**Title: Know the System**

**Brief Overview:**

This unit is a review of solving systems of linear equations. The systems will be solved graphically (using the TI-83), numerically (using matrices), and algebraically (verify algebraically). The highlight of the lesson will be solving real-world problems culminating in a performance task. The students will communicate their findings in a final lab report.

**Links to Standards:**

- **Mathematics as Problem Solving**
  Students will demonstrate the ability to use problem-solving approaches to investigate and understand linear systems. Students will apply the process of mathematical modeling to real-world problem situations.

- **Mathematics as Communication**
  Students will communicate mathematical ideas about linear systems in a performance task assessment. The final lab report gives the opportunity for students to reflect and clarify what they have learned about linear systems.

- **Mathematics as Reasoning**
  Students will reinforce logical reasoning skills by comparing and contrasting different methods for solving linear systems. Students will evaluate the most appropriate method for solving a given situation.

- **Mathematical Connections**
  Students will investigate the connections between linear systems and real-world applications. Students will recognize the algebraic, numerical, and graphical representations for solving systems of linear equations.

- **Algebra**
  Students will use graphs and matrices to solve linear systems.

- **Discrete Mathematics**
  Students will represent and solve real-world problems using matrices.

**Links to Maryland High School Mathematics Core Learning Goals:**

- **1.1.1:** The student will recognize, describe, and extend patterns and functional relationships that are expressed numerically, algebraically, and geometrically.

- **1.1.2:** The student will represent patterns and functional relationships in a table, as a graph, and/or by mathematical expression.

- **1.2.5:** The student will apply formulas and use matrices to solve real-world problems.

**Grade/Level:**

Grades 9-12; Algebra I, Algebra II

## Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Graphing linear equations
- Reading and identifying the elements of a matrix
- Naming the dimensions of a matrix
- Operating a graphing calculator

## Objectives:

Students will be able to:

- solve linear systems of equations graphically, numerically, and algebraically.
- select the most appropriate method for solving real-world applications.

## Materials/Resources/Printed Materials:

- Activity sheets and drill sheets
- Graphing calculators (this unit is specific to the TI-83)
- TI-82/TI-83 Overhead Projector

## Development/Procedures:

During this unit, students will solve linear systems of equations graphically, numerically, and algebraically and choose the most appropriate method for solving real-world applications. On Day One, students will review solving linear systems of equations by graphing on the TI-83. Day Two will introduce the solving of equations using matrices. Students will use the TI-83 matrix operations to solve system of linear equations. Day Three will introduce the solving of real-world problems by graphing or using matrices (student's choice). Day Four (and maybe Five) will include a performance assessment where the students will write a system of linear equations and solve it using the most appropriate method. The performance task uses a real-life problem to assess student understanding. The final day will have the students creating a lab report that will summarize their findings on solving linear systems.

Drills and activities are provided for each day which support the daily objectives.

## Performance Assessment:

The performance assessment is included.

Scoring Rubric: For each numbered question in the assessment points will be assigned depending on the quality of the answers. 2 points will be given for correct answers. 1 point will be given for partially correct answers. 0 points will be given for incorrect answers.

## Extension/Follow Up:

See CBL lesson titled "Learn Your Lines" by Conrad Judy, North County High School, Anne Arundel County and Tammy Berg, North East High School, Cecil County.

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Day 1 Linear Systems

Drill/ Warm-up                                        Name: ____________________

(This drill is intended to review the concepts of writing a linear equation in the form $y = mx + b$ and use algebra to show that a given coordinate is or is not a solution to a given linear equation.)

DIRECTIONS: A. Write each linear equation in the form $y = mx + b$.
B. Algebraically show that the given coordinate is or is not a solution of the linear equation.

1. A. $2y = 14x + 8$ ; B. ( 1 , 11)

2. A. $2x - y = 1$ ; B. ( 2 , 3 )

3. A. $\frac{1}{2}x + y = 3$ ; B. ( -3, 4 )

4. A. $x - \frac{1}{2}y = 4$ ; B. ( 3 , -2 )

5. Write a linear equation for this situation and use the equation to answer the following questions.

x = ________________ y = ________________

**A cellular phone company charges $17 per month for fees, and $0.29 per minute for local phone calls**. Write a linear equation in the form $y = mx + b$ that models the monthly cost.

______________________________

**If you budget your money correctly, you will be able to spend $24 per month for local cellular phone access**. Write and solve an equation that will give the maximum amount of minutes you will be able to talk locally without going over your budgeted allowance for a cell phone.

Equation: _________________

How many minutes will you be able to talk on the phone? __________
Is this a reasonable amount of time? _____________

6. How could you use your calculator to answer the above questions?

Additional drill or optional quiz, as needed
Linear Systems

Drill/ Warm-up Name: ____________________

(This drill is intended to review the concepts of writing a linear equation in the form y = mx + b and use algebra to show that a given coordinate is or is not a solution to a given linear equation.)

DIRECTIONS: A. Write each linear equation in the form y = mx + b.
B. Algebraically show that the given coordinate is or is not a solution of the linear equation.

1. A. $6y = 2x - 18$; B. (0, -3)    2. A. $x - y = 3$; B. (0, 3)

3. A. $x + \frac{1}{3}y = 2$; B. (-3, 3)    4. A. $x + 5y = 10$; B. (6, 0)

5. Write a linear equation for this situation and use the equation to answer the following questions.

x = ________________ y = ________________

a. **A taxi cab company charges $1.70 flat rate and $0.79 per mile for a cab ride**. Write a linear equation in the form y = mx + b that models the cost of a cab ride:

______________________________

b. **If you budget your money correctly, you will be able to spend $20 for you and a friend to ride to Baltimore and back.** Write and solve an equation that will give the maximum amount of miles you will be able to ride without going over your budgeted allowance.

______________________________

c. How many miles will you be able to ride in the cab? ___________

d. Is a cab a reasonable mode of transportation for you and your friend?_______

Why or why not? _____________________________________________________

e. How much money do you need to go to Glen Burnie instead? _____________

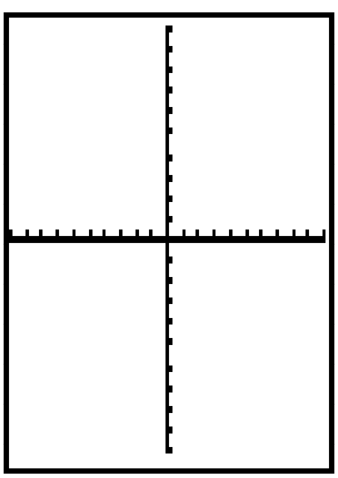
# Activity 1- Solving Linear Systems of Equations by Graphing

Directions:
A. Use a graphing calculator to graph the system of equations to determine the solution. Sketch the graph on the axes provided. Estimate the coordinates of the system's solution .

B. Use **2nd CALC 5:** to find the point of intersection. Press **ENTER** as the cursor blinks on a line ('first curve?') and **ENTER** again as the cursor blinks on the other line ('second curve?'). Move the cursor close to the point of intersection ('guess?') and press **ENTER.** Write the coordinate of the solution to the system in the space provided. If the system is inconsistent write 'no solution'. If the system is consistent, use set builder notation and write the equation of the line in standard form.

C. Use paper and pencil to check the solutions by substituting the x-coordinate of the solution for the x variable and the y- coordinate for the y variable in BOTH equations. Double check your graph and algebra if your graphic solution does not check.

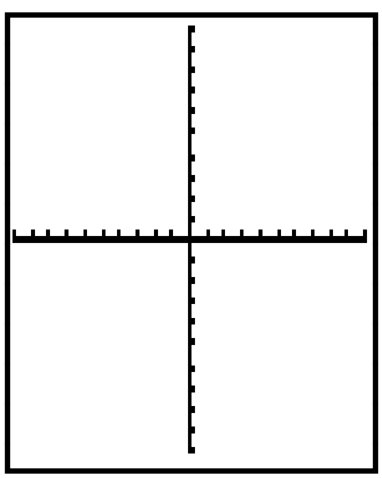
1. $y = -2x + 4$
   $y = x + 1$

A


B. ( ___ , ___ )

C.

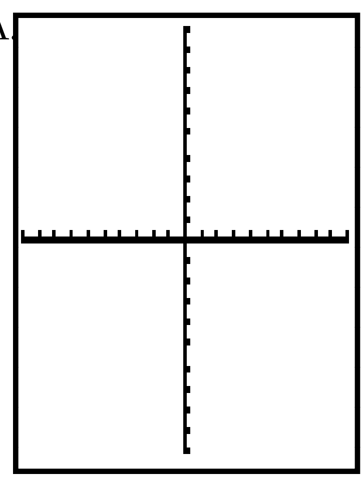
2. $y = 2x - 3$
   $2y = -x + 14$

A.


B. ( ___ , ___ )

C.

3. $y = 5x + 1$
   $1 = 5x - y$

A.


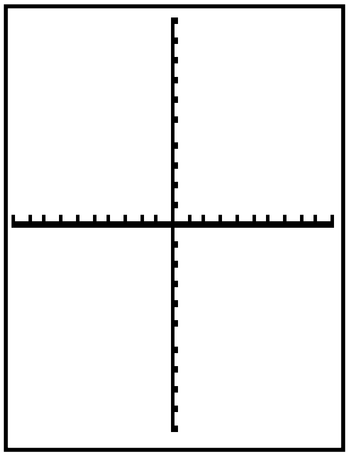
B. ( ___ , ___ )

C.

4. $\frac{1}{2}x + y = 3$

$x + 2y = 6$

A.



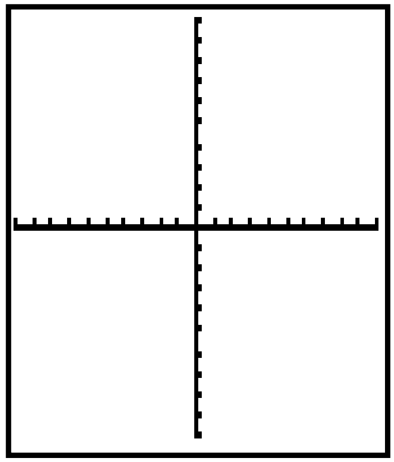
B. ( ___ , ___ )

C.

5. $y = -2x + 1$

$3x + 2y = 5$

A.



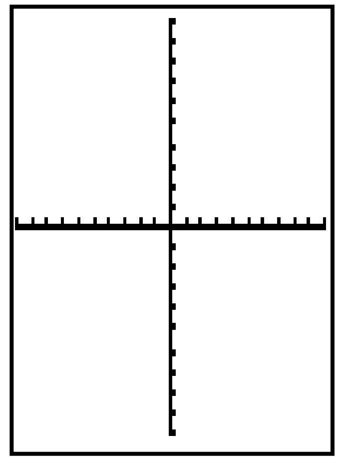
B. ( ___ , ___ )

C.

6. $y = -2x + 2$

$2x + y = 4$

A.



B. ( ___ , ___ )

C.

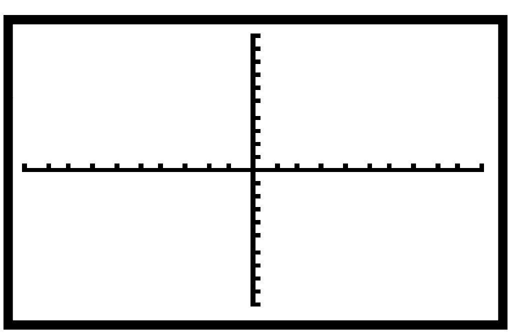
CHECK UNDERSTANDING

a. Is ( -2 , 1 ) a solution to $y = 3x + 4$? ________

b. Is ( -2 , 1 ) a solution to $y = x + 3$? ________

c. Is ( -2 , 1 ) a solution to the system $y = 3x + 4$
$y = x + 3$ ? _________

d. Show the answer to ‘c’ graphically and explain your answer.



____________________________________

____________________________________

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Day 2 Linear Systems

Drill/Warm up                                        Name: _______________

DIRECTIONS: A. Graph each system on your graphing calculator.
B. Label each system as having one solution, no solutions, or infinitely many solutions.

1. $y = x + 2$
   $2y = 2x + 4$

   __________________

2. $y = x + 4$
   $y = x + 6$

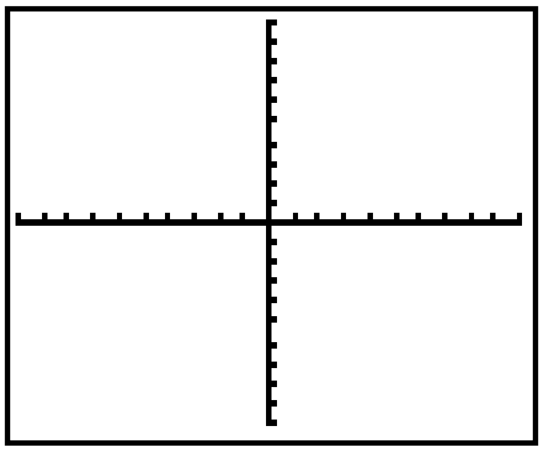
   __________________

3. $y = 2x + 3$
   $-6x + y = 1$

   __________________

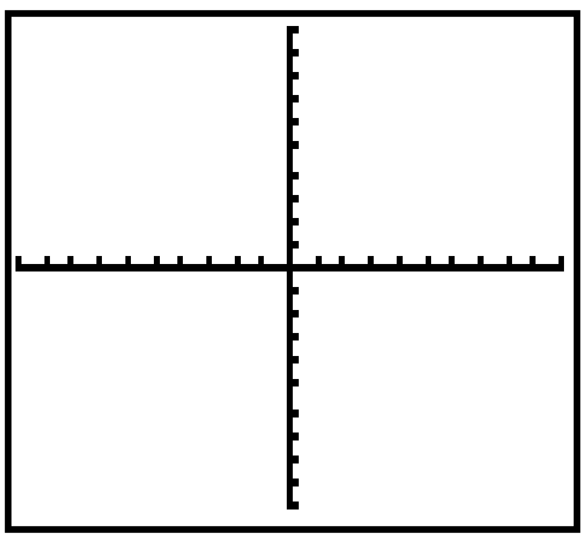
4. Find the solution to the system graphically. Sketch the system and state your answer.

A. $y = 3x - 1$
   $y = -x + 5$



( __ , __ )

B. $y = x + 4$
   $y = -2x + 1$
   $y = 2x + 5$



( __ , __ )

# ACTIVITY 2 - SOLVING LINEAR SYSTEMS OF EQUATIONS NUMERICALLY USING MATRICES

1. Use your graphing calculator to solve this system of linear equations.

$$\frac{2}{3}x + \frac{1}{4}y = 3$$
$$\frac{1}{2}x - \frac{3}{8}y = 7$$

( __ , __ )

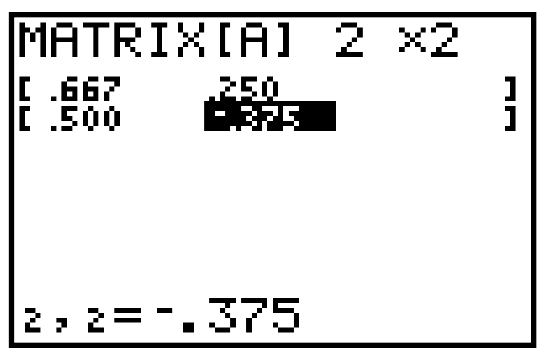
2. As you may have noticed, problem number one may be cumbersome simply because of the fractions. In such case it would be nice to discover a different method of solving systems that would help you to get the answer. We use matrices for just such a case.

Follow the directions below, using your graphing calculator, to solve a system using matrices.

a. *Press **MATRIX,** arrow over to EDIT and **ENTER** to edit matrix A. (Matrix A will be a 2 X 2 matrix where the elements are the coefficients of x and y in the two above equations.)

*Press **2 ENTER 2 ENTER** to enter a 2x2 matrix (Notice element 1,1 is highlighted)

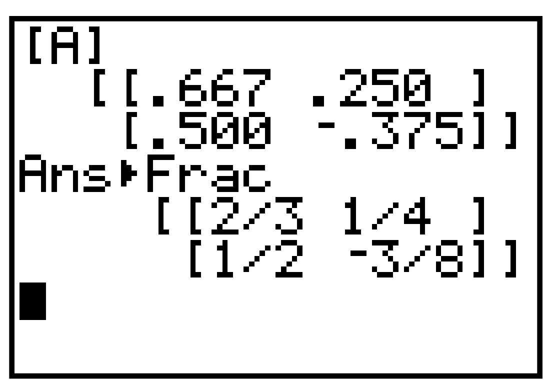
*Press **2/3 ENTER**, **1/4 ENTER** , **1/2 ENTER** ,and **-3/8 ENTER** to fill the matrix.




*Press **2nd QUIT** to go back to the home screen.

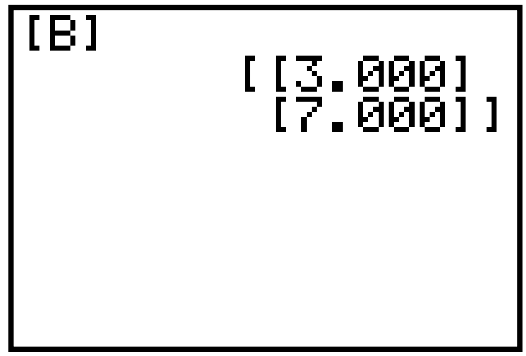
*Press **MATRIX** (1: A is highlighted) and **ENTER**, then **ENTER** (will show you matrix A)

*Press **MATH ENTER** to change the decimal elements to fractions



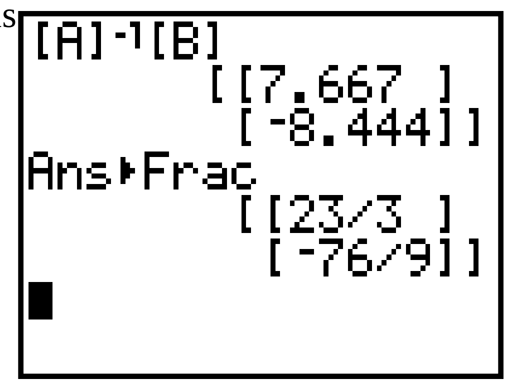
b. (Matrix b will be a matrix of the answers 3 and 7)

*Press **MATRIX,** arrow over to EDIT and down to 2: B , **ENTER** to edit matrix B. (Matrix B will be the matrix of the answers 3 and 7)
*Press **2 ENTER** , **1 ENTER** to give the dimensions of the matrix and
*Press **3 ENTER** , **7 ENTER** to enter the elements of matrix B
*Press **2nd QUIT** to go back to the home screen.
*Press **MATRIX** arrow down so that 2: B is highlighted, and **ENTER**, then **ENTER** (will show you matrix B)




c. (Remember that **multiplying by the inverse** of a number **is the same as dividing by that number**. The same fact is true for matrices. We will multiply the inverse of matrix A times matrix B)

*Press **MATRIX , ENTER,** $\mathbf{x^{-1}}$ **, MATRIX,** arrow down to 2: B **,ENTER,** and **ENTER** one last time gives you the answer matrix for x and y.
(**MATH ENTER ENTER** will rewrite your matrix as improper fractions instead of decimals)




d. The solution coordinates x and y are the elements of the answer matrix.
x = 23/3 or 7.667 and y = -76/9 or -8.44
Compare this solution with your answer in #1 above.

e. Continue to the next page to practice solving linear systems using matrices.

3. Using the procedures you learned in paragraph 2, solve the following systems of equations using matrices and your graphing calculator.

a. $\begin{aligned} 2x + 3y &= 11 \\ x + 2y &= 8 \end{aligned}$

b. $\begin{aligned} 3x + 5y &= -13 \\ 2x + 3y &= -9 \end{aligned}$

$$A=\begin{bmatrix} \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ \end{bmatrix} \qquad A=A=\begin{bmatrix} \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ \end{bmatrix}$$

$$B=\begin{bmatrix} \_\_\_ \\ \_\_\_ \end{bmatrix} \qquad B=\begin{bmatrix} \_\_\_ \\ \_\_\_ \end{bmatrix}$$

x = ___ y = ___ x = ___ y = ___

c. $\begin{aligned} 4x - 2y + 3z &= 4 \\ 3x + 5y + z &= 7 \\ 5x - y + 4z &= 7 \end{aligned}$

d. $\begin{aligned} 2x + y + z &= 3 \\ 3x - 4y + 2z &= -7 \\ x + y + z &= 2 \end{aligned}$

$$A=\begin{bmatrix} \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ \end{bmatrix}$$

$$B=\begin{bmatrix} \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{bmatrix}$$

x = ___ y = ___ z = ___ x = ___ y = ___ z = ___

e. $\begin{aligned} x + y \phantom{+ z} &= -1 \\ y + z &= 4 \\ x \phantom{+ y} + z &= 1 \end{aligned}$

f. $\begin{aligned} -x + y \phantom{- z} &= -1 \\ y - z &= 6 \\ x \phantom{+ y} + z &= -1 \end{aligned}$

$$A=\begin{bmatrix} \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ \end{bmatrix}$$

$$B=\begin{bmatrix} \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{bmatrix}$$

x = ___ y = ___ z = ___ x = ___ y = ___ z = ___

# Activity Three- Solving Real-life Applications of Linear Systems

Solve by graphing:

1. A cab ride costs $1.70 plus $0.10 per tenth of a mile traveled if you use the Bay Cab Company. The cost is $1.55 plus $0.15 per tenth of a mile traveled if you use the Capital Cab Company.

   a. Write a system of equations to show the relationships described.

   b. Use a graph to determine at what distance the cab rides cost the same.

   c. If your destination is 2 miles away, which company would you choose? Justify your answer.

   d. You have $6.00 to spend on the taxi ride and want to give the driver a $1.00 tip. How far can you go?

2. The Sunny Beach Resort is offering two weekend specials. One includes a 2-night stay with 3 meals and costs \$195. The other includes a 3-night stay with 5 meals and costs \$300. What would you pay for a 1-night stay? What would be your cost per meal?

a. Write a system of equations to show the relationships described.

b. Use a graph to determine the cost for a 1-night stay and the cost per meal.

c. How much should they charge for a 4-night stay with 7 meals?

Solving using matrices:

3. Jane has $4000 to invest. She can get 8% annual interest for a 3 month CD and 12% for a 12 month CD. To earn $440 interest, how much will she put in each CD?

a. Write a system of equations to show the relationships described.

b. Write the matrices for the system.

c. Solve the system. How much did she put in each CD?

4. Big Al's Car Rental rents compact cars for a fixed amount per day plus a fixed amount for each mile driven. Jack rented a car from Big Al's for 6 days, drove it 550 miles and paid $550, excluding taxes. Rachel rented the same car for 3 days, drove 350 miles and paid $185, excluding taxes. What was the charge per day and the charge per mile driven?

a. Write a system of equations to show the relationships described.

b. Write the matrices for the system.

c. Solve the system. What was the charge per day and the charge per mile driven?

Solve by any method:

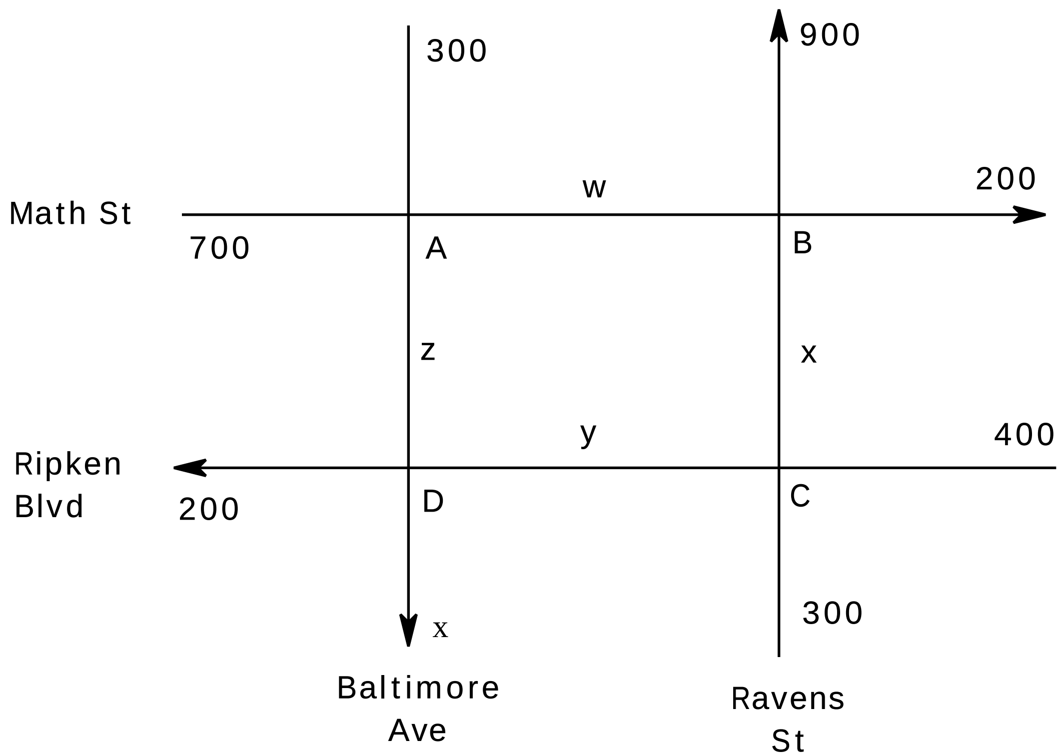
5. *Job offer:* Given the two job offers below, determine the better paying summer job. Explain your reasoning.

*Offer 1:* At Captains Hook's Fish & Chips you will earn $6.00 per hour . However, you will be required to purchase a uniform for $65. You will be expected to work 20 hours each week.

*Offer 2:* At Fred's Food Mart, you will earn $4.50 per hour. No special attire is required. You will be expected to work 20 hours each week.

# PERFORMANCE TASK

During rush hour, heavy traffic congestion is encountered at the traffic intersections shown in the figure below. All the streets are one-way.




The city wants to improve the flow of traffic at these corners. The traffic engineers first gather data. As the figure shows, 700 cars per hour come down Math Street to intersection A; 300 cars per hour come to intersection A on Baltimore Ave. A total of w cars leave A on Math Street, while z cars leave A on Baltimore Ave. The number of cars entering A must equal the number leaving, so that the equation for intersection A is

$$w + z = 700 + 300$$

or $$w + z = 1000$$

1. For intersection B, w cars enter B on Math Street, and x cars enter B on Ravens Street. The figure shows that 900 cars leave B on Ravens Street while 200 leave on Math Street. Find the equation for intersection B:

2. For intersection C, 400 cars enter on Ripken Blvd, 300 on Ravens Street, while x cars leave on Ravens Street and y cars leave on Ripken Blvd. Find the equation for intersection C:

3. Finally, intersection D has y cars entering at Ripken Blvd and z cars entering on Baltimore Ave. There are x cars leaving D on Baltimore Ave and 200 cars leaving on Ripken Blvd. Find the equation for intersection D. (Remember, your final answer should be in standard form.)

4. You now have a system of 4 linear equations ( one each for intersections A, B, C, D) and 4 unknowns (w, x, y, and z). Using any method you desire, solve the linear system for w, x, y, and z. (Hint: Try using your graphing calculators).

5. Does your solution make sense? Explain your reasoning.

# TEACHER NOTES

## DAY 1

### DRILL/WARM-UP

**1.** This drill is intended to review the concepts of writing a linear equation in the form $y = mx + b$ and uses algebra to show that a given coordinate is or is not a solution to a given linear equation.

**2.** A class discussion is encouraged to clarify what a reasonable answer for the last part of #5.

### ACTIVITY1

**1.** Use this chart to review the vocabulary needed in discussing linear systems and their solutions.

| GRAPHS | # OF SOLUTIONS | SLOPES | TYPE OF SYSTEM |
|---|---|---|---|
| Lines intersect in one point | One | Not the Same | Consistent and Independent |
| Lines are parallel | None | Same | Inconsistent |
| Lines Coincide (the same line) | Infinitely many | Same | Consistent and Dependent |

**2.** You will need to review the notation for stating the solution to a consistent and dependent system: $\{(x,y): Ax + By = C\}$

## DAY 2

### DRILL/WARM-UP

**1.** Drill question #4b is intended to promote interest in an alternate way for finding the solution to a system, namely, matrices.

### ACTIVITY 2

**1.** Problem 1 is a true motivation for learning a simpler way to solve a system of equations.

**2.** Question #2c: You may need to review notation for an inverse. Use an example such as dividing fractions to remind students that division will give the same value as multiplying by an inverse.

## DAY 3

**DRILL/WARM-UP**

**1.** This activity is intended to take and entire lesson. You may wish to assign problem #1 part a & b as the drill activity.

**2.** You may need to remind the students that when graphing, they need to write the equations in the form y=mx +b. When they use matrices to solve a system, they will need to write their equations in standard form.

Day 1 Linear Systems

Drill/ Warm-up Name: KEY

(This drill is intended to review the concepts of writing a linear equation in the form y = mx + b and use algebra to show that a given coordinate is or is not a solution to a given linear equation.)

DIRECTIONS: A. Write each linear equation in the form y = mx + b.
B. Algebraically show that the given coordinate is or is not a solution of the linear equation.

1. A. $2y = 14x + 8$; B. (1, 11)

$y = 7x + 4$
$11 = 7 \cdot 1 + 4$
$11 = 11$ ✓ yes

2. A. $2x - y = 1$; B. (2, 3)

$y = 2x - 1$
$3 = 2 \cdot 2 - 1$
$3 = 4 - 1$
$3 = 3$ ✓ yes

3. A. $\frac{1}{2}x + y = 3$; B. (-3, 4)

$y = -\frac{1}{2}x + 3$
$4 = \frac{3}{2} + 3$
$4 \neq 4.2$ NO

4. A. $x - \frac{1}{2}y = 4$; B. (3, -2)

$y = 2x - 8$
$-2 = 2 \cdot 3 - 8$
$-2 = -2$ ✓ yes

5. Write a linear equation for this situation and use the equation to answer the following questions. x = # minutes y = total monthly cost

**A cellular phone company charges \$17 per month for fees, and \$0.29 per minute for local phone calls.** Write a linear equation in the form y = mx + b that models the monthly cost.

$y = 0.29x + 17$ x = # minutes of local calls

**If you budget your money correctly, you will be able to spend \$24 per month for local cellular phone access.** Write and solve an equation that will give the maximum amount of minutes you will be able to talk locally without going over your budgeted allowance for a cell phone.

Equation: $24 = 0.29x + 17 \longrightarrow x = \frac{7}{.29} \approx 24.14$

How many minutes will you be able to talk on the phone? 24 minutes
Is this a reasonable amount of time? yes, if you only use it for emergencies. (answers will vary)

6. How could you use your calculator to answer the above questions?
Graph $y_1 = .29x + 17$. Fix window so you can see the line. Trace until $y = 24$ and the x-value is your answer.

Day 2 Linear Systems

Drill/Warm up

Name: ______________

DIRECTIONS: A. Graph each system on your graphing calculator.
B. Label each system as having one solution, no solutions, or infinitely many solutions.

1. $y = x + 2$
$2y = 2x + 4$

infinitely many
(same line)

2. $y = x + 4$
$y = x + 6$

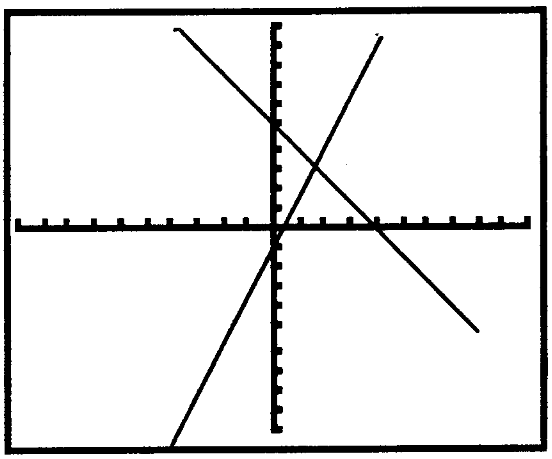
no solutions
(lines are parallel)

3. $y = 2x + 3$
$-6x + y = 1$

one solution
(different slopes)

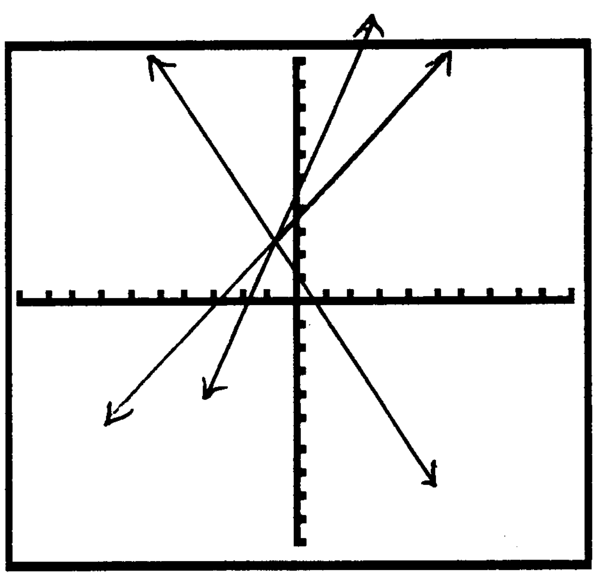
4. Find the solution to the system graphically. Sketch the system and state your answer.

A. $y = 3x - 1$
$y = -x + 5$



(1.5, 3.5)

B. $y = x + 4$
$y = -2x + 1$
$y = 2x + 5$



(-1, 3)

# ACTIVITY 2 - SOLVING LINEAR SYSTEMS OF EQUATIONS NUMERICALLY USING MATRICES

1. Use your graphing calculator to solve this system of linear equations.

$12\left(\frac{2}{3}x + \frac{1}{4}y = 3\right)$ $\quad 8x + 3y = 36$ $\quad \left(y = -\frac{8}{3}x + 12\right.$

$8\left(\frac{1}{2}x - \frac{3}{8}y = 7\right)$ $\quad 4x - 3y = 56$ $\quad \left.y = \frac{4}{3}x - 18\frac{2}{3}\right)$ $\quad \left(7\frac{2}{3}, -8\frac{4}{9}\right)$

$12x = 92$

$x = \frac{23}{3} = 7\frac{2}{3}$ $\qquad 7.\overline{6}, -8.\overline{4}$

2. As you may have noticed, problem number one may be cumbersome simply because of the fractions. In such case it would be nice to discover a different method of solving systems that would help you to get the answer. We use matrices for just such a case.

Follow the directions below, using your graphing calculator, to solve a system using matrices.

a. *Press **MATRIX,** arrow over to EDIT and **ENTER** to edit matrix A. (Matrix A will be a 2 X 2 matrix where the elements are the coefficients of x and y in the two above equations.)
*Press **2 ENTER 2 ENTER** to enter a 2x2 matrix
(Notice element 1,1 is highlighted)
*Press **2/3 ENTER, 1/4 ENTER , 1/2 ENTER** ,and **-3/8 ENTER** to fill the matrix.

```
MATRIX[A] 2 ×2
[ .667    .250      ]
[ .500              ]

2,2=-.375
```

*Press **2nd QUIT** to go back to the home screen.
*Press **MATRIX** (1: A is highlighted) and **ENTER**, then **ENTER** (will show you matrix A)
*Press **MATH ENTER** to change the decimal elements to fractions

```
[A]
  [[.667  .250 ]
   [.500  -.375]]
Ans▸Frac
     [[2/3  1/4 ]
      [1/2  -3/8]]
█
```

b. (Matrix b will be a matrix of the answers 3 and 7)

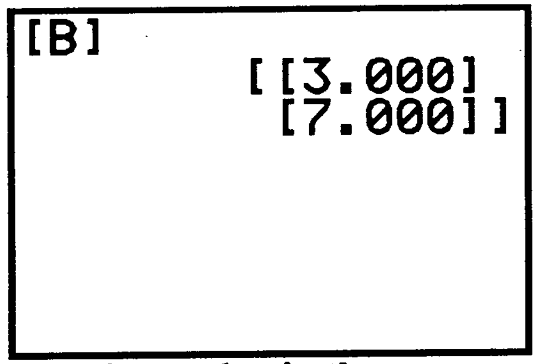
*Press **MATRIX,** arrow over to EDIT and down to 2: B , **ENTER** to edit matrix B. (Matrix B will be the matrix of the answers 3 and 7)

*Press **2 ENTER** , **1 ENTER** to give the dimensions of the matrix and

*Press **3 ENTER** , **7 ENTER** to enter the elements of matrix B

*Press **2nd QUIT** to go back to the home screen.

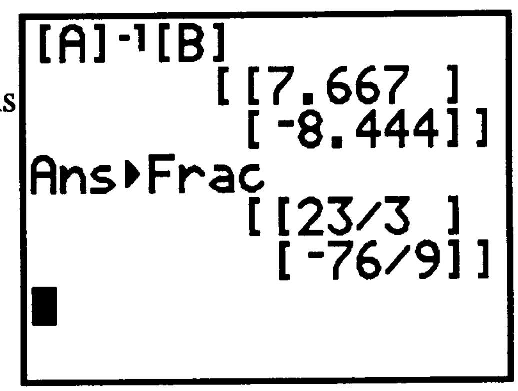
*Press **MATRIX** arrow down so that 2: B is highlighted, and **ENTER,** then **ENTER** (will show you matrix B)




c. (Remember that **multiplying by the inverse** of a number **is the same as dividing by that number**. The same fact is true for matrices. We will multiply the inverse of matrix A times matrix B)

*Press **MATRIX , ENTER,** $x^{-1}$ , **MATRIX,** arrow down to 2: B ,**ENTER,** and **ENTER** one last time gives you the answer matrix for x and y.

(**MATH ENTER ENTER** will rewrite your matrix as improper fractions instead of decimals)




d. The solution coordinates x and y are the elements of the answer matrix.

$x = 23/3$ or 7.667 and $y = -76/9$ or -8.44

Compare this solution with your answer in #1 above.

e. Continue to the next page to practice solving linear systems using matrices.

3. Using the procedures you learned in paragraph 2, solve the following systems of equations using matrices and your graphing calculator.

a. $2x + 3y = 11$
$x + 2y = 8$

A= $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

B= $\begin{bmatrix} 11 \\ 8 \end{bmatrix}$

x = $-2$ y = $5$

b. $3x + 5y = -13$
$2x + 3y = -9$

A= $\begin{bmatrix} 3 & 5 \\ 2 & 3 \end{bmatrix}$

B= $\begin{bmatrix} -13 \\ -9 \end{bmatrix}$

x = $-6$ y = $1$

c. $4x - 2y + 3z = 4$
$3x + 5y + z = 7$
$5x - y + 4z = 7$

A= $\begin{bmatrix} 4 & -2 & 3 \\ 3 & .5 & 1 \\ 5 & -1 & 4 \end{bmatrix}$

B= $\begin{bmatrix} 4 \\ 7 \\ 7 \end{bmatrix}$

x = ___ y = ___ z = ___

d. $2x + y + z = 3$
$3x - 4y + 2z = -7$
$x + y + z = 2$

$A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & -4 & 2 \\ 1 & 1 & 1 \end{bmatrix}$

$B = \begin{bmatrix} 3 \\ -7 \\ 2 \end{bmatrix}$

x = ___ y = ___ z = ___

e. $x + y = -1$
$y + z = 4$
$x + z = 1$

A= $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

B= $\begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix}$

x = $-2$ y = $1$ z = $3$

f. $-x + y = -1$
$y - z = 6$
$x + z = -1$

$A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & +1 \end{bmatrix}$

$B = \begin{bmatrix} -1 \\ 6 \\ -1 \end{bmatrix}$

x = $3$ y = $2$ z = $-4$

# Activity Three- Solving Real-life applications of linear systems

Solve by graphing:

1. A cab ride costs $1.70 plus $0.10 per tenth of a mile traveled if you use the Bay Cab Company. The cost is $1.55 plus $0.15 per tenth of a mile traveled if you use the Capital Cab Company.

a. Write a system of equations to show the relationships described.

Let $y$ = cab ride cost $x$ = tenths of miles travelled

Bay: $y = .1x + 1.70$
Capitol: $y = .15x + 1.55$

b. Use a graph to determine at what distance the cab rides cost the same.

.3 miles

c. If your destination is 2 miles away, which company would you choose? Justify your answer. Bay cab charges $3.70, Capitol $4.55

→ Choose Bay

d. You have $6.00 to spend on the taxi ride and want to give the driver a $1.00 tip. How far can you go? $5 = 1.70 + .1x$ ($x$ is tenths of a mile)

$x = 33$ tenths; you can go 3.3 miles.

2. The Sunny Beach Resort is offering two weekend specials. One includes a 2-night stay with 3 meals and costs \$195. The other includes a 3-night stay with 5 meals and costs \$300. What would you pay for a 1-night stay? What would be your cost per meal?

a. Write a system of equations to show the relationships described.

Let $x$ = cost for each night, $y$ = cost per meal

$195 = 2x + 3y$

$300 = 3x + 5y$

b. Use a graph to determine the cost for a 1-night stay and the cost per meal.

1 night = \$75

1 meal = \$15

c. How much should they charge for a 4-night stay with 7 meals?

$4 \cdot 75 + 7 \cdot 15 = \$405$

Solving using matrices:

3. Jane has \$4000 to invest. She can get 8% annual interest for a 3 month CD and 12% for a 12 month CD. To earn \$440 interest, how much will she put in each CD?

a. Write a system of equations to show the relationships described.

$x =$ 3 mo. CD $\quad y =$ 12 mo. CD

$$x + y = 4000$$

$$.08x + .12y = 440$$

b. Write the matrices for the system.

$$\begin{bmatrix} 1 & 1 \\ .08 & .12 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4000 \\ 440 \end{bmatrix}$$

c. Solve the system. How much did she put in each CD?

She put \$1000 into the 3 mo. CD and \$3000 into the 12 mo. CD.

4. Big Al's Car Rental rents compact cars for a fixed amount per day plus a fixed amount for each mile driven. Jack rented a car from Big Al's for 6 days, drove it 550 miles and paid $550, excluding taxes. Rachel rented the same car for 3 days, drove 350 miles and paid $185, excluding taxes. What was the charge per day and the charge per mile driven?

a. Write a system of equations to show the relationships described.

d = cost per day    m = cost per mile

$6d + 550m = 337$

$3d + 350m = 187$

b. Write the matrices for the system.

$$\begin{bmatrix} 6 & 550 \\ 3 & 350 \end{bmatrix} \begin{bmatrix} d \\ m \end{bmatrix} = \begin{bmatrix} 337 \\ 187 \end{bmatrix}$$

c. Solve the system. What was the charge per day and the charge per mile driven?

Charge per day was $36

The charge per mile was $0.22

Solve by any method:

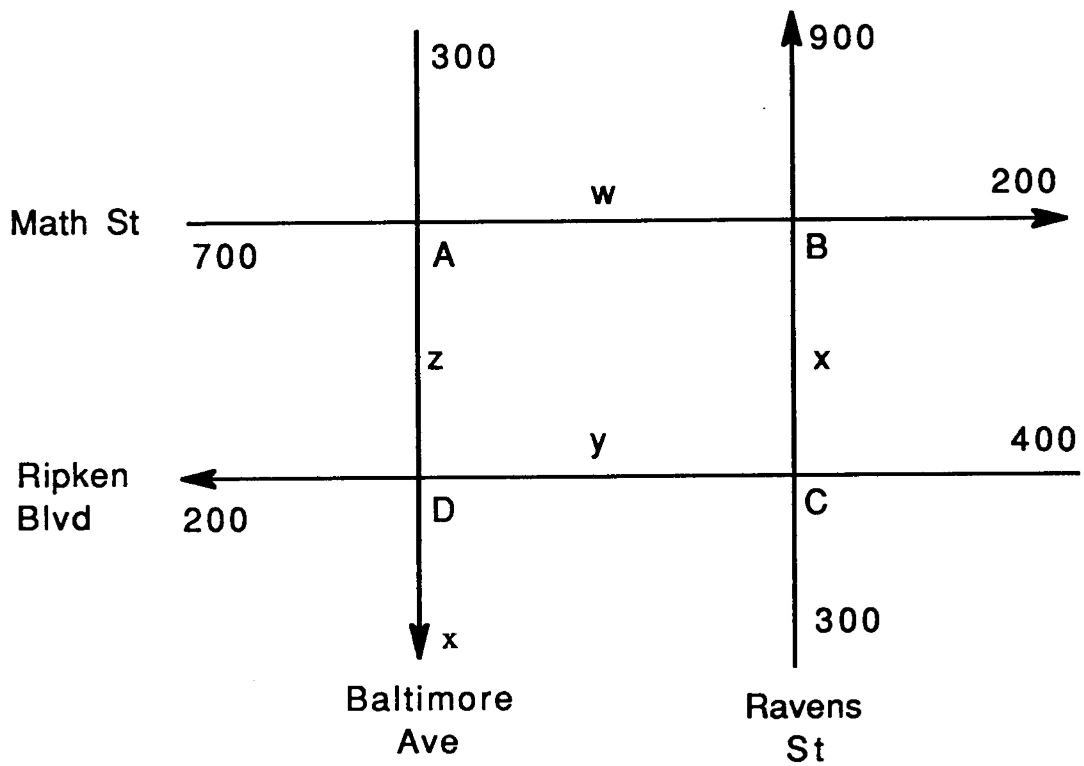
5. *Job offer:* Given the two job offers below, determine the better paying summer job. Explain your reasoning.

*Offer 1:* At Captains Hook's Fish & Chips you will earn $6.00 per hour . However, you will be required to purchase a uniform for $65. You will be expected to work 20 hours each week.

*Offer 2:* At Fred's Food Mart, you will earn $4.50 per hour. No special attire is required. You will be expected to work 20 hours each week.

# PERFORMANCE TASK

During rush hour, heavy traffic congestion is encountered at the traffic intersections shown in the figure below. All the streets are one-way.




The city wants to improve the flow of traffic at these corners. The traffic engineers first gather data. As the figure shows, 700 cars per hour come down Math Street to intersection A; 300 cars per hour come to intersection A on Baltimore Ave. A total of w cars leave A on Math Street, while z cars leave A on Baltimore Ave. The number of cars entering A must equal the number leaving, so that the equation for intersection A is

$$w + z = 700 + 300$$

or $$w + z = 1000$$

1. For intersection B, w cars enter B on Math Street, and x cars enter B on Ravens Street. The figure shows that 900 cars leave B on Ravens Street while 200 leave on Math Street. Find the equation for intersection B:

$w + x = 200 + 900$

$w + x = 1100$

2. For intersection C, 400 cars enter on Ripken Blvd, 300 on Ravens Street, while x cars leave on Ravens Street and y cars leave on Ripken Blvd. Find the equation for intersection C:

$$300 + 400 = y + x$$

$$x + y = 700$$

3. Finally, intersection D has y cars entering at Ripken Blvd and z cars entering on Baltimore Ave. There are x cars leaving D on Baltimore Ave and 200 cars leaving on Ripken Blvd. Find the equation for intersection D. (Remember, your final answer should be in standard form.)

$$y + z = x + 200$$

$$-x + y + z = 200$$

4. You now have a system of 4 linear equations ( one each for intersections A, B, C, D) and 4 unknowns (w, x, y, and z). Using any method you desire, solve the linear system for w, x, y, and z. (Hint: Try using your graphing calculators).

$$\begin{aligned} w + z &= 1000 \\ w + x &= 1100 \\ x + y &= 700 \\ -x + y + z &= 200 \end{aligned}$$

$$\begin{array}{cccc} w & x & y & z \end{array}$$
$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1000 \\ 1100 \\ 700 \\ 200 \end{bmatrix}$$

5. Does your solution make sense? Explain your reasoning.

Answers will vary.