Title: Strike a Pose: Modeling In The Real World (There's Nothing To It!)

Brief Overview:

In this unit, students will analyze data sets and determine appropriate linear, quadratic, and exponential models. Algebra II students will evaluate models by analyzing residuals. Algebra I students will be introduced to the concept of residuals in the third lesson. Students will use the models to make predictions.

NCTM Content Standard/National Science Education Standard:

Instructional programs from prekindergarten through grade 12 should enable all students to—

- Understand patterns, relations, and functions.
- Represent and analyze mathematical situations and structures using algebraic symbols.
- Use mathematical models to represent and understand quantitative relationships.
- Analyze change in various contexts.

Grade/Level:

Grades 9-12, Algebra I and/or Algebra II

Duration/Length:

Three 50-minute class periods. Lesson 2 may take longer.

Student Outcomes:

Students will:

- Write the equation of a line of best fit using two points.
- Enter data in lists and create scatter plots.
- Understand how the constants in linear, quadratic, and exponential functions affect the graph.
- Use TI-Interactive graphing to approximate linear and quadratic models.
- Use the TI-83 Plus to create linear, quadratic, and exponential regressions.
- Graph and analyze residuals (Algebra II).
- Determine whether data is better approximated by linear, quadratic, or exponential functions.
- Use models to make predictions.

Materials and Resources:

- Worksheets
- Function Toolbox
  (Source: Howard County Public Schools Algebra II Curriculum Guide)
- Class set of TI-83 Plus calculators with TI-Interactive Graphing installed

Development/Procedures:

Lesson 1

Preassessment – Discuss with students the standard form of a linear equation. Ask students what m represents and wait for responses such as slope and rate of change. Ask students what b represents and wait for responses such as y-intercept and starting value. For Algebra II students, discuss residuals and ask students what types of patterns in graphs of residuals might indicate that another model would be more accurate.

Launch – Ask students to raise their hands if they play different sports (name as many as possible). Ask students to estimate how many students at your school play sports. How many do you think will play next year? How did you choose your number?

Teacher Facilitation – Ask students to read the data and part I. Circulate while students create scatter plots. After students compare their scatter plots, read II with them. Ask the class what a good line of best-fit looks like. Ask the class whether they find the slope or the y-intercept first. Students will write their equations on the board. Ask students which equations are almost the same and why some might be slightly different.

Read III with the students. Demonstrate the steps with the calculator on the overhead projector. When you reach part D, ask students to share answers for 1 and 2. For part E, check that students start with B=1.892 or something reasonably close. For part F, most students will get the same answer.

If the class is an Algebra II class, you can then do a linear regression and evaluate the residuals. Show the steps on the overhead calculator. Discuss what residuals are and why patterns in residuals are significant.

Circulate while students work on part IV. If students are "stuck," provide a prompt. After students have written, ask them to share with their groups, then discuss the answers as a class.

Student Application – Students will analyze the data and models throughout the lesson and with the questions in part IV.

Embedded Assessment – The teacher should circulate and ask questions of both the whole class and of individual students. Check progress on worksheets.

Reteaching/Extension – The homework assignment is a good review for students who need help. The teacher should be available to help with the homework after school. Question 3 on the homework extends the lesson so that student are comparing the two linear equations and determining the solution to the system.

Lesson 2

Preassessment - Students should know the shape of a linear and a quadratic model. They should be able to graph a scatter plot by hand and sketch a line or curve of best fit. They should have already found a linear regression equation either from Lesson 1 or from another activity in class. For Algebra II students, discuss residuals and how their criteria can help determine whether a model is a good fit. Teachers should have all students install TI-Interactive Graphing on their personal calculator prior to this lesson or use a class set that already has it installed.

Launch – Start class with a discussion of gas prices and how they change. How much do you think gas cost 40 years ago? About how much does it cost right now? Ask students what affects gas prices and why the general public is interested in keeping the price low. Why would you (the student) want to keep gas prices low? Teachers could support this discussion with actual figures.

Teacher Facilitation – Ask students to examine the data. Have a graphing calculator and overhead screen set-up. Ask students to complete Part I and the Analysis; circulate around the room to answer questions about the calculator. Ask students to describe the relationship between the number of weeks and price.

Ask students to complete Part II and the Analysis. Circulate to answer questions. Facilitate discussion about whether the linear regression is a good model or not. Ask students what model they chose from the function toolbox as a better model. Various choices should be discussed with the conclusion that quadratic would be the best choice. Algebra II students should complete the Part II extension and Analysis. Ask the students to give the criteria for residuals and then have them discuss why the linear model is not a good choice. Ask students to choose another model based on the pattern of the residuals.

Ask students to complete # 's 1-3 of Part III. Ask students to compare the equation from #3 with the vertex form of a quadratic equation and find the correspondence between (h,k) and (B,C). Ask the students to determine an approximate vertex from looking at the data. Students should complete # 's 4 and 5 and Analysis. Ask the students how they would add just there A value to fit the data better. Students should complete # 6 and write their model. Ask students to send one representative from each group with their A, B, and C values. Students should then average the values for A, B, and C respectively and write a Class Equation.

Ask students to complete the Analysis section at the bottom of Part III using the Class Equation. Allow time for group work and discussion. Facilitate discussion for questions 3 and 4 of the Analysis.

Student Application – Students will analyze the data and models throughout the lesson and with questions in Part IV.

Embedded Assessment – The teacher should circulate and ask questions of individual students. The teacher should check student progress on worksheets. The teacher should get a sense of students understanding from their answers to the analysis questions.

Reteaching/Extension - Part IV (Homework) is a good extension for students. It gives the student a new method for finding a quadratic model and asks them to compare three different models. This lesson also serves as a review of entering data in the lists, creating scatter plots and finding a linear regression model. The teacher should be available for help with calculator operations outside of the classroom instruction. The second question on the homework is for Algebra II students to spiral in their knowledge of converting standard form of a quadratic function to vertex form.

Lesson 3

Preassessment – Show students examples of scatter plots with curves of best fit. Ask students how they know if the graph is a good model for the data.

Launch – Ask students why people might immigrate to the United States and what factors affect the number of people who immigrate.

Teacher Facilitation – Read the introduction with the students and review the data. Make sure they understand the scales for both columns. Circulate while students create scatter plots. Check answers to analysis and briefly discuss with the whole class. Ask students what values they might expect for m and b in the linear regression, then lead them through the steps on the overhead calculator. Ask students for answers to the analysis question. Emphasize the distance from the points to the line.

Ask students how to find the distance between two points on a number line. They should tell you to subtract their values. Explain that the class will subtract the y values of the data (actual) minus the y values of the line (predicted). Demonstrate the steps on the overhead calculator as students work on their own calculators. Circulate as students answer analysis questions. Discuss answers with the class.

Discuss the part V pre-analysis question and ask students to make their predictions for *a* and *b*. Demonstrate the steps for the exponential regression as students work on their own calculators and discuss answers to the analysis question.

Ask students to predict which model is more accurate by sight. Demonstrate part V as students work. Circulate while students work on the final analysis. If students are “stuck,” provide a prompt. After students have completed the analysis, ask them to share with their groups; then discuss the answers as a class.

Student Application – Students will analyze the data and models throughout the lesson and with the analysis questions.

Embedded Assessment – The teacher should circulate and ask questions of both the whole class and of individual students. Check progress on worksheets.

Reteaching/Extension – The homework assignment is a good review for students who need help. The teacher should be available to help with the homework after school. Asking students for reasons for the dramatic increase in Chinese immigration and decrease in Austrian immigration extends the lesson to connect with their knowledge of world history and current affairs.

Summative Assessment:

The homework assignments serve as summative assessments.

Authors:

Sharon Bellino
Winston Churchill High School
Montgomery County, MD

Emily Burrell
Wakefield High School
Arlington County, VA

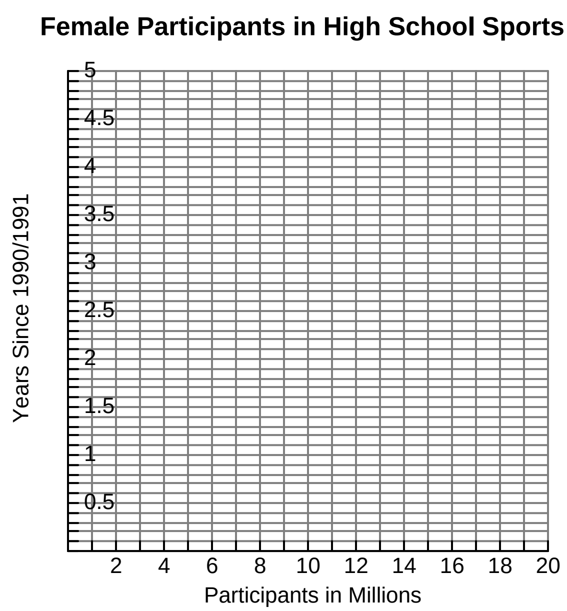
Lesson 1

Who's Playing Ball?

The table below shows the numbers of students participating in high school sports from the 1990-1991 school year to the 2000-2001 school year.

Years	Years Since 1990/1991	Number of **Female** Participants (in millions)	Number of **Male** Participants (in millions)
1990-1991	0	1.891	3.405
1991-1992	1	1.942	3.431
1992-1993	2	1.996	3.415
1993-1994	3	2.131	3.474
1994-1995	4	2.239	3.535
1995-1996	5	2.369	3.635
1996-1997	6	2.471	3.705
1997-1998	7	2.571	3.764
1998-1999	8	2.652	3.831
1999-2000	9	2.677	3.863
2000-2001	10	2.783	3.92

I. Make a scatter plot of female participants per year. Compare scatter plots with group members.

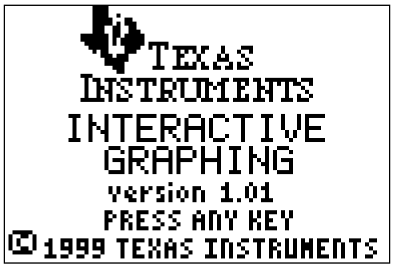


II. Use the scatter plot to write a linear model for your data.

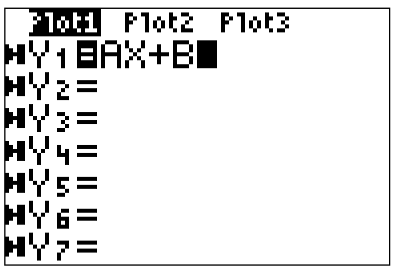
A. Draw a line of best fit.
B. Choose 2 points on your line, find your slope, then find your y-intercept.
C. Equation: ____________
D. Send a representative from your group to write your equation on the board.

III. Use TI-83 Plus to write a linear model for female participants per decade.

A. Enter Data: Press STAT **,** Choose **EDIT**, Enter the Years Since 1990/1991 in L_1 and the Number of Female Participants (in Millions) in L_2.
B. Graph Data: Press ZOOM **,**then **9**.
C. Turn on TI-Interactive Graphing: Press **APPS**, choose **INTERACT**.



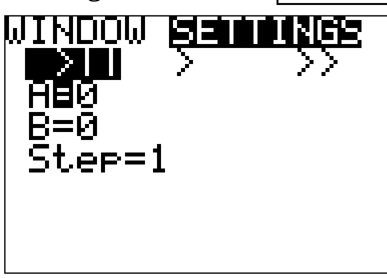
D. Enter the linear equation: Press Y = ,then enter Y_1=AX + B.



1.What do you think A represents?

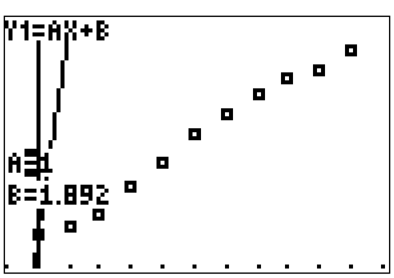
2. What do you think B represents?

E. Set starting values: Press WINDOW , press ⏶ to get to **SETTINGS**.



Set **A=1**. What is a good starting value for B? Set **B= _______**. Set **Step=0.1**

F. Press GRAPH .



1. Does the slope need to be steeper or less steep? ______________
2. Highlight the = next to A. Use the **left arrow** to decrease and the **right arrow** to increase slope.
3. Adjust the B if necessary.
4. Equation: ____________

Algebra II Only

G. Find a linear regression and evaluate the residuals.

1. Press $\boxed{\text{2nd}}$, then $\boxed{0}$, and choose **Diagnostics On**.
2. Press $\boxed{\text{STAT}}$, choose **CALC**, choose **LinReg**.
3. Press $\boxed{\text{VARS}}$, choose **Y-VARS**, choose **FUNCTION**, choose $\mathbf{Y_1}$.

   Press $\boxed{\text{ENTER}}$

4. Linear Regression Equation: ______________
5. Put Residuals in L_3.
   Press $\boxed{\text{STAT}}$, choose **EDIT**, highlight $\mathbf{L_3}$.

   Press $\boxed{\text{2nd}}$, then $\boxed{\text{STAT}}$,choose **RESID**, press $\boxed{\text{ENTER}}$

6. Graph the residuals (L_1, L_3).
   Press $\boxed{\text{2nd}}$, then $\boxed{\text{Y=}}$, choose **2**, switch plot **On**, for Ylist enter $\mathbf{L_3}$.

   Press $\boxed{\text{ZOOM}}$, choose **9**.
   Do the residuals show a pattern? Explain.
   ____

IV. Explain, Justify, Predict.

A. Compare the linear equations found in IIC, IIF4, and IIG4. Explain which method gives a more accurate model and why.

B. Predict the number of female participants in high school sports in the 2010/2011 school year. Explain how you found your answer.

C. Do you think a linear model is reasonable? Why or why not?

Algebra I Only

IV. Explain, Justify, Predict.

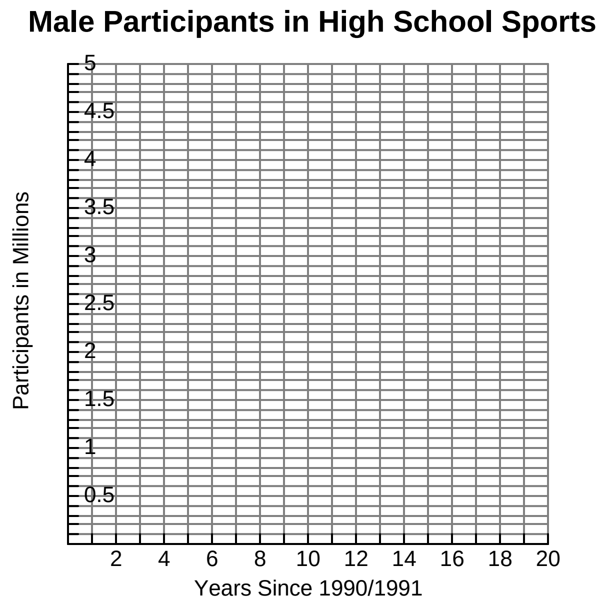
A. Compare the linear equations found in IIC and IIF4. Explain which method gives a more accurate model and why.

B. Predict the number of female participants in high school sports in the 2010/2011 school year. Explain how you found your answer.

C. Do you think a linear model is reasonable? Why or why not?

V. Homework:

1.Write a linear model to express the number of male participants in high school sports in terms of the years since 1990/1991. You may use either method we used in class. A graph is provided if you prefer to graph by hand.



2. Predict the number of male participants in high school sports in the 2010/2011 school year.

3. Explain: Do you think there will ever be more female than male participants in high school sports? Why or why not? If yes, then in what year?

Lesson 1 KEY

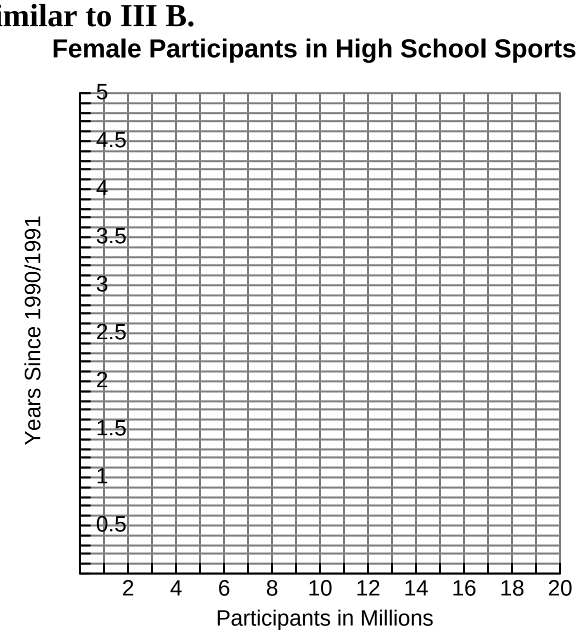
Who's Playing Ball?

The table below shows the numbers of students participating in high school sports from the 1990-1991 school year to the 2000-2001 school year.

Years	Years Since 1990/1991	Number of **Female** Participants (in millions)	Number of **Male** Participants (in millions)
1990-1991	0	1.891	3.405
1991-1992	1	1.942	3.431
1992-1993	2	1.996	3.415
1993-1994	3	2.131	3.474
1994-1995	4	2.239	3.535
1995-1996	5	2.369	3.635
1996-1997	6	2.471	3.705
1997-1998	7	2.571	3.764
1998-1999	8	2.652	3.831
1999-2000	9	2.677	3.863
2000-2001	10	2.783	3.92

I. Make a scatter plot of female participants per year. Compare scatter plots with group members.

Graph should be similar to III B.



II. Use the scatter plot to write a linear model for your data.

A. Draw a line of best fit. **Check, answer will vary.**

B. Choose 2 points on your line, find your slope, then find your y-intercept. **Answers will vary.**

C. Equation: **Answer should be reasonably close to y =.095x + 1.862**

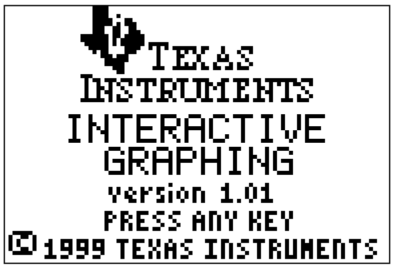
D. Send a representative from your group to write your equation on the board.

III. Use TI-83 Plus to write a linear model for female participants per decade.

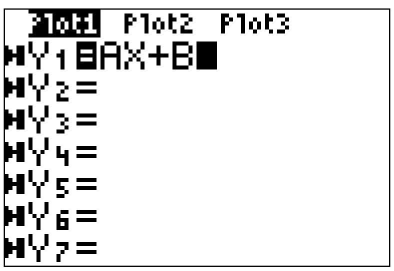
A. Enter Data: Press STAT, Choose **EDIT**, Enter the Years Since 1990/1991 in L_1 and the Number of Female Participants (in Millions) in L_2.

B. Graph Data: Press ZOOM ,then **9**.

C. Turn on TI-Interactive Graphing: Press **APPS**, choose **INTERACT**.



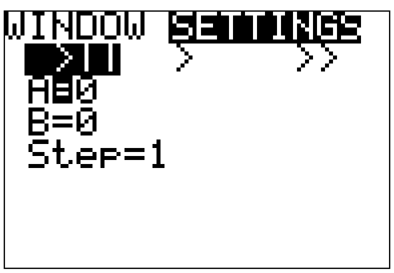
D. Enter the linear equation: Press Y = ,then enter Y_1=AX + B.



1.What do you think A represents? **Acceptable answers include m, slope, rate of change, participants per decade.**

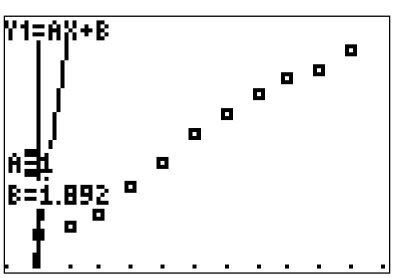
2. What do you think B represents? **Acceptable answers include y-Intercept, starting value and # of Participants in 1990/1991.**

E. Set starting values: Press WINDOW, press ^ to get to **SETTINGS**.



Set **A=1**. What is a good starting value for B? Set **B= 1.891**. Set **Step=0.1**

F. Press GRAPH.



1. Does the slope need to be steeper or less steep? **It must be less steep.**
2. Highlight the = next to A. Use the **left arrow** to decrease and he **right arrow** to increase slope.
3. Adjust the B if necessary.

4. Equation: **Sample: $y=0.1x+1.892$**

Algebra II Only

G. Find a linear regression and evaluate the residuals.

Press 2nd , then 0 and choose **Diagnostics On**.

Press STAT , choose **CALC**, choose **LinReg**.

Press VARS , choose **Y-VARS**, choose **FUNCTION**, choose **Y_1**.

Press ENTER

Linear Regression Equation: **<u>y=.057x + 3.348</u>**

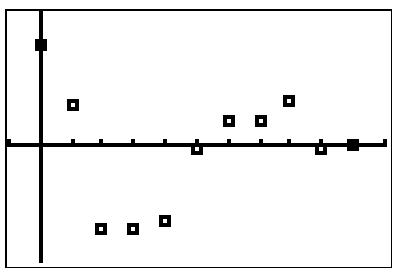
Put Residuals in L_3.
Press STAT choose **EDIT**, highlight **L_3**.

Press 2nd , then STAT ,choose **RESID**, press ENTER

Graph the residuals (L_1, L_3).
Press 2nd , then Y= , choose **2**, switch plot **On**, for Ylist enter **L_3**.

Press ZOOM , choose **9**.

Residuals for Linear Regression



Do the residuals show a pattern? Explain.
The residuals do not show a pattern such as in a linear or Quadratic relationship.

IV. Explain, Justify, Predict.

A. Compare the linear equations found in IIC, IIF4, and IIG4. Explain which method gives a more accurate model and why. **Answers will vary.**

B. Predict the number of female participants in high school sports in the 2010/2011 school year. Explain how you found your answer.
Students may find answers from tables, graphs, or by substituting 20 for x. Answers should be close to 3.768 million female participants in high school sports.

C. Do you think a linear model is reasonable? Why or why not?
Students should base their answers on an evaluation of the residuals.

Algebra I Only

IV. Explain, Justify, Predict.

A. Compare the linear equations found in IIC and IIF4. Explain which method gives a more accurate model and why.

Answers will vary.

B. Predict the number of female participants in high school sports in the 2010/2011 school year. Explain how you found your answer.

Students may find answers from tables, graphs, or by substituting 20 for x. Answers should be close to 3.768 million female participants in high school sports.

C. Do you think a linear model is reasonable? Why or why not?

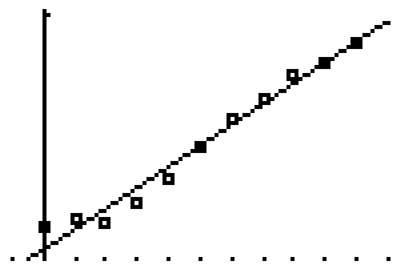
Answers will vary.

V. Homework:

1. Write a linear model to express the number of male participants in high school sports in terms of the years since 1990/1991. You may use either method we used in class. A graph is provided if you prefer to graph by hand.

Answer should be close to y = .057x + 3.348

Scatter Plot w/ Linear Regression:



2. Predict the number of male participants in high school sports in the 2010/2011 school year.

Answers should be close to 4.494 million male participants in high school sports.

a. Explain: Do you think there will ever be more female than male participants in high school sports? Why or why not? If yes, then in what year?

Students should recognize that the female line is increasing more rapidly than the male line and will eventually cross the male line. According to a linear model, this will happen between the 2029/2030 and the 2030/2031 school years.

Lesson 2

Gas Prices are Sky-High!!

The table below shows national average gas prices from selected weeks between March 17, 2003 and August 25, 2003. (Source: U.S. Department of Energy, Energy Information Administration)

U.S. Regular Retail Gasoline Conventional Prices (¢/gal)

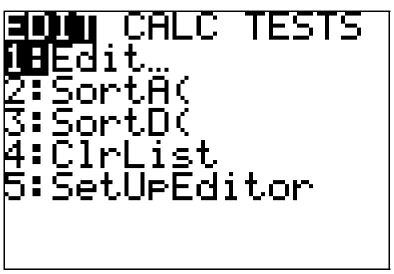
Week	Price Per Gallon (¢/gal)
1	167.3
2	162.6
3	157.7
4	155.7
5	152.1
6	150.4
7	148.6
8	144.1
9	142.7

Week	Price Per Gallon (¢/gal)
12	142.8
16	144.3
17	144.8
20	148.8
21	151.6
22	155
23	159.4
24	169.3

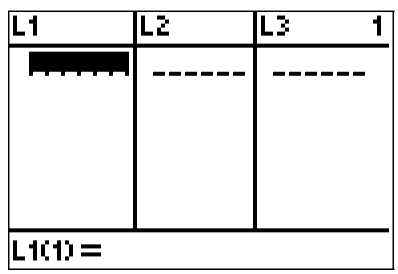
Part I: Create a scatter plot for the data.

To create a scatter plot:

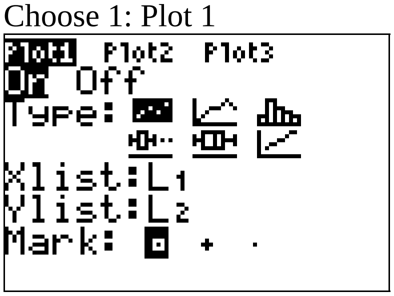
1. Press STAT, Choose 1:Edit



2. Type Week into L1 and Price into L2



3. Press 2nd Y= (STAT PLOT)
   Choose 1: Plot 1



4. Press ZOOM 9

Analysis:
Compare your scatter plot with your group members. Do the scatter plots look the same?

Describe the relationship between the number of weeks and price.

Part II: Find a line of best fit for the data using linear regression and graph it on the scatter plot.

To find the linear regression equation:

1. Press $\boxed{\text{STAT}}$
2. Right arrow ▸ over to CALC
3. Choose 4:LinReg (ax+b)
4. Press $\boxed{\text{ENTER}}$

Write your equation in slope-intercept form (y = mx + b):________________________________

To graph the linear regression on the scatter plot:

1. Press $\boxed{\text{Y=}}$
2. Press $\boxed{\text{VARS}}$
3. Choose 5:Statistics
4. Right arrow ▸ over to EQ
5. Choose 1:RegEQ
6. Press $\boxed{\text{ZOOM}}$ 9

Analysis:
Is the linear regression equation a good model for this data set? Why or why not?

Is there a different model from our function toolbox that you believe would be more appropriate? Justify you answer.

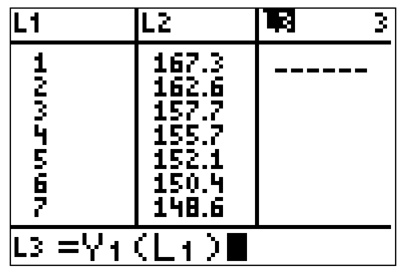
<u>ALGEBRA 2 Part II Extension</u>:

Find a linear regression model that shows the correlation coefficient (r) and variance values (r^2). To do this you must turn on the diagnostics feature.

To turn on the diagnostics feature:
1. Press 2nd 0 (Catalog)
2. Press x^{-1}(D)
3. Scroll down to DiagnosticOn, Press ENTER
4. Press ENTER again

Write your model in slope-intercept form:____

Create a table with Predicted Gas Prices and Residuals.



To fill in a list using your model:
1. Press STAT , choose EDIT
2. Arrow up and highlight L3, press ENTER
3. Press VARS , ▸ to Y-VARS
4. Choose 1:Function, choose 1:Y1, then type (L1)
5. Press ENTER

To find residuals (the quick way):
1. Arrow up and highlight L4, Press ENTER
2. Press 2nd STAT (LIST), choose RESID, Press ENTER

Fill in the table with your predicted values (L3) and residuals (L4).

L1 Weeks	L2 Actual Price ($)	L3 = Y1(L1) Predicted Price ($)	L4 = RESID Residuals
1	167.3		
2	162.6		
3	157.7		
4	155.7		
5	152.1		
6	150.4		
7	148.6		
8	144.1		
9	142.7		
12	142.8		
16	144.3		
17	144.8		
20	148.8		
21	151.6		
22	155		
23	159.4		
24	169.3		

Graph the Residuals.

1. Press $\boxed{\text{Y=}}$
2. Arrow up ▲ to Plot 1 and turn it off by pressing $\boxed{\text{ENTER}}$ to un-highlight
3. Arrow down ▼ to the = sign for Y1 and turn it off by pressing $\boxed{\text{ENTER}}$
4. Press $\boxed{\text{2nd}}$ $\boxed{\text{Y=}}$, choose 2:Plot 2
5. Press $\boxed{\text{ENTER}}$ to turn it On
6. Change your Ylist to L4 to view, Press $\boxed{\text{ZOOM}}$ 9

Analysis:
Based on your knowledge of residuals, is the linear regression model appropriate for this data? Explain.

What does the shape of the residuals suggest? Is there another model that would be more appropriate? If so, what model would you choose?

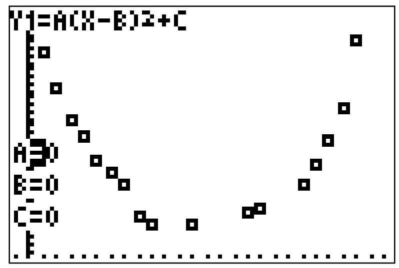
Part III: Find a Quadratic model for the data using Interactive Graphing.

To find a quadratic model using Interact:

1. Press blue APPS key, choose Interact
2. Press Y=
3. Clear Y1 and type in Y1 = A(X – B)^2 + C

Analysis:
How do B and C in the above equation correspond to the vertex form of a quadratic equation?____

What is the approximate vertex of the data set?_________



4. Press GRAPH
5. To change A, B or C: highlight the = sign, type a number and press ENTER

   Start with A = 1
   Choose values for B and C based on the answer to # 3.
   B = __________   C = __________

Analysis:
Describe the shape of your model with respect to the scatter plot of the original data.
____

6. Based on your knowledge of how A affects the quadratic model, adjust it accordingly. Change B and C if necessary. Record your quadratic model below.

   YOUR Quadratic Model:_______________________________________

Send a representative from the group to the board to record the values you chose for A, B and C. Average the values for A, B, and C respectively and record a class equation. Round your answer to three decimal places.

CLASS Quadratic Model:_______________________________________

Analysis: Answer the following questions using the CLASS quadratic model.

1. What is the minimum gas price during this time period?___________________
2. What is the approximate week that the lowest gas price occurs?_____________
3. Predict how much gas will cost a year later in March 2004 (week 52)._______
   Is this reasonable? Explain.
4. Is the quadratic equation an appropriate model for gas prices over a longer period of time?________ If not, sketch an example of what that model might look like.

HOMEWORK

Part IV: Find a quadratic model using quadratic regression and graph it on the scatter plot.

To find a model using quadratic regression:

1. Enter data into L1 and L2
2. Press $\boxed{\text{STAT}}$
3. Right arrow ▸ over to CALC
4. Choose 5: QuadReg
5. Press $\boxed{\text{ENTER}}$

Write your quadratic regression equation in standard form ($y = ax^2 + bx + c$):________________________________.

To graph the quadratic regression on the scatter plot:

1. Press $\boxed{\text{Y=}}$
2. Press $\boxed{\text{VARS}}$
3. Choose 5:Statistics
4. Right arrow ▸ over to EQ
5. Choose 1:RegEQ
6. Press $\boxed{\text{ZOOM}}$ 9

Analysis:
Of the three models we have used for this data, which one do you think is the best for approximating the curve of best fit? Explain.

Algebra 2 Extension: Is it possible to convert from the quadratic regression (standard form) to the interact model (vertex form)? If so, try it.

Lesson 2 KEY

Gas Prices are Sky-High!!

The table below shows national average gas prices from selected weeks between March 17, 2003 and August 25, 2003. (Source: U.S. Department of Energy, Energy Information Administration)

U.S. Regular Conventional Retail Gasoline Prices (¢/gal)

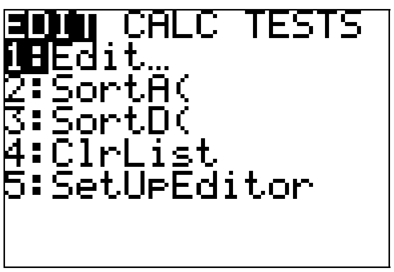
Week	Price Per Gallon (¢/gal)
1	167.3
2	162.6
3	157.7
4	155.7
5	152.1
6	150.4
7	148.6
8	144.1
9	142.7

Week	Price Per Gallon (¢/gal)
12	142.8
16	144.3
17	144.8
20	148.8
21	151.6
22	155
23	159.4
24	169.3

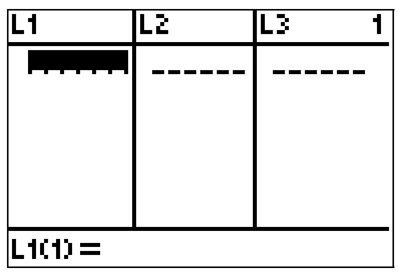
Part I: Create a scatter plot for the data.

To create a scatter plot:

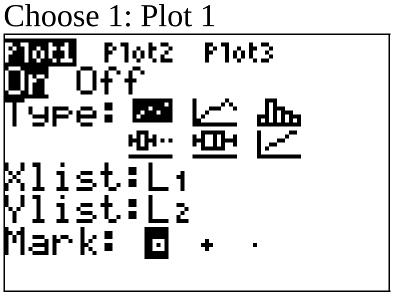
1. Press STAT, Choose 1:Edit



2. Type Week into L1 and Price into L2



3. Press 2nd Y= (STAT PLOT)
   Choose 1: Plot 1



4. Press ZOOM 9

Analysis:

Compare your scatter plot with your group members. Do the scatter plots look the same?

The plots should look the same if the students have all entered the data correctly

Describe the relationship between the number of weeks and price

> **Students should describe the relationship as decreasing and then increasing, quadratic, symmetrical, etc. Answers may vary depending on their previous knowledge.**

Part II: Find a line of best fit for the data using linear regression and graph it on the scatter plot.

To find the linear regression equation:

1. Press $\boxed{\text{STAT}}$
2. Right arrow ▸ over to CALC
3. Choose 4:LinReg (ax+b)
4. Press $\boxed{\text{ENTER}}$

Write your equation in slope-intercept form (y = mx + b):_**y= -.042x + 153.266**

To graph the linear regression on the scatter plot:

1. Press $\boxed{\text{Y=}}$
2. Press $\boxed{\text{VARS}}$
3. Choose 5:Statistics
4. Right arrow ▸ over to EQ
5. Choose 1:RegEQ
6. Press $\boxed{\text{ZOOM}}$ 9

Analysis:
Is the linear regression equation a good model for this data set? Why or why not?

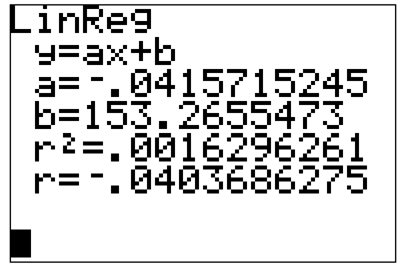
No.
Answers could include: it doesn't fit the data well, the shape is wrong, it decreases the whole time instead of decreasing and then increasing.

Is there a different model from our function toolbox that you believe would be more appropriate? Justify you answer.

Answers could include: quadratic, absolute value, etc.

<u>ALGEBRA 2 Part II Extension</u>:

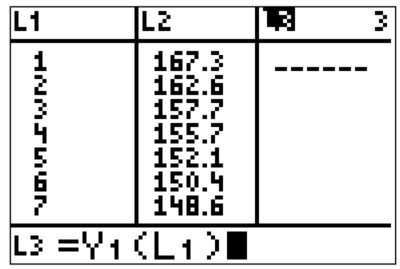
Find a linear regression model that shows the correlation coefficient (r) and variance values (r^2). To do this you must turn on the diagnostics feature.



To turn on the diagnostics feature:
1. Press 2nd 0 (Catalog)
2. 2.Press x^{-1}(D)
3. Scroll down to DiagnosticOn, Press ENTER
4. Press ENTER again

Write your model in slope-intercept form: $y = -.042x + 153.266$

Create a table with Predicted Gas Prices and Residuals.



To fill in a list using your model:
1. Press STAT, choose EDIT
2. Arrow up and highlight L3, press ENTER
3. Press VARS, ▸ to Y-VARS
4. Choose 1:Function, choose 1:Y1, then type (L1)
5. Press ENTER

To find residuals (the quick way):
1. Arrow up and highlight L4, Press ENTER
2. Press 2nd STAT (LIST), choose RESID, Press ENTER

Fill in the table with your predicted values (L3) and residuals (L4).

L1 Weeks	L2 Actual Price ($)	L3 = Y1(L1) Predicted Price ($)	L4 = RESID Residuals
1	167.3	153.22	14.08
2	162.6	153.18	9.42
3	157.7	153.14	4.56
4	155.7	153.1	2.60
5	152.1	153.06	-0.96
6	150.4	153.02	-2.62
7	148.6	152.97	-4.37
8	144.1	152.93	-8.83
9	142.7	152.89	-10.19
12	142.8	152.77	-9.97
16	144.3	152.6	-8.30
17	144.8	152.56	-7.76
20	148.8	152.43	-3.63
21	151.6	152.39	-0.79
22	155	152.35	2.65
23	159.4	152.31	7.09
24	169.3	152.27	17.03

Graph the Residuals.

7. Press $\boxed{\text{Y=}}$
8. Arrow up ▲ to Plot 1 and turn it off by pressing $\boxed{\text{ENTER}}$ to un-highlight
9. Arrow down ▼ to the = sign for Y1 and turn it off by pressing $\boxed{\text{ENTER}}$
10. Press $\boxed{\text{2nd}}$ $\boxed{\text{Y=}}$, choose 2:Plot 2
11. Press $\boxed{\text{ENTER}}$ to turn it On
12. Change your Ylist to L4 to view, Press $\boxed{\text{ZOOM}}$ 9

Analysis:
Based on your knowledge of residuals, is the linear regression model appropriate for this data? Explain.

No. They form a pattern. They are not scattered evenly. The range of the residuals is too large. The correlation coefficient and variance values are poor. Other answers may be appropriate.

What does the shape of the residuals suggest? Is there another model that would be more appropriate? If so, what model would you choose?

The shape of the residual graph indicates that a quadratic model may be a good choice.

Part III: Find a Quadratic model for the data using Interactive Graphing.

To find a quadratic model using Interact:

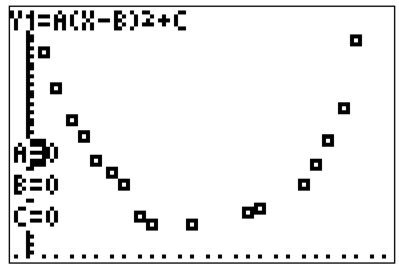
1. Press blue APPS key, choose Interact
2. Press [Y=]
3. Clear Y1 and type in Y1 = $A(X - B)^2 + C$

Analysis:
How do B and C in the above equation correspond to the vertex form of a quadratic equation?

B corresponds to h and C corresponds to k. If (h,k) is the vertex then (B,C) would be the corresponding vertex.

What is the approximate vertex of the data set? **(12,142)**

4. Press [GRAPH]
5. To change A, B or C: highlight the = sign, type a number and press [ENTER]



Start with A = 1
Choose values for B and C based on the answer to # 3.
B = **12** C = **142**

Analysis:
Describe the shape of your model with respect to the scatter plot of the original data.

With an A value of 1, the model is narrower than the data set.

6. Based on your knowledge of how A affects the quadratic model, adjust it accordingly. Change B and C if necessary. Record your quadratic model below.

YOUR Quadratic Model: **Answers should be near** $y = .18(x-13)^2 + 142$

Send a representative from the group to the board to record the values you chose for A, B and C. Average the values for A, B, and C respectively and record a class equation. Round your answer to three decimal places.

CLASS Quadratic Model: **Answers will vary depending on collected data.**

Answer the following questions using the CLASS quadratic model:

5. What is the minimum gas price during this time period? **Answers will vary.**

6. What is the approximate week that the lowest gas price occurs? **Answers will vary.**

7. Predict how much gas will cost a year later in March 2004 (week 52).
Is this reasonable? Explain. **Answers will vary.**

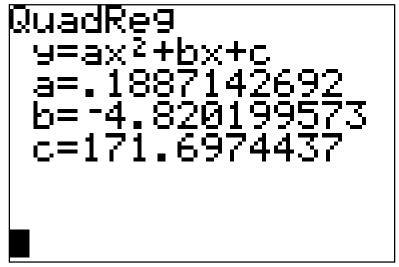
8. Is the quadratic equation an appropriate model for gas prices over a longer period of time? **<u>NO</u>** If not, sketch an example of what that model might look like. **Graphs should show gas prices rising and falling, i.e. sine/cosine waves.**

HOMEWORK

Part IV: Find a quadratic model using quadratic regression and graph it on the scatter plot.

To find a model using quadratic regression:

1. Enter data into L1 and L2
2. Press STAT
3. Right arrow ▸ over to CALC
4. Choose 5: QuadReg
5. Press ENTER

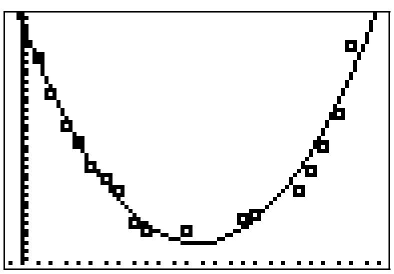


Write your quadratic regression equation in standard form ($y = ax^2 + bx + c$):

$y = .189x^2 - 4.82x + 171.697$

To graph the quadratic regression on the scatter plot:

1. Press Y=
2. Press VARS
3. Choose 5:Statistics
4. Right arrow ▸ over to EQ
5. Choose 1:RegEQ
6. Press ZOOM 9



Analysis:

Of the three models we have used for this data, which one do you think is the best for approximating the curve of best fit? Explain.

Answers may vary. Students should hopefully recognize that the linear graph is a poor model as compared to either the interactive model or the quadratic regression.

Algebra 2: Is it possible to convert from the quadratic regression (standard form) to the interact model (vertex form)? If so, try it.

Yes, by completing the square. $y = .189(x - 12.751)^2 + 140.966$

Lesson 3

Coming to America

The table below gives the immigration figures (in thousands) from China to the United States from the 1950's through the 1980's. The x values are defined as the number of decades from the 1950's, i.e. 1951-1960 = 0.

Immigrants to U.S. from China

Decade	X	# of immigrants (in thousands)
1951-1960	0	9.656
1961-1970	1	34.765
1971-1980	2	124.325
1981-1990	3	388.687

I. Create a scatter plot for the data.

To create a scatter plot:

1. Press STAT, Choose 1:Edit
2. Type X values into L1 and # of immigrants into L2
3. Press 2nd Y= (STAT PLOT), Choose 1: Plot 1
4. Press ZOOM 9

Analysis:
Compare your scatter plot with your group members. Describe the relationship between the number of decades since 1950 and the number of Chinese immigrating to the U.S.

II. Find a line of best fit for the data using linear regression.

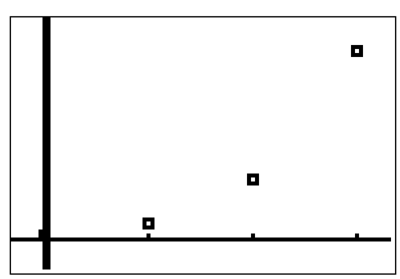
To find the linear regression equation:

1. Press STAT
2. Right arrow ▸ over to CALC
3. Choose 4:LinReg (ax+b)
4. Press ENTER

A. Write your equation in slope-intercept form (y = mx + b):___________________________

B. Graph the linear regression on the scatter plot:

1. Press Y=
2. Press VARS
3. Choose 5:Statistics
4. Right arrow ▸ over to EQ



5. Choose 1:RegEQ
6. Press $\boxed{\text{ZOOM}}$ 9

C. Sketch the line of best fit on the graph to the right.

Analysis:
Is the linear regression a good model? Explain.

III. Compare actual immigration with values predicted by the linear regression equation.

Fill in L3 (predicted values) by plugging your X values into your linear regression equation.

To fill in a list using your model:
1. Press $\boxed{\text{STAT}}$, choose EDIT
2. Arrow up and highlight L3, press $\boxed{\text{ENTER}}$
3. Press $\boxed{\text{VARS}}$, ▸ to Y-VARS
4. Choose 1:Function, choose 1:Y1, then type (L1)
5. Press $\boxed{\text{ENTER}}$

Fill in L4 by finding the difference between the actual immigration figures and the predicted
immigration figures.

To find differences:
1. Arrow up and highlight L4, Press $\boxed{\text{ENTER}}$
2. Type $\boxed{\text{2nd}}$ 2 - $\boxed{\text{2nd}}$ 3 (L2-L3), Press $\boxed{\text{ENTER}}$

Fill in the table below with your values for L3 and L4.

L1 = X	**L2 = Y (actual immigration)**	**L3 = Y1(L1) (predicted immigration)**	**L4 = L2-L3 (actual – predicted)**
0	9.656		
1	34.765		
2	124.325		
3	388.687		

Analysis:
Based on the differences between the actual population and the predicted population, is the linear regression a good model for the data? Justify your answer.

Do you think a quadratic model could fit the data? Why or why not? Is there another model that would fit better than either linear or quadratic?

IV. Find an Exponential Model for the data.

Pre-Analysis:
Given $y = a(b)^x$, what do *a* and *b* represent? What do they represent in the context of the problem?

Predict values for *a* and *b* with respect to the data. $a =$ __________ $b =$ __________

Find a curve of best fit for the data using exponential regression.

1. Press STAT, Right arrow ▸ over to CALC
2. Choose 0:ExpReg, Press ENTER

Write your equation in standard form ($y = a(b)^x$) Round to three decimal places.

Exponential Regression Equation:_____________________________________

Analysis: Compare your *a* and *b* values from your exponential regression model to those that you predicted. Are they different? Why?

V. Compare actual immigration with values predicted by the exponential regression equation.

1. Clear your linear regression equation from o and transfer your exponential regression equation to Y1.

2. Fill in L3 (predicted values) by plugging your X values into your exponential regression equation.

   Arrow up and highlight L3, press ENTER
   Press VARS, ▸ to Y-VARS, Choose 1:Function, choose 1:Y1, then type (L1), Press ENTER

3. Fill in L4 by finding the difference between the actual immigration figures and the predicted immigration figures.

4. Fill in the table below with your values for L3 and L4.

L1 = X	L2 = Y (actual immigration)	L3 = Y1(L1) (predicted immigration)	L4 = L2-L3 (actual – predicted)
0	9.656		
1	34.765		

2	124.325		
3	388.687		

Analysis:

Based on the differences between the actual population and the predicted population, is the exponential regression a good model for the data? Justify your answer.

According to your exponential model, in what decade will the number of immigrants be 4,800,000 (4800 thousand)?

What factors do you think could have caused this dramatic increase in immigration from China to the U.S.?

Can you think of another country whose immigration numbers (to the U.S.) have grown exponentially?

HOMEWORK

The table below gives the immigration figures (in thousands) from Austria to the United States from the 1950's through the 1980's. The x values are defined as the number of decades from the 1950's, i.e. 1951-1960 = 0.

Immigrants to U.S. from Austria

Decade	X	# of immigrants (in thousands)
1951-1960	0	67.105
1961-1970	1	20.622
1971-1980	2	9.477
1981-1990	3	4.637

1. Enter your data in to L1 and L2 and create a scatter plot.

2. Choose an appropriate model for the data and do the regression.

   What model did you choose?___________________________

   Write your regression equation:_________________________

3. Graph the regression on the scatter plot. Is your choice a good model for the data? Justify your answer.

Analysis:

According to your model, in what decade will the number of immigrants from Austria be 9 people (.009 thousand)?

In the 1941-1950 decade, 24,480 people immigrated from Austria to the U.S. What factors do you think would have caused the number to jump in the next decade to 67,106 and then start decreasing?

Can you think of other countries whose immigration numbers have decreased exponentially?

Lesson 3 KEY

Coming to America

The table below gives the immigration figures (in thousands) from China to the United States from the 1950's through the 1980's. The x values are defined as the number of decades from the 1950's, i.e. 1951-1960 = 0.

Immigrants to U.S. from China

Decade	X	# of immigrants (in thousands)
1951-1960	0	9.656
1961-1970	1	34.765
1971-1980	2	124.325
1981-1990	3	388.687

I. Create a scatter plot for the data.

To create a scatter plot:

1. Press STAT , Choose 1:Edit
2. Type X values into L1 and # of immigrants into L2
3. Press 2nd Y= (STAT PLOT), Choose 1: Plot 1
4. Press ZOOM 9

Analysis:
Compare your scatter plot with your group members. Describe the relationship between the number of decades since 1950 and the number of Chinese immigrating to the U.S.
Students should describe the positive trend of the data and may guess that it would fit an exponential or linear model.

II. Find a line of best fit for the data using linear regression.

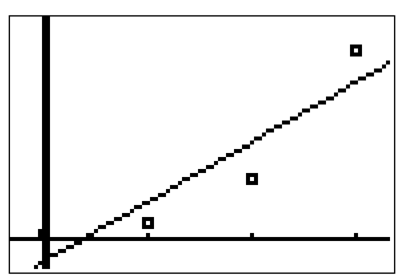
To find the linear regression equation:

1. Press STAT
2. Right arrow ▸ over to CALC
3. Choose 4:LinReg (ax+b)
4. Press ENTER

A. Write your equation in slope-intercept form (y = mx + b): **<u>y = 122.665x – 44.639</u>**

B. Graph the linear regression on the scatter plot:

1. Press Y=
2. Press VARS
3. Choose 5:Statistics
4. Right arrow ▸ over to EQ
5. Choose 1:RegEQ



6. Press ZOOM 9

C. Sketch the line of best fit on the graph to the right.

Analysis:
Is the linear regression a good model? Explain.

Students should explain that while the linear model shows a positive trend, the data points are too far from the line.

III. Compare actual immigration with values predicted by the linear regression equation.

Fill in L3 (predicted values) by plugging your X values into your linear regression equation.

To fill in a list using your model:

1. Press STAT , choose EDIT
2. Arrow up and highlight L3, press ENTER
3. Press VARS , Y= to Y-VARS
4. Choose 1:Function, choose 1:Y1, then type (L1)
5. Press ENTER

Fill in L4 by finding the difference between the actual immigration figures and the predicted
immigration figures.

To find differences:

1. Arrow up and highlight L4, Press ENTER
2. Type 2nd 2 - 2nd 3 (L2-L3), Press ENTER

Fill in the table below with your values for L3 and L4.

L1 = X	L2 = Y (actual immigration)	L3 = Y1(L1) (predicted immigration)	L4 = L2-L3 (actual – predicted)
0	9.657	***-44.64***	***54.296***
1	34.764	***78.026***	***-43.26***
2	124.326	***200.69***	***-76.36***
3	388.686	***323.36***	***65.33***

Analysis:
Based on the differences between the actual population and the predicted population, is the linear regression a good model for the data? Justify your answer.

Students should recognize that points are too far from the line as shown in L4 (L2-L3).

Do you think a quadratic model could fit the data? Why or why not? Is there another model that would fit better than either linear or quadratic?

Students may dismiss the quadratic model because there is no indication that immigration from China would decline. They should recognize that the pattern of the data fits an exponential curve.

IV. Find an Exponential Model for the data.

Pre-Analysis:

Given $y = a(b)^x$, what do a and b represent? What do they represent in the context of the problem?

a **is the initial amount and the y-intercept.** ***b*** **is the constant base;** ***b*** **determines whether the function is increasing or decreasing.** ***a*** **represents the initial # of immigrants from China in the 1951-1960 decade.** ***b*** **should be greater than 1 since the data is increasing.**

Predict values for *a* and *b* with respect to the data. a = **9.657** b = **any number > 1 acceptable**

Find a curve of best fit for the data using exponential regression.

1. Press STAT , Right arrow ▸ over to CALC
2. Choose 0:ExpReg, Press ENTER

Write your equation in standard form ($y = a(b)^x$) Round to three decimal places.

Exponential Regression Equation: $y = 9.940(3.442)^x$

Analysis: Compare your *a* and *b* values from your exponential regression model to those that you predicted. Are they different? Why?

Students should notice that ***a*** **is similar, but may have underestimated** ***b*****.**

V. Compare actual immigration with values predicted by the exponential regression equation.

1. Clear your linear regression equation from Y= and transfer your exponential regression equation to Y1.

2. Fill in L3 (predicted values) by plugging your X values into your exponential regression equation.
   Arrow up and highlight L3, press ENTER
   Press VARS , ▸ to Y-VARS, Choose 1:Function, choose 1:Y1, then type (L1),

   Press ENTER

3. Fill in L4 by finding the difference between the actual immigration figures and the predicted immigration figures.

4. Fill in the table below with your values for L3 and L4.

L1 = X	L2 = Y	L3 = Y1(L1)	L4 = L2-L3

	(actual immigration)	(predicted immigration)	(actual – predicted)
0	9.657	***9.9398***	***-.2828***
1	34.764	***34.21***	***.55439***
2	124.326	***117.74***	***6.5875***
3	388.686	***405.22***	***-16.53***

Analysis:

Based on the differences between the actual population and the predicted population, is the exponential regression a good model for the data? Justify your answer.

Yes, the exponential regression is a good model. The exponential model is closer to the data since the differences between the exponential model and the data are smaller than the differences between the linear model and the data.

According to your exponential model, in what decade will the number of immigrants be 4,800,000 (4800 thousand)?

2001-2010

What factors do you think could have caused this dramatic increase in immigration from China to the U.S.?

A number of events made immigration from China increasingly easier. In 1952, the McCarren-Walter Act made it illegal to use race as a barrier to immigration. National quotas (regulations on how many immigrants were allowed in from any one country) were ended in 1965. In 1979, President Carter visited China, thus resuming diplomatic relations. The Immigration Act of 1990 also made immigration from China easier.

Can you think of another country whose immigration numbers (to the U.S.) have grown exponentially?

Answers will vary, students may mention Central and South American countries, Mexico and India.

HOMEWORK

The table below gives the immigration figures (in thousands) from Austria to the United States from the 1950's through the 1980's. The x values are defined as the number of decades from the 1950's, i.e. 1951-1960 = 0.

Immigrants to U.S. from Austria

Decade	X	# of immigrants (in thousands)
1951-1960	0	67.105
1961-1970	1	20.622
1971-1980	2	9.477
1981-1990	3	4.637

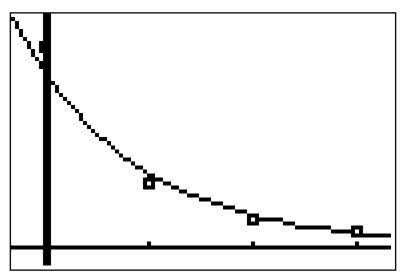
1. Enter your data in to L1 and L2 and create a scatter plot.

2. Choose an appropriate model for the data and do the regression.

   What model did you choose? **Exponential Decay**

   Write your regression equation: $y = 58.735(0.415)^x$

3. Graph the regression on the scatter plot. Is your choice a good model for the data? Justify your answer.



Yes, this is a good model since the points are reasonably close to the line.

Analysis:

According to your model, in what decade will the number of immigrants from Austria be 9 people (.009 thousand)?

2051-2060

In the 1941-1950 decade, 24,480 people immigrated from Austria to the U.S. What factors do you think would have caused the number to jump in the next decade to 67,106 and then start decreasing?

Answers may refer to World War II.

Can you think of other countries whose immigration numbers have decreased exponentially?

Students may mention other European countries affected by World War II.