Title: "A Slice of the Cone"

Link to Outcomes:

- **Problem Solving** Students will demonstrate the ability to classify conics and to solve real-life mathematical and scientific problems cooperatively with the use of technology.
- **Communication** Students will demonstrate the ability to identify a group of quadratic polynomial equations and make generalizations.
- **Reasoning** Students will demonstrate the ability to reason logically in generalizing equations of specific conics.
- **Connections** Students will see the relationship algebraic and geometric concepts have on conics.
- **Cooperation** Students will demonstrate the ability to solve problems using cooperative learning skills.
- **Functions** Students will demonstrate the ability to work real-life problems that may or may not be modeled by functions.
- **Technology** Students will use the TI-82 graphics calculator to verify graphs of conic sections.
- **Algebra** Students will develop quadratic models from the definitions of the particular conic under investigation.
- **Geometry** Students will recognize a particular conic by its graph.
- **Real-World Applications** Students will demonstrate their ability to apply mathematical concepts to real-world situations, using astronomy, architecture, history, and bicycling.

Brief Overview:

Quadratic equations are often used as algebraic models for real-life situations. This unit is designed to provide a cooperative learning environment in which students solve, analyze, and apply several quadratic models.

Students will be assigned to cooperative learning groups (4-5/group) to identify and differentiate conics according to their equations and graphs. This will be further facilitated by the use of the TI-82 graphics calculator.

Applying previous knowledge of quadratic equations, the students will use ice cream cones and flashlights to investigate how each conic is produced by the way a plane cuts the cone.

Grade/Level:

Grades 10-12; Algebra II, Algebra III, or Analytic Geometry, Pre-calculus

Duration/Length:

This unit can range from three to five days depending on the course and the level of the students.

Prerequisite Knowledge:

- Students should be able to solve for one variable in terms of another.
- Students should be able to write equations of a straight line.
- Students should be able to use the distance formula to find the distance between two points.
- Students should be able to use the midpoint formula to find the midpoint of a line segment.
- Students should be able to recognize the geometric shapes of a circle, parabola, ellipse, and a hyperbola.
- Students should be able to understand the concepts of translations, rotations, and reflections.
- Students should know the Pythagorean Theorem.
- Students should be able to complete the square.
- Students should be able to solve quadratic equations by factoring, completing the square, and using the quadratic equation.
- Students should be able to solve systems of equations.
- Students should be able to identify x and y intercepts.
- Students should know what is meant by the discriminant and how to calculate it.
- Students should know how to solve square root equations.
- Students should know how to find the area of a circle.
- Students should know the speed of sound.
- Students should know how many feet are in a mile.

Objectives:

- Identify a particular conic by its graph.
- Identify a particular conic by its equation.
- Take a real-life situation and identify the conic.
- Determine the center, the radius, and the eccentricity of a circle.
- Determine the vertex, focus, the length of the focal radius, directrix, and the eccentricity of a parabola.
- Determine the vertices, the foci, the length of the focal radii, the center, the length of the major axis, the length of the minor axis, and the eccentricity of an ellipse.
- Determine the vertices, the foci, the length of the focal radii, the center, the transverse axis, the conjugate axis, the equations of the asymptotes, and the eccentricity of a hyperbola.

Materials/Resources/Printed Materials:

- Flashlights
- TI-82 graphics calculator
- Graph paper
- Instructions to the graphics calculator
- Instructions for the group activities
- Analysis worksheet
- Follow-up worksheets
- Extension worksheets

Development/Procedures:

The solving of conics involves the use of several skills listed in the prerequisites. The activities have been structured so as to measure the integration of the skills, and the visual recognition of conics and their equations on an individual and group basis. Students are encouraged to remain grounded in the physical world by using applied problems.

Teachers will be able to assess students' ability to comprehend the horizontal relationship between conics, their respective equations, and their natural use and presence in the world. They will be circulating in the classroom as students work.

Exercises are sequenced from simple, experimental exercises through cooperative activities and worksheets to ensure recognition of conics and solutions to problems. The tasks are to be completed within a 45 minute period. The hierarchy of activities is as follows:

1. Activity #1: Shapes Around You
2. Worksheet #1: To record findings from activity #1
3. Activity #2: Conic Matching to Their Equations in Standard Form
4. Activity #3: TI-82 Graphics Calculator Instructions
5. Worksheet #2: Conic Identification From the Equation in General Form
6. Answer sheet to worksheet #2 for teachers
7. Activity #4: A Real Life Situation Problem Involving Profit
8. Worksheet #3: Sketching the Graphs of the Conics
9. Worksheet #4: Charting the Different Parts Of The Conics
10. Worksheet #5: Cooperative Treasure Hunt Involving Four Real Life Problems
11. Extensions for Remediation
12. Extensions for Enrichment

This lesson can be used by a wide range of teachers of students with varied math ability levels. Some activities can be completed without the aid of technology.

Evaluation:

Group participation and the completion of worksheets are monitored by the teacher. Groups will consist of 4-5 members.
Each group member will be prepared to give an oral presentation of each conic and its equation using correct terminology.

Each group member will provide a written description of the method of changing the general form to standard form of the equation by completing the square. The members will demonstrate the use of technology such as the TI-82 graphics calculator and/or computer printout to explain solutions and facilitate computation.

Extension/Follow Up:

The extensions and follow-up activities serve a two-fold purpose:

1. remediation for students who still have problems with particular conics;
2. enrichment for students who have mastered the skills and need further challenge.

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ACTIVITY #1: Shapes Around You

Purpose:

Cooperative learning activity for identifying conics in their lives.

Materials Needed:

- flashlight
- empty toilet tissue rolls
- empty paper cylinders
- ice cream cones-sugar or plain
- a darkened classroom

Directions:

Students are divided into pairs. Each one is to work with a partner to complete the activities.

Overview:

Circles, parabolas, hyperbolas, and ellipses are known as conic sections because they are formed as cross sections of a cone when the cone is intersected by a plane. A circle can be considered a special type of ellipse.

To demonstrate this concept, you are to perform the following experiment:

Procedure:

One student will hold the flashlight, turn it on, and hold it with the light against the wall.

The light from the flashlight held in the air is a conic. When light hits a flat surface, like a wall, a conic section can be formed by the boundaries of light.

In a darkened room, shine the flashlight against the wall so that the edge forms a circle, a parabola, an ellipse, and a hyperbola.

Worksheet for Activity #1

1. **Describe how you can do this.**

2. **Draw or sketch what you and your partner observe.**

3. **Describe two other objects that are in the form of each conic.**

ellipse	**hyperbola**
_____	_____
_____	_____
circle	**parabola**
_____	_____
_____	_____

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Activity #2

Conic Matching to Their Equations in Standard Form

Directions: Write the appropriate letter of the graph that the equation of the conic represents below.

1. $y^2 = 4x$

2. $(x - 2)^2 + (y - 1)^2 = 25$

3. $\frac{x^2}{3} - \frac{y^2}{4} = 1$

4. $x^2 + y^2 = 9$

5. $\frac{x^2}{4} + \frac{y^2}{3} = 1$

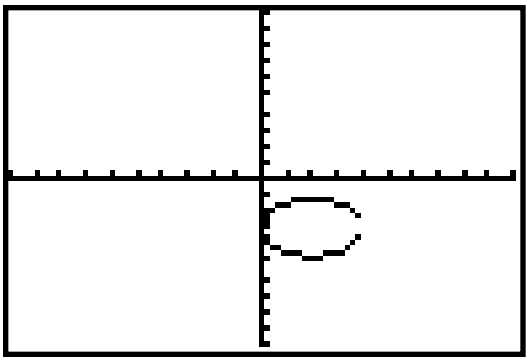
6. $\frac{(x - 2)^2}{4} + \frac{(y + 3)^2}{3} = 1$

7. $\frac{(x + 3)^2}{3} - \frac{(y - 2)^2}{4} = 1$

8. $(x + 1)^2 = 4(y - 1)$

9. $(y - 2)^2 = -4(x + 3)$

10. $(x + 1)^2 + (y - 3)^2 = 16$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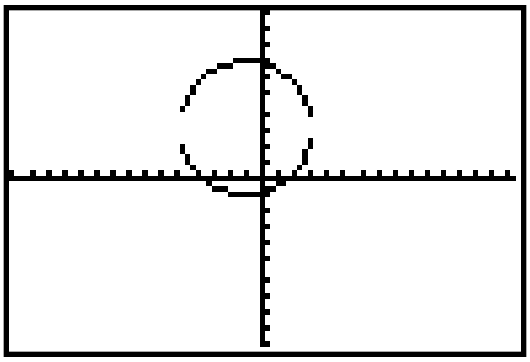
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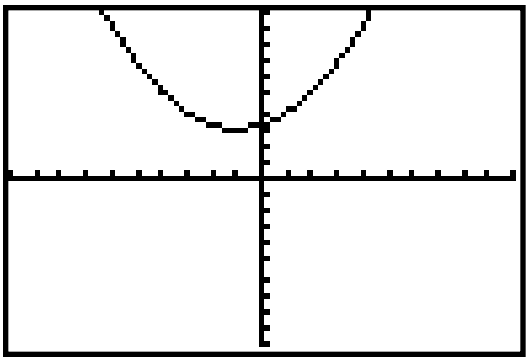
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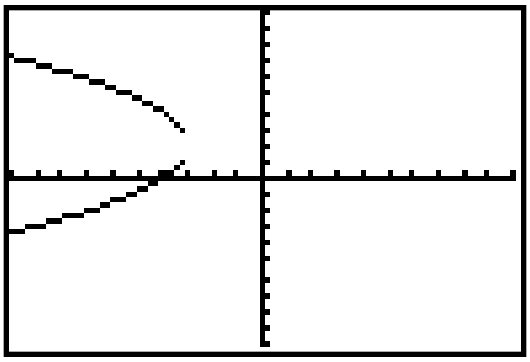
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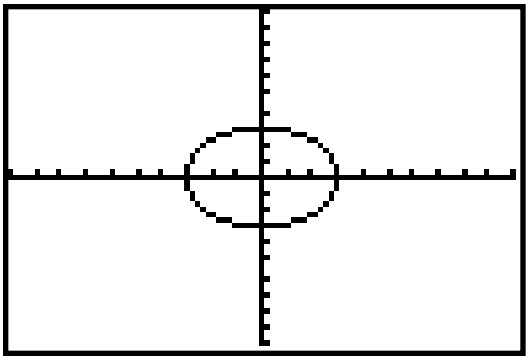
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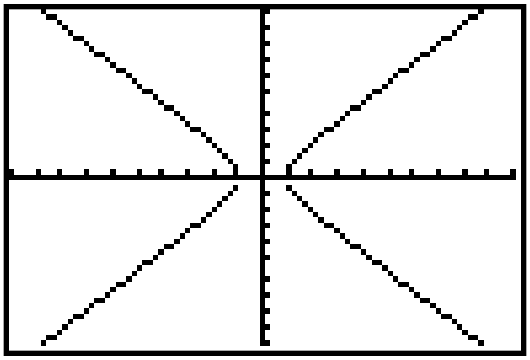
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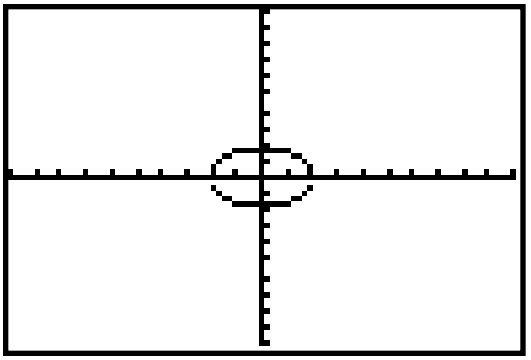
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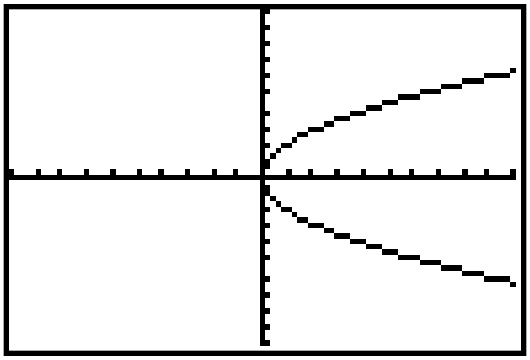
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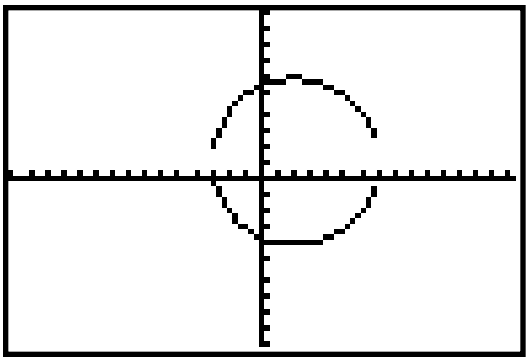

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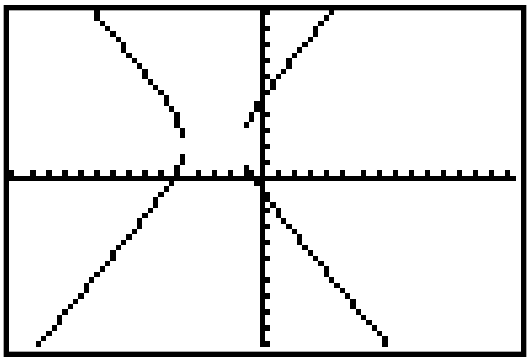

J

Answer Sheet to Activity #2

1. H
2. I
3. F
4. E
5. G
6. A
7. J
8. C
9. D
10. B

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Worksheet #2

Conic Identification From the Equation in General Form

Directions: Students will be divided into groups of four to five. The students will express each conic section in standard form by completing the square. Then they will identify each conic as either a circle, parabola, ellipse, or hyperbola by its standard form. Students will check by using the graphics calculator.

1. $9x^2 + 4y^2 + 36x - 24y + 36 = 0$
2. $x^2 - 9y^2 + 2x - 54y - 80 = 0$
3. $x^2 - 9y^2 + 36y - 72 = 0$
4. $x^2 + y^2 - 10x - 2y + 10 = 0$
5. $16x^2 + 25y^2 - 36x - 50y + 61 = 0$
6. $x^2 + y^2 - 16x + 4y + 67 = 0$
7. $9y^2 - x^2 + 2x + 54y + 62 = 0$
8. $9x^2 + 25y^2 - 36x - 50y + 61 = 0$
9. $x^2 - 2x + 8y + 9 = 0$

10. $y^2 - 4y - 4x = 0$

11. $12x^2 + 20y^2 - 12x + 40y - 37 = 0$

12. $9x^2 - y^2 + 54x + 10y + 55 = 0$

13. $x^2 + y^2 - 4x - 2y - 4 = 0$

14. $36x^2 + 9y^2 + 48x - 36y + 43 = 0$

15. $16y^2 - x^2 + 2x + 64y + 63 = 0$

16. $x^2 - 2x + 8y + 17 = 0$

Note: The program POLCONIC.82P for the TI-82 can be used to check the type of conic it is. This program is included on the next page.

The general equation of a conic is:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0.$$

The type of conic is determined by the discriminant. If the discriminant is negative the conic is an ellipse or a circle, but if A = C then it is a circle. If the discriminant is equal to zero the conic is a parabola. A discriminant greater than zero is a hyperbola.

Program: POLCONIC

```
: 2→K
: Func: Degree
: FnOff: PlotsOff
: Prompt A,B,C,D,E,F
: ClrDraw
: If A=0 and B=0 and C=0:Goto L
: 0→θ
: Lbl 1
: sin θ→S: cos θ→T
: DT+ES→G
: If H=0:Goto 2
: G²/H²-4F/H→I
: If I<0:Goto 9
: (√I-G/H)/2→R
: Goto 3
: Lbl 2
: If G=0:Goto 9
: -F/G→R
: Lbl 3
: P▸Rx(R,θ)→X
: P▸Ry(R,θ)→Y
: Pt-On(X,Y)
: Lbl 9
: θ+K→θ
: If θ<360:Goto 1
: Return
: Lbl L
: If D=0:Goto Y
: If E=0:Goto X
: DrawF (-F-DX)/X
: Return
: Lbl X
: Line(-F/D,Ymin,-F/D,Ymax)
: Return
: Lbl Y
: If E=0:Return
: Line(Xmin,-F/E,Xmax,-F/E)
```

Answer Sheet to Worksheet #2

1. ellipse	**2. hyperbola**
3. hyperbola	**4. circle**
5. ellipse	**6. circle**
7. hyperbola	**8. ellipse**
9. parabola	**10. parabola**
11. ellipse	**12. hyperbola**
13. circle	**14. ellipse**
15. hyperbola	**16. parabola**

Name_____
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Worksheet #3

Conic Sketches By First Identifying the Eccentricity

Directions: Take an educated guess as to whether the equation represents a circle, parabola, ellipse, or hyperbola; find the eccentricity of each; and then check with the graphics calculator.

1. $x^2 + 4x + 4y - 16 = 0$
2. $y^2 - 2x - 2y + 94 = 0$
3. $15x^2 + 15y^2 - 150x - 30y + 330 = 0$
4. $x^2 + y^2 - 20x - 16y + 64 = 0$
5. $16x^2 + y^2 + 160x - 22y + 505 = 0$
6. $4x^2 - 9y^2 + 32x - 144y - 548 = 0$
7. $y^2 - 16x - 14y + 17 = 0$
8. $x^2 + y^2 - 12x - 24y + 36 = 0$
9. $25x^2 + 16y^2 + 50x - 320y + 1225 = 0$
10. $x^2 + 9y^2 - 28x - 36y + 151 = 0$
11. $-25x^2 + y^2 + 50x + 20y + 50 = 0$
12. $4x^2 - y^2 - 8x + 4y - 9 = 0$

Note: An eccentricity of zero is a parabola, one is a circle, less than 1 is an ellipse, and greater than 1 is a hyperbola.

WORKSHEET 4

DIRECTIONS: Fill in the sheet with a partner. Use the accompanying sheet for the particular conics.

	Conic	Focus	Vertex	Directrix	Center	Eccentricity
1.						
2.						
3.						
4.						
5.						

	Major Axis	Minor Axis	Asymptotes	Radius	Discriminant	Transverse	Conjugate
1.							
2.							
3.							
4.							
5.							

TI-82 Graphics Calculator

Graphing Conics

A second degree equation needs to be fed into the graphics calulator as two separate functions.

Before using the graphics calculator, solve the quadratic polynomial equation for y.

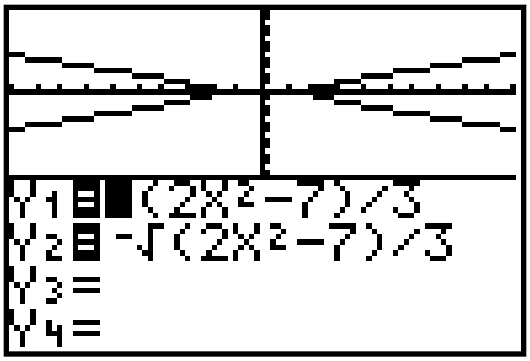
Lets take the quadratic polynomial equation $2x^2 - 3y^2 = 7$ and solve for y.

$$-3y^2 = 7 - 2x^2$$
$$y^2 = -\frac{(7 - 2x^2)}{3}$$
$$y^2 = \frac{(2x^2 - 7)}{3}$$
$$y = \pm\sqrt{\frac{(2x^2 - 7)}{3}}$$

Procedure:

Using the example above.

1. Press [ON]
2. Press [Y=]
3. Press [CLEAR]
   Clear out all prior equations
4. In Y_1 = Press [2nd].
5. [2nd] [x^2]
6. Press [(]
7. Press [2]
8. Press [X,T,θ]
9. Press [x^2]
10. Press [-]
11. Press [7]
12. Press [)]
13. Press [÷]
14. Press [3]
15. In Y_2 = repeat instructions 6-14 by pressing [2nd]. Remember to press [-], [2nd], [x^2] first


Y1 = √(2X²−7)/3
Y2 = −√(2X²−7)/3
Y3 =
Y4 =

WORKSHEET 5: COOPERATIVE TREASURE HUNT

Directions: Teams will be assigned one of the following problems to solve as a group. As your team completes a problem, pick up one the three remaining problems until all four have been completed. Teams solving all four problems will receive a MYSTERY PRIZE.

PROBLEM I : THE HUMAN CANNONBALL

A human cannonball is shot from a cannon and follows the path given by:

$$h(x) = -\frac{1}{100}x^2 + x + 8$$

Where h(x) is the height in feet (above ground) and x is the horizontal distance traveled in feet.

- Write the function in completed square form. Explain how this form can be used to find the maximum weight of the human cannonball.

- The human cannonball lands in a net that is is 8 feet above the ground. How many feet does the human cannonball travel?

COOPERATIVE TREASURE HUNT
Problem 2: Bicycle Chain Wheel

The pedals of a bicycle drive a chain wheel which drives a smaller sprocket wheel on the rear axle. Many chain wheels are circular. Some, however, are slightly elliptical, which tends to make pedaling easier. The front chain wheel on a bicycle is 9 inches at its widest and 8 1/2 inches at its narrowest.

- Find an equation for the outline of this elliptical chainwheel.

- What is its eccentricity?

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COOPERATIVE TREASURE HUNT

Problem 3: Locating An Explosion

Two microphones, 1 mile apart, recorded an explosion. Microphone A received the sound 2 seconds before Microphone B. Is this enough information to decide where the sound came from?

Name_____
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COOPERATIVE TREASURE HUNT

Problem 4: Desert Irrigation

Each of two circular fields have an area of about 2,400,000 square yards. Write an equation that represents the boundary of one of the fields. Let (0,0) represent the center.

Name_____
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Extension-Remediation #1

1. Solve for p:

   a. $\frac{4}{3}y = 4py$ b. $28x = 4px$ c. $-36y = 4py$

   d. $-80x = 4px$ e. $\frac{2}{3}y = 4py$ f. $\frac{8}{5}x = 4px$

2. Write the equation of each straight line described below:

   a. a horizontal line 3 units below the x-axis.

   b. a vertical line 6 units to the right of the y-axis.

   c. a vertical line containing the point (- 4, 2).

   d. a horizontal line passing through the point (-12,7)

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Extension-Remediation #2

1. Find the distance between the given points in a coordinate plane

   a. (8, -3) and (-1, -3)

   b. (4, 5) and (4, 11)

   c. (0, 0) and (5, 12)

   d. (-2, 7) and (2, -5)

2. Solve each system of equations by substitution.

   a. $2x + y = 3$
      $x - y = 3$

   b. $3x + 4y = -3$
      $2x + y = -7$

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Extension-Remediation #3

1. Find the missing value of the relationship $c^2 = a^2 + b^2$ for given values of a, b, c.

   a. Find c, given a = 6, b= 5.

   b. Find c, given a = 3, b = 2.

   c. Find b, given a = 4, c = 8.

   d. Find a, given b = 14, c= 17.

2. Rewrite the given equation of an ellipse in standard form.

   a. $4x^2 + 9y^2 = 36$ b. $16x^2 + 25y^2 = 400$

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Extension-Remediation #4

1. Explain how the graph of $g(x) = (x - 3)^2 + 4$ is different from the basic graph of $f(x) = x^2$.

2. Solve by completing the square.

   a. $x^2 - 4x + 1 = 2$ b. $4x^2 + 3x - 11 = 1$

Name_____
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Remedial-Extension #5

1. Compute the discriminant of the following quadratic equations.

   a. $y = 3x^2 - 2x + 8$ b. $y = 4 + 9x - 5x^2$

2. Solve y in terms of x and graph each "branch" of the relation $x^2 + y^2 = 0$.

Answer Sheet to Remediation-Extensions #1-#5

#1:

1. **a.** **1/3** **b.** **7** **c.** **- 9**

d. **-20** **e.** **1/6** **f.** **2/5**

2. **a.** **y = - 3** **b.** **x = 6** **c.** **x = - 4**

d. **y = 7**

#2:

1. **a.** **0** **b. None** **c.** **2.4**

d. **-3**

2. **a.** **(2, -1)** **b.** **(-5, 3)**

#3:

1. **a.** $\sqrt{61}$ **b.** $\sqrt{13}$ **c.** $4\sqrt{3}$

d. $\sqrt{93}$

2. **a.** $\frac{x^2}{9} + \frac{y^2}{4} = 1$ **b.** $\frac{x^2}{25} + \frac{y^2}{16} = 1$

#4:

1. **The graph of g(x) is a translation of the graph of f(x) shifted 3 units to the right and 4 units up.**

2. **a.** $x = 2 + \sqrt{5}$
$x = 2 - \sqrt{5}$

b. $\frac{-3 + \sqrt{201}}{8}$
$\frac{-3 - \sqrt{201}}{8}$

#5

1. **a.** **-92** **b.** **161**

2. **y = x**
y = -x

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Enrichment-Extension #1

Using CONIC GRAPH PAPER Sketch a hyperbola whose foci are the centers of the concentric circles. The difference of the distances between each point on the hyperbola and the foci is 10.

Name_____
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Enrichment-Extension #2

Using CONIC GRAPH PAPER sketch the graph of $x^2 - y^2 + 18x + 8y + 65 = 0$.

Name_____
Date_____
Period_____

Enrichment-Extension #3

SOLVE THE PUZZLE: ONE MAN ON AN ISLAND

A man lives on an island near a fresh water lake. Each morning he rides his horse to his gold mine—one mile away. Each evening he rides back on his horse. Because he has no well, he always rides to the island's shore to get water on each trip to and from the gold mine. For several weeks, he has been hunting for the shortest path, but no matter how he travels, the distance is always the same. Why?