Title: Investing

Brief Overview:

This activity will use interest calculations as an introduction to exponential functions. Students will investigate the effects of changing the principal and/or interest rates.

Links to NCTM Standards:

- **Mathematics as Problem Solving**
  Students will use a spreadsheet to investigate the effect on total interest earned in a savings account when changing the parameters of rate and principal.

- **Mathematics as Communication**
  Students will explain mathematical ideas orally and in writing.

- **Mathematics as Reasoning**
  Students will make and test conjectures involving parameters affecting the account balance. They will be able to defend their conclusions about the parameters.

- **Mathematical Connections**
  Students will utilize the spreadsheet in a financial application using both simple and explicit formulas.

- **Functions**
  Students will model and analyze a real-world financial application using recursion and explicit formulation, as well as using tables and graphs. Students will analyze the effects of parameter changes on the graphs of the savings account balance.

- **Discrete Mathematics**
  Students will investigate interest earned over discrete amounts of time.

Links to Maryland High School Mathematics Core Learning Goals

- **1.1.1**
  The student will recognize, describe, and extend patterns and functional relationships that are expressed numerically, algebraically , and geometrically.

- **1.1.2**
  The student will represent patterns and functional relationships in a table, as a graph, and/or by mathematical expression.

- **1.2.4**
  The student will describe how the graphical model of a non-linear function represents a given problem and will estimate the solution.

- **1.2.5**
  The student will apply formulas to solve real-world problems.

Grade/Level:

Grades 9-12; Algebra II and beyond

Duration/Length:

This lesson will take approximately one 90 minute class period or two 45 minute periods, but may be extended due to the many teacher options.

Prerequisite Knowledge:

Students should have working knowledge of spreadsheet software.

Objectives:

Using a computer spreadsheet, students will be able to:

- analyze data both numerically and graphically.
- manipulate data to investigate changes in parameters and outcomes.
- draw conclusions and communicate their findings orally and verbally.
- recognize an exponential function.
- compose an algorithm.
- investigate the effects of compounding.

Materials/Resources/Printed Materials:

- Computer with spreadsheet software
- Copies of student worksheets and assessment

Development/Procedures:

- First, the class should do the Warm-up Activity using an overhead projector. For number 1, the teacher may need to review I = PRT with students. Number 2 can lead into a discussion on the development of the simple formula for interest.

- Students will need to be at a computer to work on the rest of the lesson. The first activity is about Simple Interest. There are three separate resources enclosed as follows: a.) spreadsheet with formulas, b.) spreadsheet with examples, and c.) blank spreadsheet. These three resources can be copied and given to students or simply used for teacher reference. Spreadsheet 'a' can be copied and given to students to assist in inputting formulas; spreadsheet 'b' is for teacher use as examples; and spreadsheet 'c' can be copied for students to fill out and turn in for a grade. Students can investigate the relationship between the balance, principle, and interest. Using the graphing capabilities of the software, students can graph two balances over time, then compare and analyze. A sample graph is included. Possible questions could involve the shape of the curves (exponential), the point of intersection and what it represents, and y-intercept.

- The next investigation deals with compounding interest over time. Three types of hardcopies and a graph are included for this activity. Teacher can discuss the number of compounding periods (n) with class and have them calculate hours, minutes, and/or seconds. Class can manipulate years, principle, and interest. Graphs can be generated to compare various data. A sample graph is included of the two extreme compounding periods, annually and daily, based on the given example. Possible questions could include: How does the compounding period affect balance over time? Does it make a big enough difference? What if your principle is $ 1 million?

- Give the performance assessment. Students will need to have access to the spreadsheet software.

Extension/Follow Up:

- Use a spreadsheet to show there is a constant ratio between consecutive terms in an interest problem and make the connection to both geometric sequences and exponential functions.
- The Compounding Interest Activity can lead into a discussion of compounding interest continuously, Balance = P * e^(r*t), and limits.
- Discuss doubling time and test the Rule of 72 (under 10%) and Rule of 70 (over 10%).
- Calculate the effective interest rate and use this to compare accounts offering various interest rates and compounding periods.
- Have students look up average interest rates in past years using various sources such as parents, Internet, or library.
- Have students make an advertisement for a bank's savings account to draw investors to their institution.
- Have students investigate other economic factors that affect savings account balance such as taxes and the Consumer Price Index

Authors:

Kristine Coan
Frederick High School
Frederick County, MD

Karen Hunkele
Catoctin High School
Frederick County, MD

Dale Harriman
Long Reach High School
Howard County, MD

George Reidy
School Without Walls
Washington, DC

WARM-UP

1. Calculate the simple interest earned and the balance on $1,000 earned at 5% (Use I = PRT) for:

   a. 1 year

   b. 2 years

   c. 5 years

2. Complete the interest table below for a principle of $1,000 earning 5% interest compounded yearly. Add each year's interest to the current principle to get the new principle.

YEARS	CURRENT PRINCIPLE		INTEREST EARNED		NEW PRINCIPLE
1	**1,000**	**+**	**1,000*.05**	**=**	**1,050**
2	**1,050**	**+**		**=**	
3		**+**		**=**	
4		**+**		**=**	
5		**+**		**=**	

SIMPLE INTEREST WORKSHEET -- FORMULAS

Simple Interest		
Principle	400	800
Interest	9	4.5
Rate/100	=$B$4/100	=$C$4/100
Years	**BALANCE 1**	**BALANCE 2**
1	=$B$3	=$C$3
=A8+1	=B8+B8*$B$5	=C8+C8*$C$5
=A9+1	=B9+B9*$B$5	=C9+C9*$C$5
=A10+1	=B10+B10*$B$5	=C10+C10*$C$5
=A11+1	=B11+B11*$B$5	=C11+C11*$C$5
=A12+1	=B12+B12*$B$5	=C12+C12*$C$5
=A13+1	=B13+B13*$B$5	=C13+C13*$C$5
=A14+1	=B14+B14*$B$5	=C14+C14*$C$5
=A15+1	=B15+B15*$B$5	=C15+C15*$C$5
=A16+1	=B16+B16*$B$5	=C16+C16*$C$5
=A17+1	=B17+B17*$B$5	=C17+C17*$C$5
=A18+1	=B18+B18*$B$5	=C18+C18*$C$5
=A19+1	=B19+B19*$B$5	=C19+C19*$C$5
=A20+1	=B20+B20*$B$5	=C20+C20*$C$5
=A21+1	=B21+B21*$B$5	=C21+C21*$C$5
=A22+1	=B22+B22*$B$5	=C22+C22*$C$5
=A23+1	=B23+B23*$B$5	=C23+C23*$C$5
=A24+1	=B24+B24*$B$5	=C24+C24*$C$5
=A25+1	=B25+B25*$B$5	=C25+C25*$C$5
=A26+1	=B26+B26*$B$5	=C26+C26*$C$5

SIMPLE INTEREST EXAMPLES

Simple Interest					
	Maternal	Paternal			
Principle	$500.00	$750.00			
Interest Rate	5.50	3.50			
Rate/100	0.055	0.035			
Years	BALANCE 1	BALANCE 2			
1	$500.00	$750.00			
2	$527.50	$776.25			
3	$556.51	$803.42			
4	$587.12	$831.54			
5	$619.41	$860.64			
6	$653.48	$890.76			
7	$689.42	$921.94			
8	$727.34	$954.21			
9	$767.34	$987.61			
10	$809.55	$1,022.17			
11	$854.07	$1,057.95			
12	$901.05	$1,094.98			
13	$950.60	$1,133.30			
14	$1,002.89	$1,172.97			
15	$1,058.05	$1,214.02			
16	$1,116.24	$1,256.51			
17	$1,177.63	$1,300.49			
18	$1,242.40	$1,346.01			
19	$1,310.73	$1,393.12			
20	$1,382.82	$1,441.88			
21	$1,458.88	$1,492.34			
22	$1,539.12	$1,544.57			
23	$1,623.77	$1,598.63			
24	$1,713.08	$1,654.59			

SIMPLE INTEREST WORKSHEET

Simple Interest				
Principle				
Interest Rate				
Rate/100				
Years	**BALANCE 1**	**BALANCE 2**	**BALANCE 3**	**BALANCE 4**
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				
GRAPH:				

COMPOUNDED INTEREST – FORMULAS

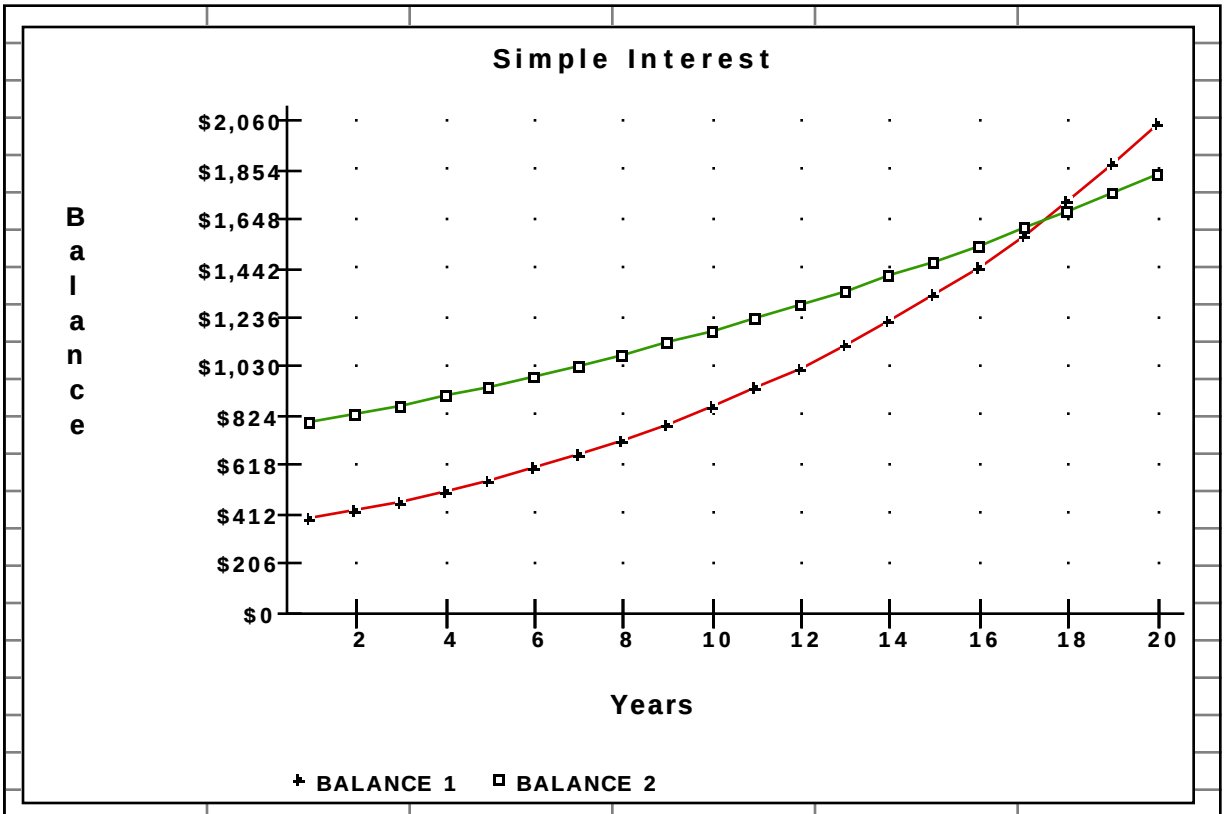
Interest by Compounding Periods per year		
Principle	400	
Interest	9	
Rate/10	=$B$4/100	
B = P*(1+(r/n))^(n*t)		
B: Balance		
P: Principle		
r: Interest rate		
n: # of compoundings per year		
t: time (years)		
time (	1	5
n		
1	=$B$3*(1+($B$5/$A18))^($A18*B$16)	=$B$3*(1+($B$5/$A18))^($A18*C$16)
2	=$B$3*(1+($B$5/$A19))^($A19*B$16)	=$B$3*(1+($B$5/$A19))^($A19*C$16)
4	=$B$3*(1+($B$5/$A20))^($A20*B$16)	=$B$3*(1+($B$5/$A20))^($A20*C$16)
6	=$B$3*(1+($B$5/$A21))^($A21*B$16)	=$B$3*(1+($B$5/$A21))^($A21*C$16)
12	=$B$3*(1+($B$5/$A22))^($A22*B$16)	=$B$3*(1+($B$5/$A22))^($A22*C$16)
52	=$B$3*(1+($B$5/$A23))^($A23*B$16)	=$B$3*(1+($B$5/$A23))^($A23*C$16)
365	=$B$3*(1+($B$5/$A24))^($A24*B$16)	=$B$3*(1+($B$5/$A24))^($A24*C$16)
hours		
mins		
secs		

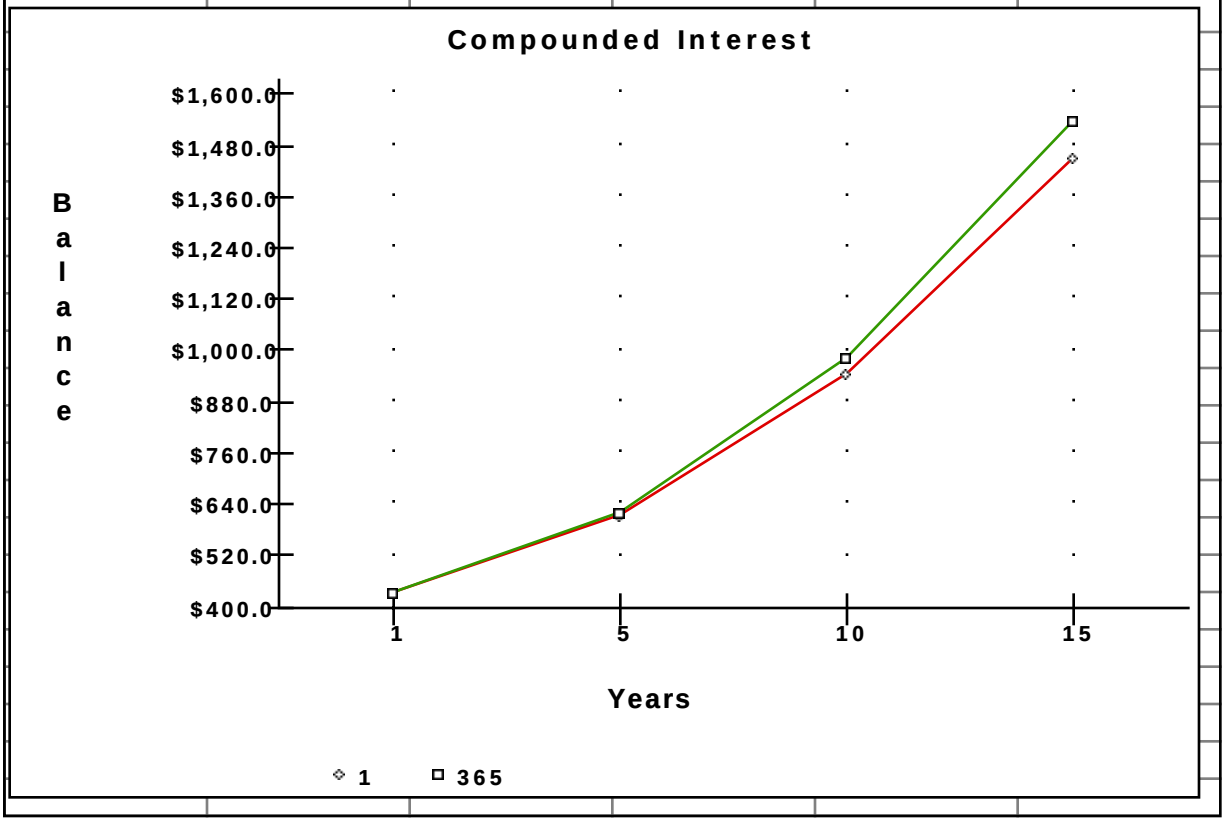
Interest by Compounding Periods per year				
Principle	$400.00			
Interest Rate	9.00			
Rate/100	0.090			
B = P*(1+(r/n))^(n*t)				
B: Balance				
P: Principle				
r: Interest rate				
n: # of compoundings per year				
t: time (years)				
time (years)	1	5	10	15
n				
1	$436.00	$615.45	$946.95	$1,456.99
2	$436.81	$621.19	$964.69	$1,498.13
4	$437.23	$624.20	$974.08	$1,520.05
6	$437.38	$625.23	$977.29	$1,527.58
12	$437.52	$626.27	$980.54	$1,535.22
52	$437.64	$627.08	$983.08	$1,541.17
365	$437.66	$627.29	$983.73	$1,542.71
hours				
mins				
secs				
continuous				

COMPOUNDED WORKSHEET

Interest by Compounding Periods per year				
Principle				
Interest Rate				
Rate/100				
B = P*(1+(r/n))^(n*t)				
B: Balance				
P: Principle				
r: Interest rate				
n: # of compoundings per year				
t: time (years)				
time (years)	1	5	10	15
n				
1				
2				
4				
6				
12				
52				
365				
hours				
mins				
secs				
continuous				
GRAPH:				

GRAPHS


Simple Interest
Balance
$2,060
$1,854
$1,648
$1,442
$1,236
$1,030
$824
$618
$412
$206
$0
2
4
6
8
10
12
14
16
18
20
Years
BALANCE 1
BALANCE 2


Compounded Interest
Balance
$1,600.0
$1,480.0
$1,360.0
$1,240.0
$1,120.0
$1,000.0
$880.0
$760.0
$640.0
$520.0
$400.0
1
5
10
15
Years
1
365

Performance Assessment

Teacher's Guide

Introduction

This activity will use interest calculations as an introduction to exponential functions. Students will investigate the effects of changing the principal and/or interest rates. This lesson will take approximately one 90 minute class period, but may be extended due to the many teacher options.

Objectives Covered

Using a computer spreadsheet, students will be able to:

- analyze data both numerically and graphically.
- manipulate data to investigate changes in parameters and outcomes.
- draw conclusions and communicate their findings orally and verbally.
- recognize an exponential function.

Tools/Materials Needed for Assessment

- Computer with spreadsheet software
- Copies of the Assessment

Administering the Assessment

This task should take approximately 45-50 minutes.

Distribute the necessary materials to the students.

SAY: *Now you are going to work independently to complete an assessment activity that involves the concepts that I have just guided you through. Please read through the assessment first.*

Pause and allow students to read silently.

SAY: *Are there any questions? You may begin.*

Allow students time to work on assessment.

Collect students' work. Make sure that spread sheet and graph print out are included.

Performance Assessment

Student Response Sheet

Your birthday is fast approaching and you have asked for money this year. Your maternal grandparents(mother's parents) have decided to invest $500 in your name. The bank closest to them offers a 5.5% interest rate. Your paternal grandparents(father's parents), not to be outdone, decide to invest $750 in your name. However, the bank in their neighborhood only offers a 3.5% interest rate.

1. Using the spreadsheet Simple Interest change the given balances to match your birthday gifts. You also need to change the interest rates to match those offered at the two banks. Print out your spreadsheet.

2. If you leave the money in the bank for the specified time, which gift is worth the most? Explain.

(a.) 6 years _____
_____.

(b.) 12 years _____
_____.

(c.) 24 years _____
_____.

3. Using the spreadsheet to graph both situations. Print your graph.

4. What type of function is illustrated in the Simple Interest graph?

A.) Linear
B.) Exponential
C.) Logarithmic
D.) Quadratic

5. Compare the two graphs and estimate the point of intersection. What does this point represent? _____
_____
_____
_____
_____.

6. Given a choice between an account offering 7.9% interest compounded daily and an account offering 8% compounded semi-annually, which account would you choose and why? Be sure to consider the amount of principle, the rate of interest, and the amount of time the money is in the account._____
_____
_____
_____
_____
_____

Performance Assessment

Scoring Guide

Your birthday is fast approaching and you have asked for money this year. Your maternal grandparents(mother's parents) have decided to invest $500 in your name. The bank closest to them offers a 5.5% interest rate. Your paternal grandparents(father's parents), not to be outdone, decide to invest $750 in your name. However, the bank in their neighborhood only offers a 3.5% interest rate.

1. Using the spreadsheet Simple Interest change the given balances to match your birthday gifts. You also need to change the interest rates to match those offered at the two banks. Print out your spreadsheet.

	A	B	C	D
1	Simple Interest			
2		Maternal	Paternal	
3	Principal	$500.00	$750.00	
4	Interest Rate	5.50	3.50	
5	Rate/100	0.055	0.035	
6				
7	Years	BALANCE 1	BALANCE 2	
8	1	$500.00	$750.00	
9	2	$527.50	$776.25	
10	3	$556.51	$803.42	
11	4	$587.12	$831.54	
12	5	$619.41	$860.64	
13	6	$653.48	$890.76	
14	7	$689.42	$921.94	
15	8	$727.34	$954.21	
16	9	$767.34	$987.61	
17	10	$809.55	$1,022.17	
18	11	$854.07	$1,057.95	
19	12	$901.05	$1,094.98	
20	13	$950.60	$1,133.30	
21	14	$1,002.89	$1,172.97	
22	15	$1,058.05	$1,214.02	
23	16	$1,116.24	$1,256.51	
24	17	$1,177.63	$1,300.49	
25	18	$1,242.40	$1,346.01	
26	19	$1,310.73	$1,393.12	
27	20	$1,382.82	$1,441.88	
28	21	$1,458.88	$1,492.34	
29	22	$1,539.12	$1,544.57	
30	23	$1,623.77	$1,598.63	
31	24	$1,713.08	$1,654.59	
32				

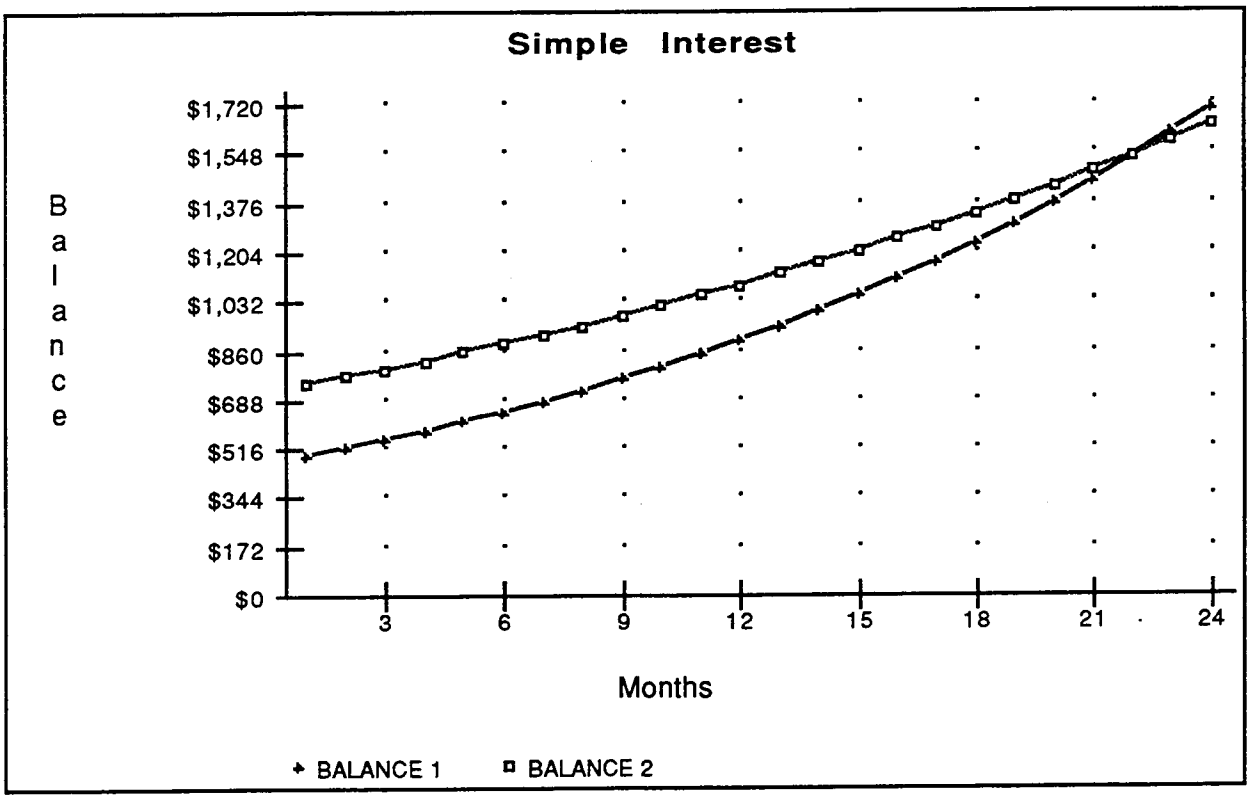
2. **If you leave the money in the bank for the specified time, which gift is worth the most? Explain.**

(a.) **6 years** *The paternal gift is worth more. The initial deposit is higher and the money has not been there long enough for the interest to matter.*

(b.) **12 years** *The paternal gift is worth more for the same reason as above, but the difference is not as extreme*

(c.) **24 years** *The maternal gift is now worth more. The higher interest rate has had enough time to have taken effect, specifically two years.*

3. **Using the spreadsheet to graph both situations. Print your graph.**



4. **What type of function is illustrated in the Simple Interest graph?**

A.) **Linear**
B.) Exponential
C.) **Logarithmic**
D.) **Quadratic**

5. **Compare the two graphs and estimate the point of intersection. What does this point represent?** *The point of intersection is approximately (22,1540). This means that at about 22 years the two gifts will both be worth about $1540.*

Scoring Key:

3 This response includes the following:
- Correct solution
- Identifies relationship between time and balance

2 This response includes the following:
- Correct solution shown
- Does not identify all relationships

1 This response includes the following:
- All other responses

0 No response

6. Given a choice between an account offering 7.9% interest compounded daily and an account offering 8% compounded semi-annually, which account would you choose and why? Be sure to consider the amount of principle, the rate of interest, and the amount of time the money is in the account. *I would choose the account with 7.9% because it would yield more interest due to the fact that it is compounded more frequently at approximately the same rate. Over a long period of time, the difference becomes more significant. Also, the larger the balance, the more significant the dollar amount difference is.* **.**

Scoring Key:

3 This response includes the following:
- Correct solution shown
- Identifies the relationship between interest, time, and principle

2 This response includes the following:
- Correct solution shown
- Does not identify all relationships

1 This response includes the following:
- All other responses

0 No response