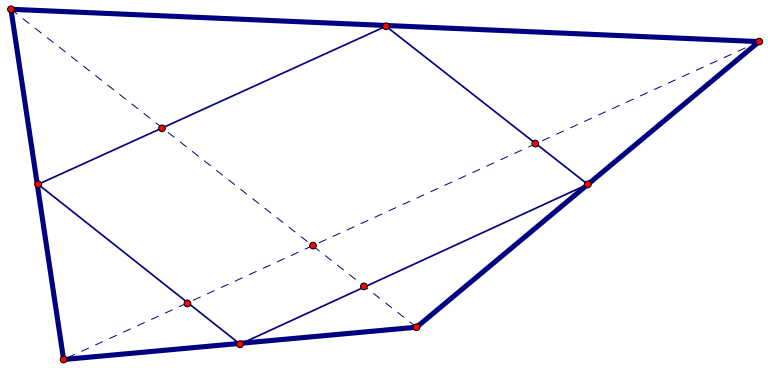
<u>Question 2</u>. The diagonal divides the original quadrilateral into two triangles. Each triangle has as a mid-segment one of the sides of the midpoint quadrilateral. Use this fact and what you know about the slope and length of the diagonal to write a paragraph explaining why the conjecture you made in Q1 is true.



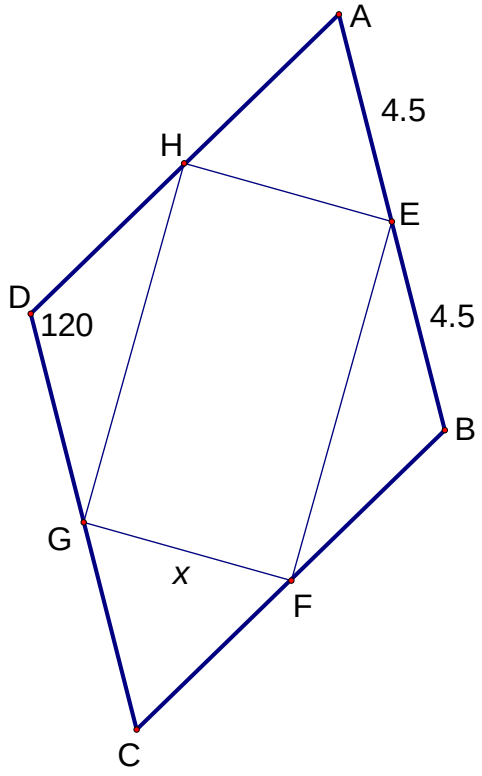
Midpoint Quadrilateral Problems

1. Prove why the midpoint quadrilateral of any quadrilateral is always a parallelogram.

2. Below is a diagram containing a quadrilateral, its diagonals, and its midpoint quadrilateral. What appears to be true about the 4 regions contained within the midpoint quadrilateral formed by the diagonals of the external figure? Can you prove this?

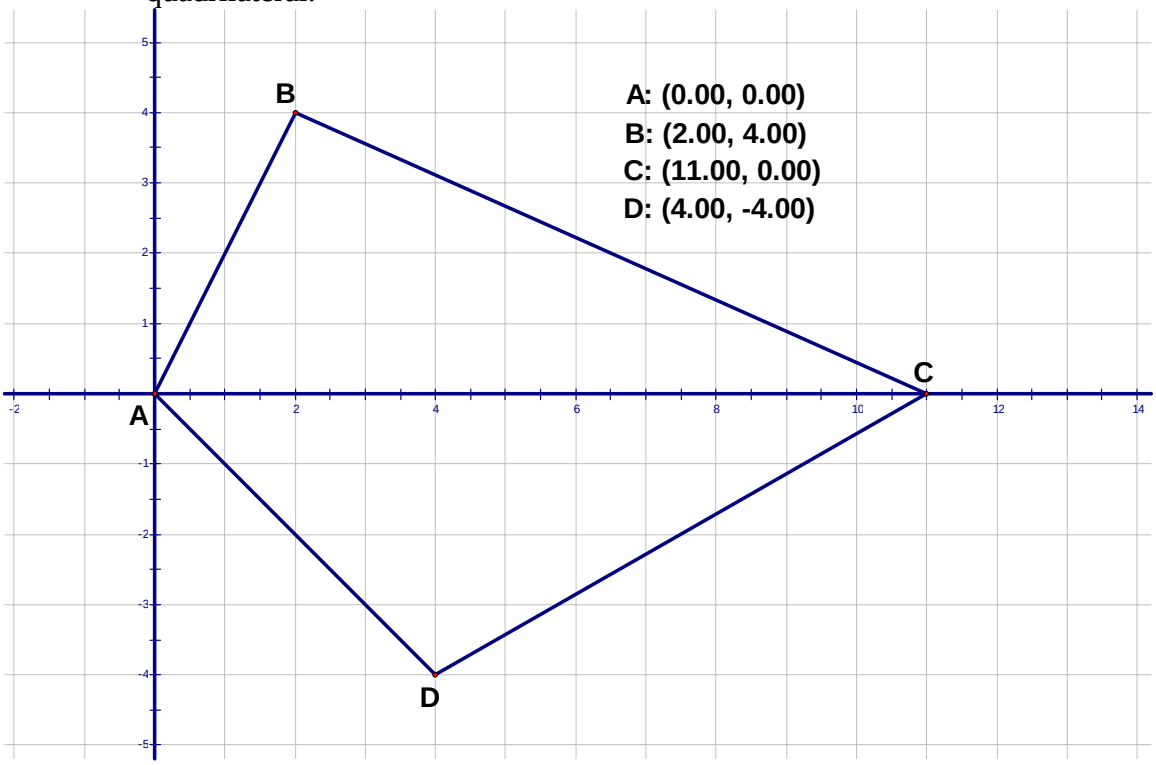


3. Parallelogram ABCD is shown below. EFGH is the midpoint quadrilateral. If AD = 9, find *x*.



4. Quadrilateral ABCD is shown below. Use the midpoint formula to locate the midpoints of all four sides and create the midpoint quadrilateral.

a. Use the slopes of the sides of the midpoint quadrilateral to verify that it is a parallelogram.
b. Find the coordinates of the intersection of the diagonals of the midpoint quadrilateral.



Lesson 7 – Investigating Special Midpoint Quadrilaterals

Preassessment/Launch – Have students summarize their discoveries from Lesson 6, namely that all midpoint quadrilaterals (the quadrilateral formed by joining the midpoints of another quadrilateral) are parallelograms.

Teacher Facilitation – Teachers will need to create in advance sketches of constructions for demonstration purposes or for class discovery. The steps for these constructions are included in this lesson.

Student Application – Students will investigate properties of the three different quadrilaterals in this activity. The students should use The Geometer's Sketchpad to investigate the properties of a quadrilateral whose midpoint quadrilateral is a rhombus, a rectangle, and a square.

Embedded Assessment – Through investigation of the quadrilaterals, with concentration on their diagonals, the students will discover the following relationships:

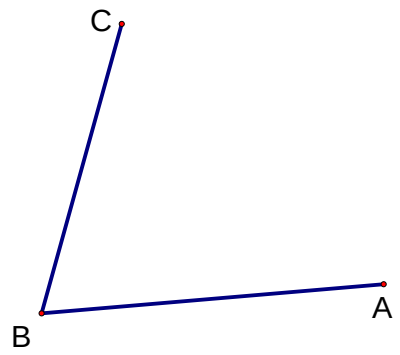
1. A quadrilateral whose midpoint quadrilateral is a rhombus has congruent diagonals.
2. A quadrilateral whose midpoint quadrilateral is a rectangle has perpendicular diagonals
3. A quadrilateral whose midpoint quadrilateral is a square has congruent and perpendicular diagonals.

Reteaching/Extension – Students should justify why each of the quadrilaterals described in this activity have a rhombus, rectangle, or square as its midpoint quadrilateral.

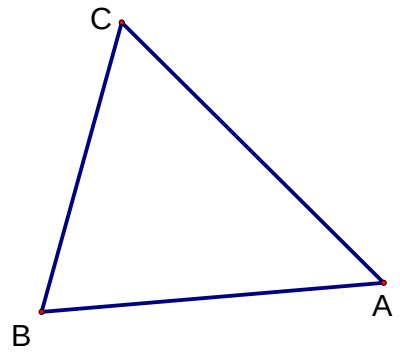
Geometer's Sketchpad Construction - Directions for Special Midpoint Quadrilaterals

Directions for Creating a "Midpoint Rhombus"

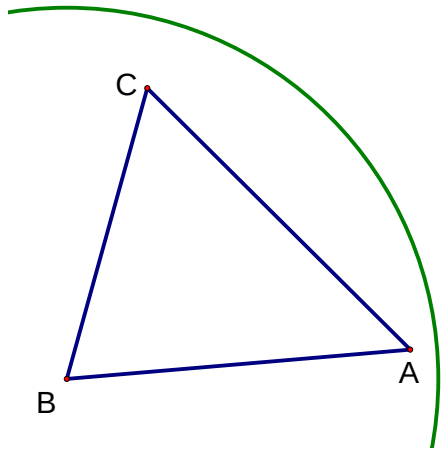
1. Create line segments AB & BC using the segment tool.



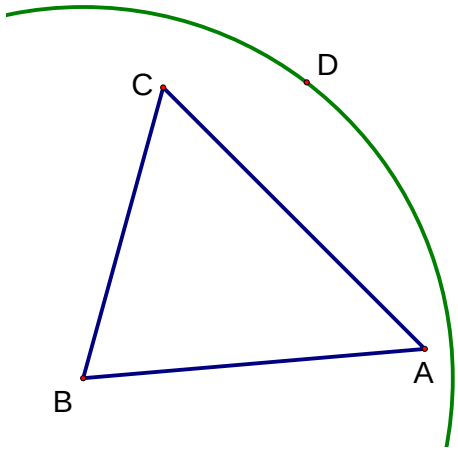
2. Create line segment AC using the segment tool.



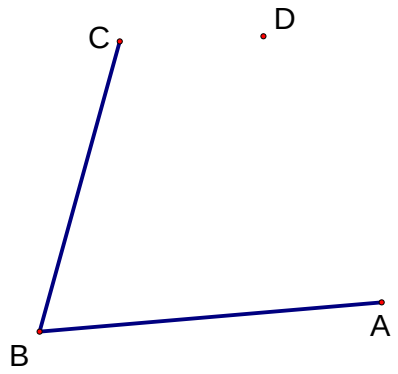
3. Select point B & line segment AC and choose **Construct → Circle by Point & Radius**.



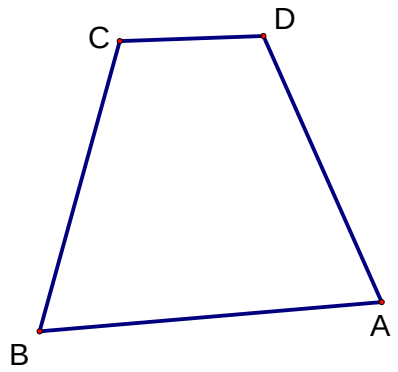
4. Create point D arbitrarily on circle B.



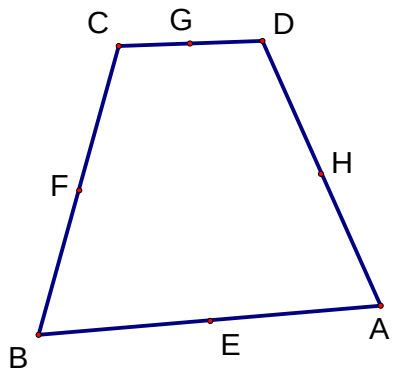
5. Hide line segment AC & circle B.



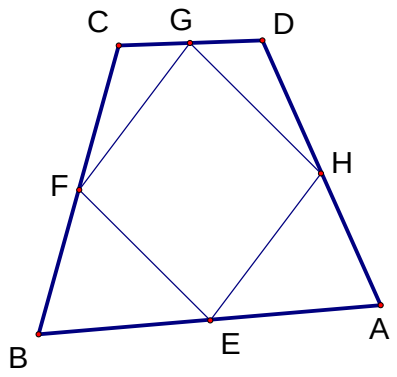
6. Create line segments AD & CD.



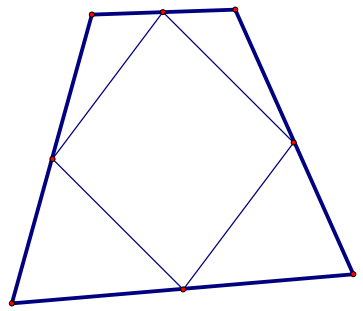
7. Select line segments AB, BC, CD, and AD and choose **Construct → Midpoints**.



8. With these midpoints still selected, choose **Construct → Segments**. The midpoint quadrilateral of quadrilateral ABCD should be a rhombus (you may want to take some measurements to convince yourself of this). This new quadrilateral is your "Midpoint Rhombus".
Suggestion: you may want to make the "midpoint rhombus" using thin segments, and the external quadrilateral using thick segments.



9. Hide all labels at this time.

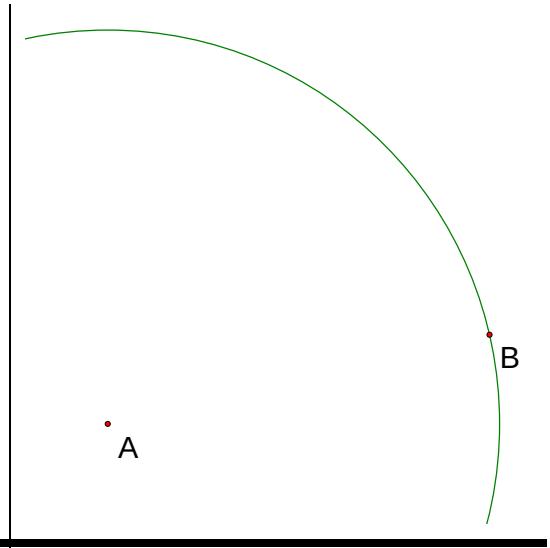


10. Have students drag the vertices of the external quadrilateral to observe properties of the midpoint quadrilateral.

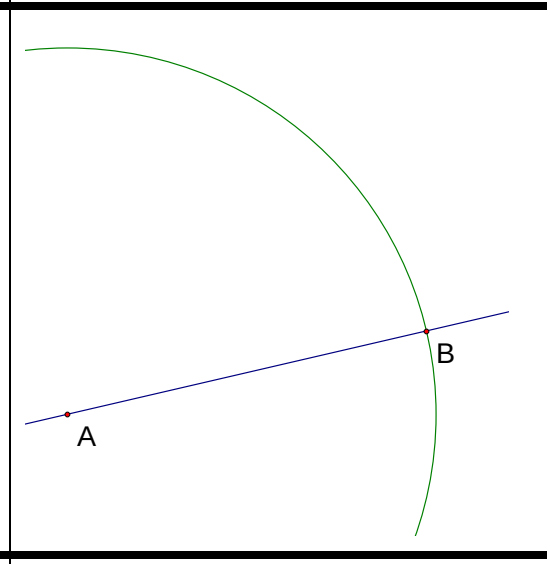
11. Have students make conjectures about both the midpoint quadrilateral and the external quadrilateral.

Directions for Creating a "Midpoint Rectangle"

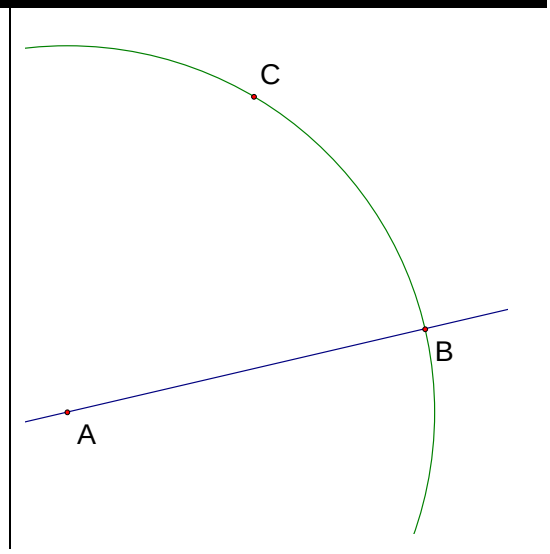
1. Create circle A with defining point B using the circle tool.



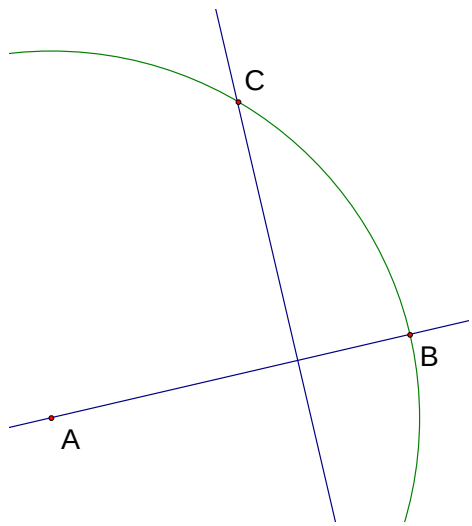
2. Create line AB.



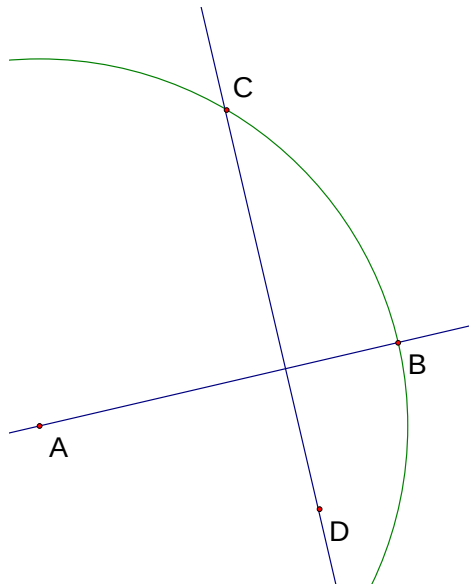
3. Create point C arbitrarily on circle A.



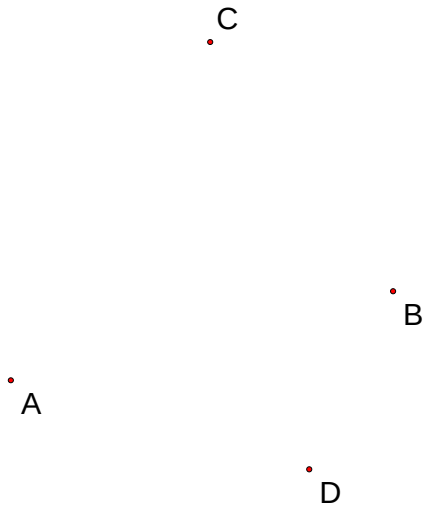
4. Select C and line AB and choose **Construct → Perpendicular Line**.



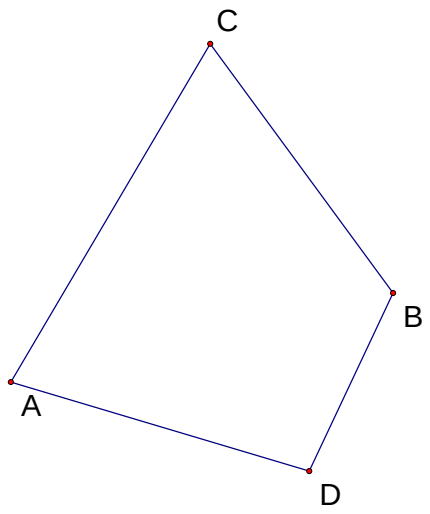
5. Create point D on this perpendicular, but not on circle A.



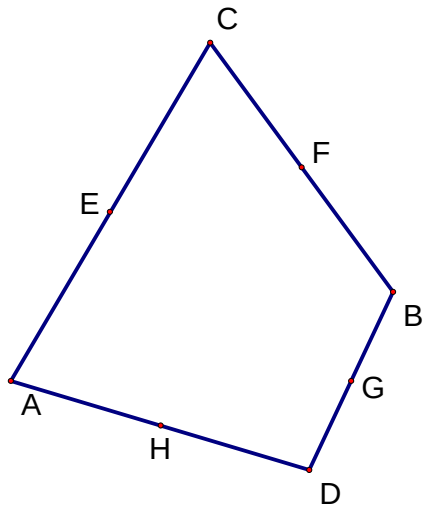
6. Hide everything except points A, B, C, and D.



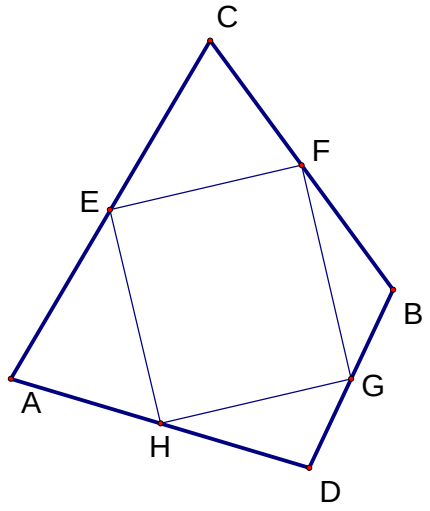
7. Create quadrilateral ADBC using the segment tool.



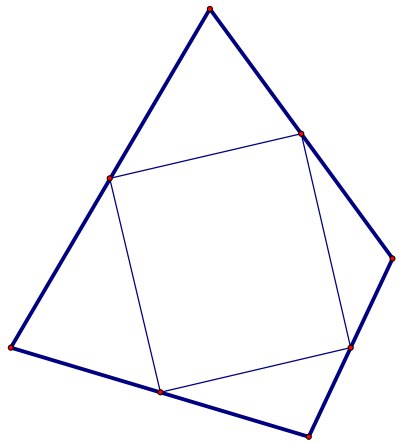
8. Select line segments AD, DB, BC, and CA and choose **Construct → Midpoints**.



9. With these midpoints still selected, choose **Construct → Segments**. The midpoint quadrilateral of quadrilateral ABCD should be a rectangle (you may want to take some measurements to convince yourself of this). This new quadrilateral is your "Midpoint Rectangle".
Suggestion: you may want to make the "midpoint rectangle" using thin segments, and the external quadrilateral using thick segments.



10. Hide all labels at this time.

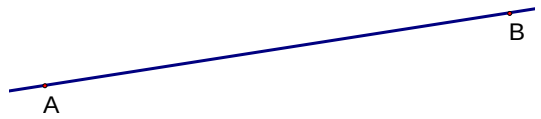


11. Have students drag the vertices of the external quadrilateral to observe properties of the midpoint quadrilateral.

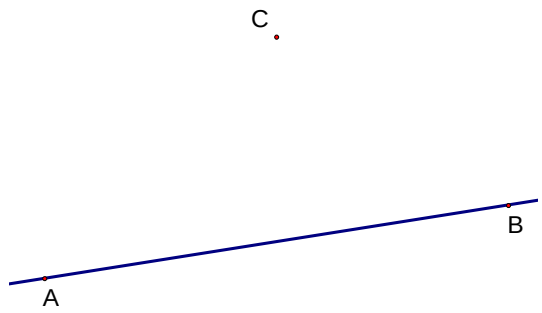
12. Have students make conjectures about both the midpoint quadrilateral and the external quadrilateral.

Directions for constructing a "Midpoint Square"

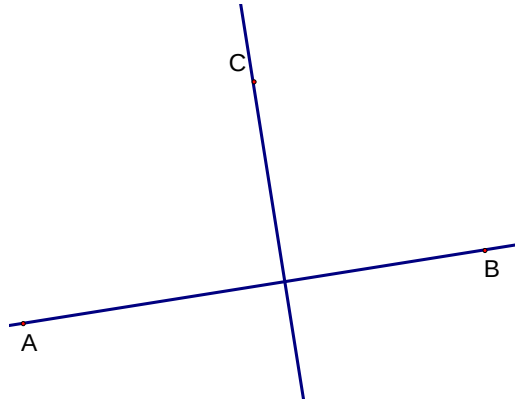
1. Create line AB using the line tool.



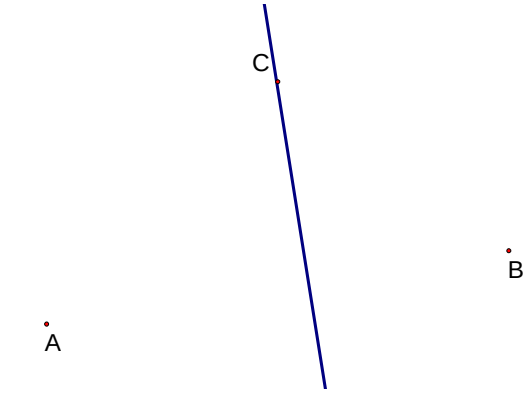
2. Create point C, not on line AB.



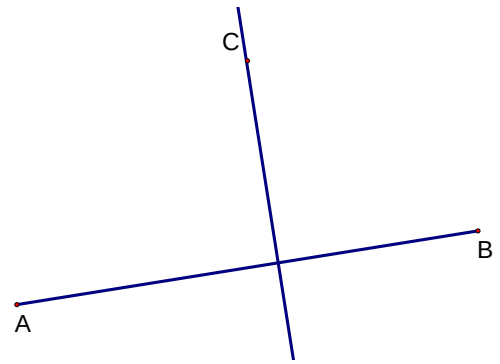
3. Select point C and line AB and choose **Construct → Perpendicular Line**.



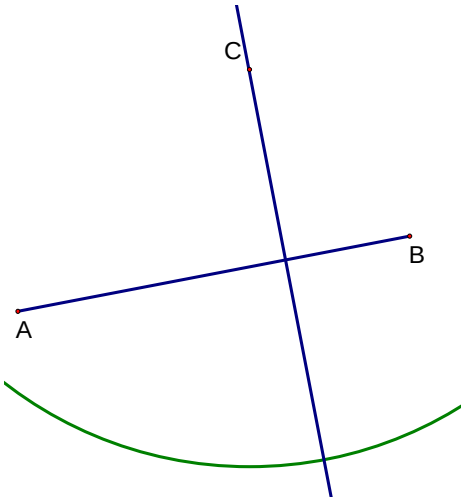
4. Hide line AB.



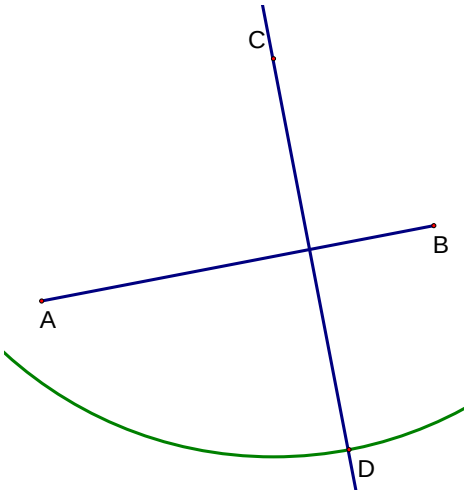
5. Create line segment AB.



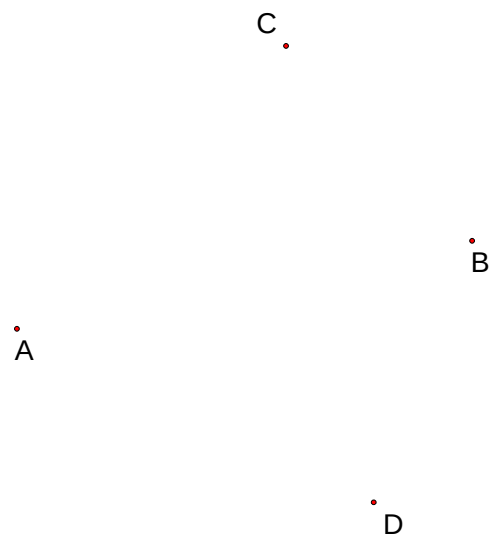
6. Select point C and line segment AB and choose
Construct → Circle by Center + Radius.



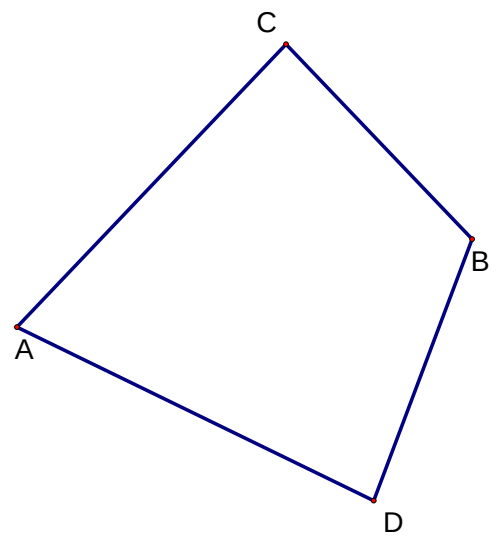
7. Create point D at the intersection of the circle and the perpendicular on the side of line segment AB opposite point C.



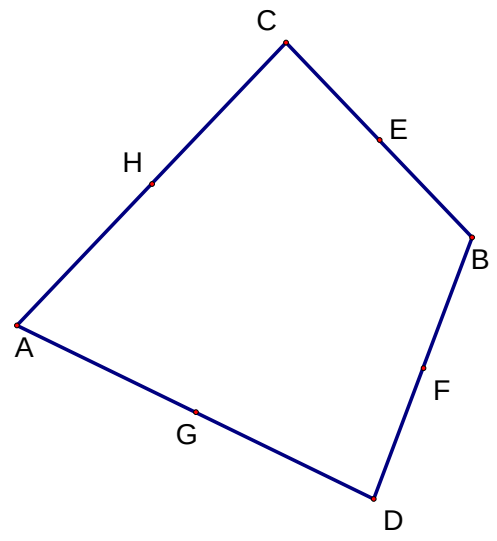
8. Hide everything except point A, B, C, and D.



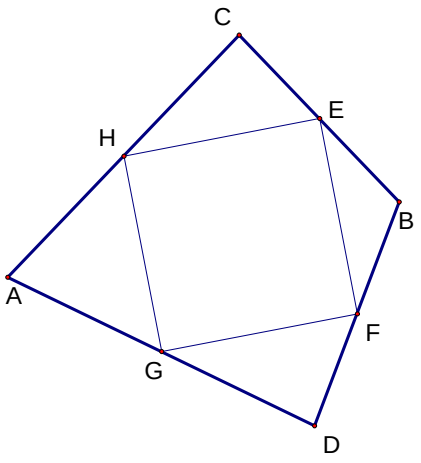
9. Create quadrilateral ADBC using the segment tool.



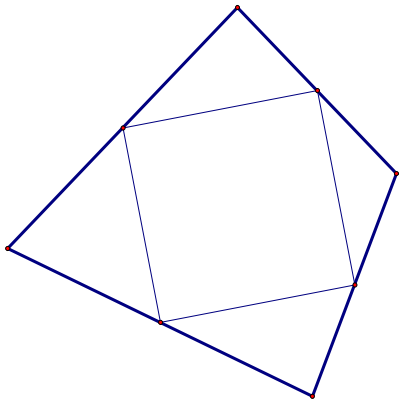
10. Select line segments AD, DB, BC, and CA and choose **Construct → Midpoints**.



11. With these midpoints still selected, choose **Construct → Segments**. The midpoint quadrilateral of quadrilateral ABCD should be a square (you may want to take some measurements to convince yourself of this). This new quadrilateral is your "Midpoint Square".
Suggestion: you may want to make the "midpoint square" using thin segments, and the external quadrilateral using thick segments.



12. hide all labels at this time.



13. Have students drag the vertices of the external quadrilateral to observe properties of the midpoint quadrilateral.

14. Have students make conjectures about both the midpoint quadrilateral and the external quadrilateral.

Special Midpoint Quadrilaterals Assignment

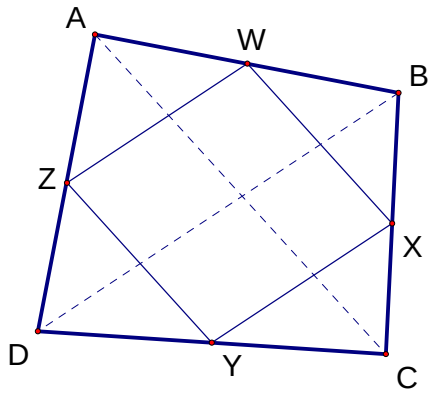
1. Given quadrilateral ABCD with congruent diagonals AC and BD. W is the midpoint of line segment AB, X is the midpoint of line segment BC, Y is the midpoint of line segment CD, and Z is the midpoint of line segment AD. Prove that the midpoint quadrilateral WXYZ is always a rhombus.

2. Given quadrilateral ABCD with perpendicular diagonals AC and BD. W is the midpoint of line segment AB, X is the midpoint of line segment BC, Y is the midpoint of line segment CD, and Z is the midpoint of line segment AD. Prove that the midpoint quadrilateral WXYZ is always a rectangle.

3. Given quadrilateral ABCD with congruent ***and*** perpendicular diagonals AC and BD. W is the midpoint of line segment AB, X is the midpoint of line segment BC, Y is the midpoint of line segment CD, and Z is the midpoint of line segment AD. Prove that the midpoint quadrilateral WXYZ is always a square.

Special Midpoint Quadrilaterals Assignment
Answer Key

1. Given quadrilateral ABCD with congruent diagonals AC and BD. W is the midpoint of line segment AB, X is the midpoint of line segment BC, Y is the midpoint of line segment CD, and Z is the midpoint of line segment AD. Prove that the midpoint quadrilateral WXYZ is always a rhombus.

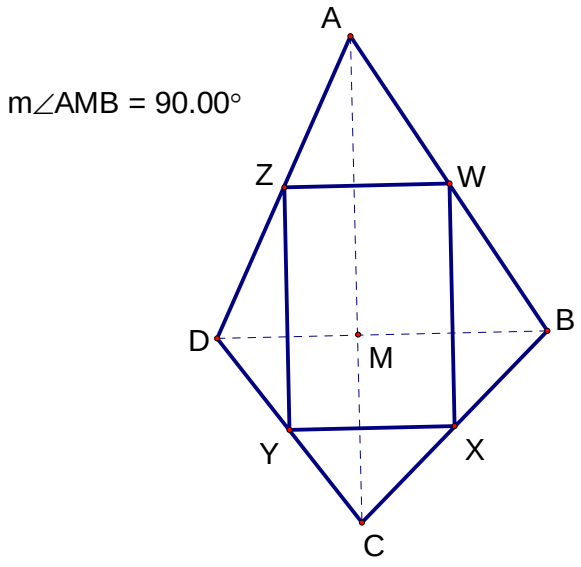


AC = 9.01 cm BD = 9.01 cm

Possible solution:

WZ = XY = ½ BD and WX = ZY = ½ AC since they are all midsegments of the triangles formed by the diagonals. Additionally, WZ || XY || BD and WX || ZY || AC (also because they are all midsegments). Since it is given that the diagonals are congruent, WZ = XY = WX = ZY by the transitive property. Since midpoint quadrilateral WXYZ has two pairs of opposite sides that are parallel and all four sides congruent, **midpoint quadrilateral WXYZ is a rhombus**.

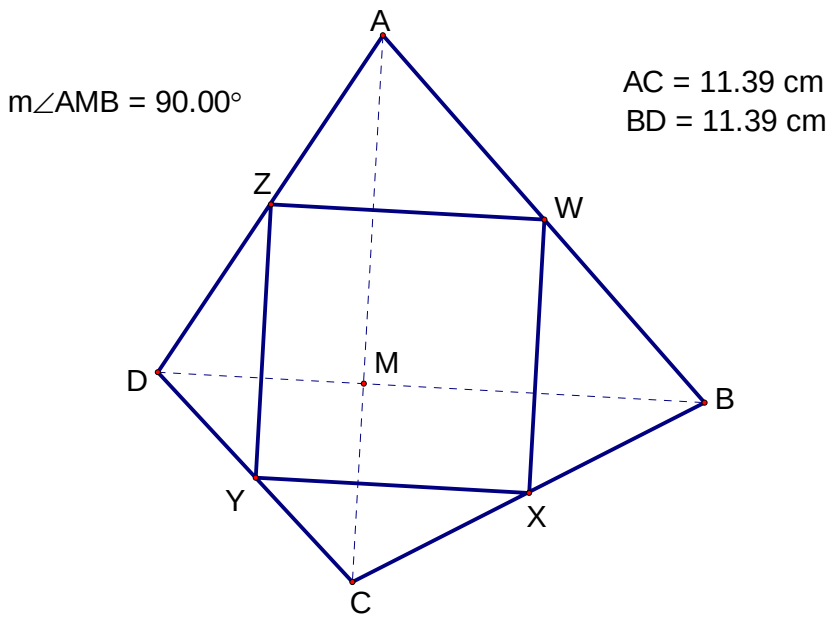
2. Given quadrilateral ABCD with perpendicular diagonals AC and BD. W is the midpoint of line segment AB, X is the midpoint of line segment BC, Y is the midpoint of line segment CD, and Z is the midpoint of line segment AD. Prove that the midpoint quadrilateral WXYZ is always a rectangle.



Possible solution:

WZ || XY || BD and WX || ZY || AC since WZ, XY, WX, and ZY are all midsegments of the triangles formed by the diagonals. Since it is given that the diagonals are perpendicular, consecutive midsegments are also perpendicular to each other (WX ⊥ XY for example). Since midpoint quadrilateral WXYZ has been shown to have two pairs of opposite sides that are parallel and at least one right angle, **midpoint quadrilateral WXYZ is a rectangle**.

3. Given quadrilateral ABCD with congruent ***and*** perpendicular diagonals AC and BD. W is the midpoint of line segment AB, X is the midpoint of line segment BC, Y is the midpoint of line segment CD, and Z is the midpoint of line segment AD. Prove that the midpoint quadrilateral WXYZ is always a square.



Possible solution:

Using the results from problems 1 and 2…

Midpoint quadrilateral WXYZ is a rhombus because the diagonals AC and BD are congruent. Additionally, midpoint quadrilateral WXYZ is a rectangle because the diagonals AC and BD are perpendicular. Since it is both a rhombus and a rectangle, **midpoint quadrilateral WXYZ is also a square**.

GEOPARDY –Geometry Chapter 6-Polygons

1-100 What do we know about the opposite angles of a parallelogram? U**CONGRUENT**
1-200 What is the sum of any pair of consecutive angles of a parallelogram? **180°**
1-300 What is the sum of the angles of any quadrilateral? **360°**
1-400 Which quadrilaterals have all angles congruent? **RECTANGLES & SQUARES**
1-500 Which quadrilaterals have only 1 pair of opposite angles congruent? **KITES**

2-100 What special quadrilaterals have all angles congruent? **RECTANGLES & SQUARES**
2-200 Which quadrilaterals have only one pair of opposite sides parallel? **TRAPEZOIDS**
2-300 What quadrilaterals have 2 pairs of opposite sides congruent? **PARALLELOGRAMS**
2-400 What special quadrilaterals have all sides congruent? **RHOMBUS & SQUARE**
2-500 ALWAYS, SOMETIMES, OR NEVER? If a quadrilateral is a square, then it is a rectangle. **ALWAYS**

3-100 What are two facts about the opposite sides of a parallelogram? **THEY ARE PARALLEL AND CONGRUENT**
3-200 What word describes any two consecutive angles of a parallelogram? **SUPPLEMENTARY**
3-300 ALWAYS, SOMETIMES, OR NEVER? A parallelogram is a quadrilateral. **ALWAYS**
3-400 If you draw one diagonal of a parallelogram, by what method can you prove that it forms 2 congruent triangles? **SSS, or ASA**
3-500 Since the diagonals of a parallelogram bisect each other, how can it be proven that they form 2 congruent ▲s? **SAS**

4-100 What are polygons called that have congruent sides and congruent angles? **REGULAR**
4-200 What is the name of a five-sided polygon that is also the name of a famous military complex in D.C.? **PENTAGON**
4-300 What kind of a polygon is a "stop" sign? **OCTAGON**
Daily Double 4-400 What is another name for a regular quadrilateral? **SQUARE**
4-500 What kind of a polygon is it if a line containing a side of the polygon, when extended, goes inside the polygon? **CONCAVE**

5-100 What is the formula for the area of a rectangle? **A = l x w or A= b x h**
5-200 What is the formula for the area of a square? **$A = s^2$**
5-300 What is the formula for the area of a parallelogram? **A = l x w or A = b x h**
5-400 For which quadrilaterals does the area equal ½ the product of the diagonals? **KITE, RHOMBUS, SQUARE**
5-500 For what quadrilaterals does the Area equal ½ the sum of the bases times the height? **TRAPEZOID**

6-100 What do you know about the diagonals of a parallelogram? **THEY BISECT EACH OTHER**
6-200 In which 3 quadrilaterals are the diagonals perpendicular? **RHOMBUS, SQUARE, KITE**
6-300 In which 2 quadrilaterals do the diagonals bisect the opposite angles? **RHOMBUS & SQUARE**
6-400 Which two parallelograms have congruent diagonals? **RECTANGLE & SQUARE**
6-500 In which quadrilateral are the diagonals congruent, perpendicular, and bisectors of the opposite angles? **SQUARE**

Name _____Block_____Date_____

GEOPARDY SCORE SHEET

Angles	Quadrilaterals	▱	Polygons	Area	Diagonals
100	100	100	100	100	100
200	200	200	200	200	200
300	300	300	300	300	300
400	400	400	400	400	400
500	500	500	500	500	500

TOTAL

Name _____Block_____Date_____

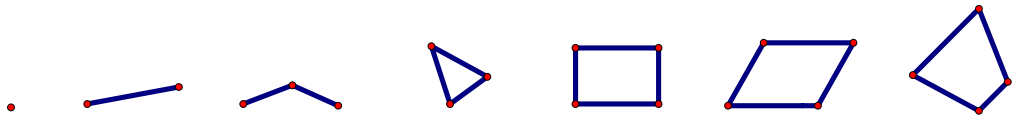
GEOPARDY SCORE SHEET

Angles	Quadrilaterals	▱	Polygons	Area	Diagonals
100	100	100	100	100	100
200	200	200	200	200	200
300	300	300	300	300	300
400	400	400	400	400	400
500	500	500	500	500	500

TOTAL

[Extension Lesson]

Lesson 8: Area of Quadrilaterals ***Discovery*** Name:_____



The Evolution of the Modern Trapezoid

In this lesson, you will learn about properties of trapezoids by discovering how they relate to other quadrilaterals. We will proceed with their predecessors – parallelograms.

I. <u>Sketch and Discover:</u> Students will use Geometer's Sketchpad to construct a parallelogram in order to discover its relationship to a trapezoid.

- Construct a parallelogram ABCD. (See Lesson 2) Drag point C until the figure resembles a rectangle.

- Determine the area of the rectangle.
  - Highlight the vertices of the figure in consecutive order. Select **Construct – Quadrilateral Interior**
  - Select **Measure – Area** (The Measure will be displayed on the upper left corner of the screen)

- Verify the Area by finding the product of the base and the height.
  - Highlight the base. Select **Measure – Length.**
  - Highlight the height. Select **Measure – Length.**
  - Select **Measure – Calculate.**
  - Highlight the value of the length of the base, press the multiplication symbol, and highlight the value of the length of the side.

- Drag point C to the right to form a parallelogram. Why does the Area remain constant while the value of the length of the side changes as you drag point C?

- Construct a segment that represents the height of the figure. Explore area given this new value.

- Make a conjecture for the formula of the area of a parallelogram. Verify your conjecture using your drawing.

- Construct a non-isosceles trapezoid. Construct the midpoint of one of the legs. Highlight this point by double-clicking on it. (It will expand to a bull's eye for a moment.)

- Now, highlight the bases and the opposite leg of the figure. Select **Transform – Rotate**. Rotate the figure 180 degrees. What figure resulted? Why?

Lesson 8: Area of Quadrilaterals ***Discovery*** **Name: _____**
Homework

- Explain why the formula for the area of any trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$.

  Use compass and straight-edge constructions, free-hand drawings, or Geometer's Sketchpad constructions to illustrate your explanation.