Title: Don't Spill Over (balancing equations)

Brief Overview:

In this unit, students will represent equivalent equations using variables. These lessons begin with the students using hands on activities to explore the topic of balanced equations using variables to represent shapes in equations.

NCTM Content Standard/National Science Education Standard:

Algebra: Represent and analyze mathematical situations and structures using algebraic symbols
Identify, write, solve, and apply equations and inequalities.

- Represent relationships using appropriate relational symbols (<, >, =) and operational symbols (+, -, x, ÷) on either side

Grade/Level:

Grades 2/3.

Duration/Length:

This unit consists of three 60-minute lessons and one summative assessment.

Student Outcomes:

Students will:

- Use variables to represent pattern blocks.
- Show equality using pattern blocks.
- Write balanced equations using variables.

Materials and Resources:

- Journal – Student Resource Sheet # 1
- Rulers for every student
- Bins of pattern blocks (hexagons, trapezoids, parallelogram, and triangles)
- Overhead projector
- Balanced Equations Chart - Student Resource Sheet # 2
- Combinations Worksheet – Student Resource Sheet # 3
- Hexagon Puzzle – Student Resource Sheet # 4
- Hexagon Puzzle directions – Student Resource Sheet # 4
- Assessment – Student Resource Sheet # 5

Development/Procedures:

Lesson 1 .....

Pre-assessment

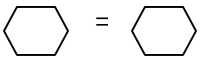
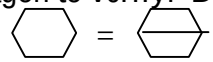
1. Direct students to write or draw what they think the word *balance* means on the first page of their journal, *Student Resource Sheet # 1*.
2. Have students share their journals with a partner. Have partners create a definition for the word balance to share with the class.
3. Create one class definition for the word balance.
   Balance means equal quantities on each side.

Launch

Distribute rulers and allow 3 minutes for students to explore ways to balance rulers on their fingers. Then model how to balance the ruler for the class.

- Ask students to explain how the ruler is able to balance on their finger. *The length of the ruler on each side is the same*.

Teacher Facilitation

1. Review the class definition of balance and how it was demonstrated while using the ruler.
2. Show students a hexagon from the pattern blocks. Place it on the overhead. Show students another hexagon.
   - Ask students if the second one is the same or equivalent to the first. (*Students should say yes*.)
   - Ask students to explain how they know the hexagons are equivalent. (*The second yellow hexagon can be put on top of the first hexagon to show that they are equal*.)
3. On the overhead, demonstrate how to draw the equation. 
4. Pass out the Balanced Equations Chart, S*tudent Resource Sheet # 2*. Have students draw the hexagon equation in the first box in the workspace column.
5. Discuss with students how hard it is to draw a hexagon perfectly.
6. Explain to students that in mathematics symbols can be used to represent other numbers or objects. These symbols are called variables. For example, we can use the letter h to represent 1 hexagon. Show how to write the equation h = h for the example above.
7. Have students copy the equation for the hexagons using variables into the first box under the equations column.
8. Take away the second hexagon. Show students a trapezoid.
   - Ask students how many trapezoids they think they would need to make 1 hexagon. Accept all answers.
9. Show students that they can make 1 hexagon with 2 trapezoids by putting the trapezoids on top of the hexagon to verify. Draw the equation on the overhead. 

10. Tell students that the variable z will be used to represent the trapezoid. Have students create the equation to represent the picture.
11. Have students record the equation h = z + z on their charts in the second box under the equations column.

Student Application

Allow students time to explore using the other shapes (parallelogram and triangle) to figure out how many of each shape are needed to make 1 hexagon and record their findings on their chart in the form of an equation using variables (parallelogram = p, and triangle = t).

h = p + p + p h = t + t + t + t + t + t

Embedded Assessment

Students will answer the question on the bottom of the Balanced Equations Chart. How do you know your answers are correct? (*My answer is correct because I put the shapes on top of the hexagon and they are equal.)*

Re-teaching

Pull struggling students into a small group. Review how you can determine that 2 trapezoids equals 1 hexagon. Assist students with the parallelogram and the triangle. Review how to write the equation using variables.

Extension

1. Have students use the triangles to make the trapezoid and the parallelogram and write the equations on the back of the Balanced Equations Chart.

2. On the computer, students use weights with assigned values to determine how much the poodle weighs.
   http://pbskids.org/cyberchase/games/algebra/algebra.html

3. On the computer, students use weighted shapes to balance a scale.
   http://illuminations.nctm.org/tools/tool_detail.aspx?id=33

Lesson 2

Pre-assessment

Direct students to complete Journal 2A, *Student Resource Sheet # 1*. Write an equation from yesterday using variables. *Students can use the pattern blocks to help them.*

Launch

Pose the scenario to the class: Amy has 1 hexagon. Kelly has 1 trapezoid and 3 triangles. Do they have the same amount?

Allow students to use pattern blocks to investigate the answer. Call on students to give their answers and explain how they figured it out. Have students write the equation using the variables to show that they are equal.

$h = z + t + t + t$

Teacher Facilitation

Explain to students that they can have different combinations of items on the same side of an equation as long as they equal the other side. Ask students if they can think of a way to use a combination of parallelograms and triangles to make a hexagon. Allow 3-5 minutes for exploration. Have students share with the class their discoveries. Have the class write the equations using variables.

$h = p + p + t + t$ $\quad$ $h = p + t + t + t + t$

Student Application

In pairs, have students complete the Combination Worksheet, S*tudent Resource Sheet # 3*. As a class, review the different combinations the students discovered. Have students prove that their equations are balanced by putting different shapes on top of the other shape to show that they are the same.

Show students the equation $z = p$
Ask students: "What is wrong with the equation?" (*It is not balance*d.)
Ask students: "How could we balance the equation?" (*Add another piece*.)
Have students figure out what piece is missing. (*Triangle)*
Have students write the equation correctly. Have students complete Journal 2B, S*tudent Resource Sheet # 1. What variable would complete this equality? How do you know your answer is correct?*

Embedded Assessment

While students are working on the Combination Worksheet, observe students to determine their understanding of balanced equations.

Re-teaching

Assist small groups in completing the worksheet. Model how to find equality using pattern blocks. Show them how to check their work by putting the shapes on top of each other.

Extension

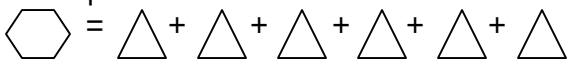
1. Have students use more than 1 shape on the left side of the equation and find balanced equations.
   Example: $h + h = z + z + z + t + t + t$

2. On the computer, students use weighted shapes to balance a scale.
http://illuminations.nctm.org/tools/tool_detail.aspx?id=33

Lesson 3 .....

Pre-assessment

Direct students to complete Journal 3, S*tudent Resource Sheet # 1*.
Students will rewrite the picture equation using variables to represent the same equation.



h = t + t + t + t + t + t

Launch

Ask students: "Can you think of another way to write this equation that might be faster?" Allow 1-2 minutes of "think time." Call on students to share their ideas and discuss why their response is correct or incorrect. Once a student comes up with the correct response, h = 6t, move on to teacher facilitation. If no one is able to figure out the correct answer, give the correct equation and move on to teacher facilitation.

***T*eacher Facilitation**

1. Explain to students that since the triangles are all exactly the same we can use a shortcut to record how many we used. Since there were 6 triangles used, we put the number 6 in front of the variable t.
2. Put the equation h = t + t + t + z on the board. Have students help you rewrite this equation using the short cut (h = 3t + z).
3. Put the following equations on the board and have students use the short cut to rewrite them.
   1. h = z + z (h = 2z)
   2. z = t + p + t (z = 2t + p) or (z = p + 2t)
   3. p + p = t + t + t + t (2p = 4t)

4. Call on students to review the answers. Have students explain how they found their answers.

Student Application

1. Explain to students that in order to complete the next activity they will need to use all of the knowledge that they have learned about balanced equations.
2. Model on the overhead how to use shapes to cover the hexagon, as shown on *Teacher Resource 01*. Have students write the equation that shows how you covered the big hexagon. *(H = h + 4t + 2z + 4p)*
3. Then, move all the shapes over onto the second hexagon but put them in different positions.

- Ask students: "Is this hexagon equal to the first hexagon?" (Y*es*)
- Ask students: "How do you know?" (*They are equal because you used the same exact pieces to cover the area*.)
- Have students tell you what the equation for the second big hexagon is. {*It will be exactly the same since all the same pieces were used. (H= h + 4t + 2z + 4p)}*

4. Distribute the Hexagon Puzzle worksheet, S*tudent Resource Sheet # 4*.
5. Students will use pattern blocks to create 2 other ways to cover the big hexagon (H).

Embedded Assessment

While the students are working on the Hexagon Puzzle worksheet, the teacher will observe students' understanding of the material learned.

Re-teaching

Question students to verbalize their thinking about what they are trying to do while completing the Hexagon Puzzle worksheet. Clarify misconceptions that students have so that they are able to complete the assignment independently.

Extension

1. Create a Pattern Picture Puzzle. Use a white piece of paper as a mat. On top of the paper, use pattern blocks to create a picture. Trace around the outside edge of the picture. Remove the pattern pieces. Trade pictures with a partner. Have your partner use pattern blocks to cover the area.

2. On the computer, students are able to design a quilt square using all six-pattern block shapes.
   http://illuminations.nctm.org/tools/patchtool/patchtool.asp

Summative Assessment:

Students will complete Student Resource Sheet # 5: Summative Assessment. Answers can be found on Teacher Resource Sheet # 2.

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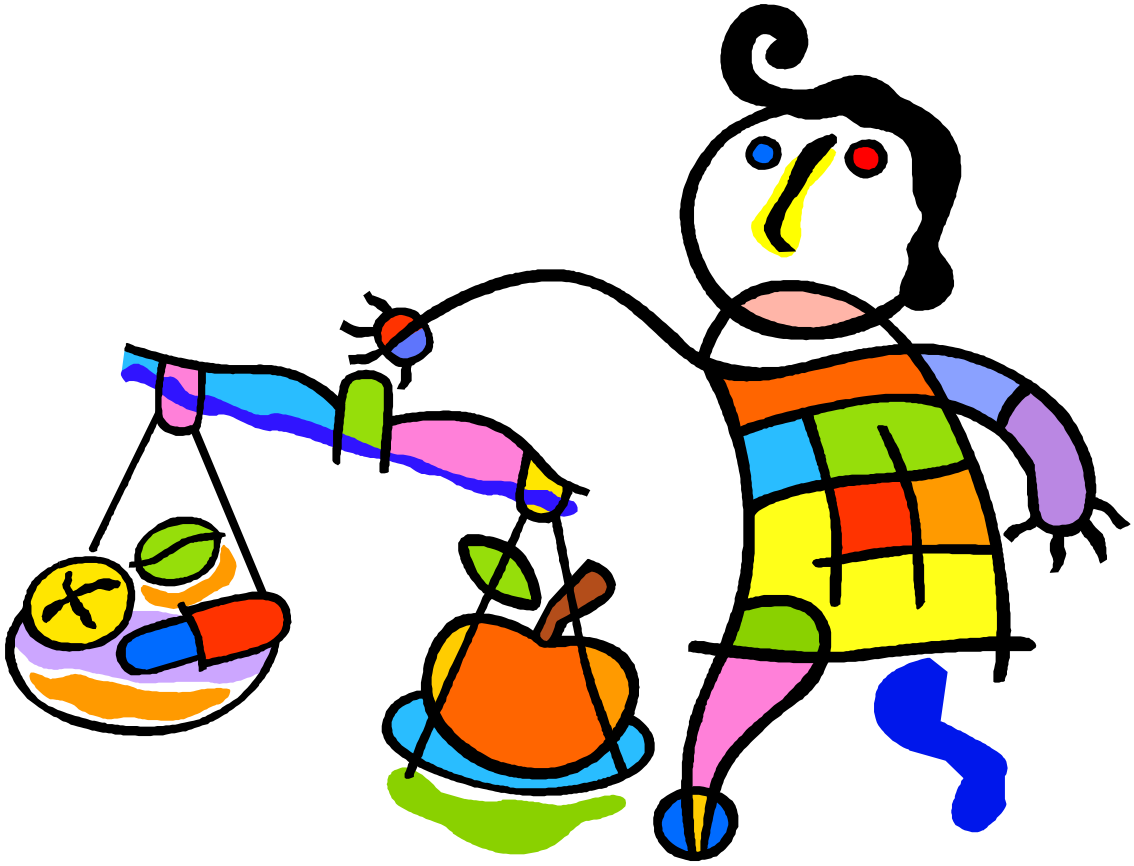
Student Resource Sheet # 1

Journal



Name _____

Day 1

Balance means

_____

_____

_____

_____

_____

Class definition:

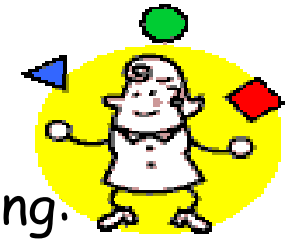
_____

_____

_____

Day 2

Part A



Write the equation for the following.

1. hexagon = ? trapezoids _____

2. hexagon = ? parallelograms_____

3. hexagon = ? triangles _____

Part B

What variable would complete this equality?

h = t + t + z + _____

How do you know your answer is correct?

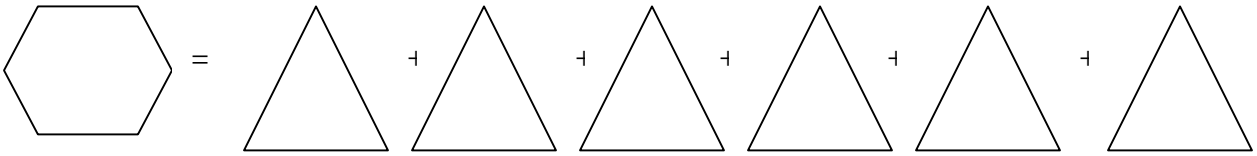
_____

_____

_____

_____

Day 3



Write the equation using variables:

_____

What is a variable?

_____

_____

_____

_____

_____

Name _____

Student Resource Sheet # 2

Balanced Equations

Work Space	Equations

Explain how you know your answers are correct:

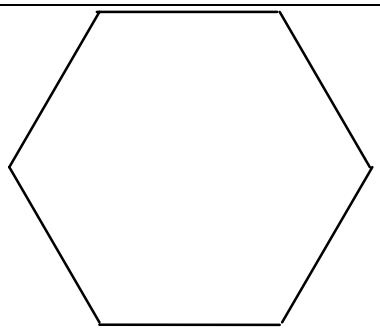
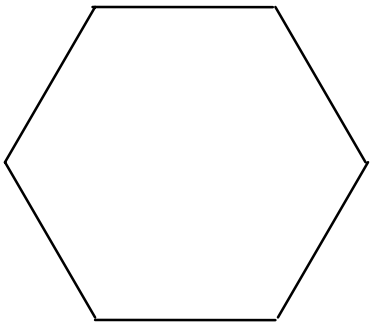
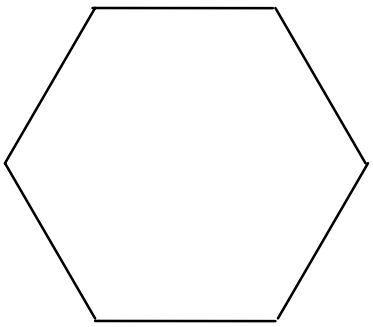
_____

_____

_____

Student Resource Sheet # 3

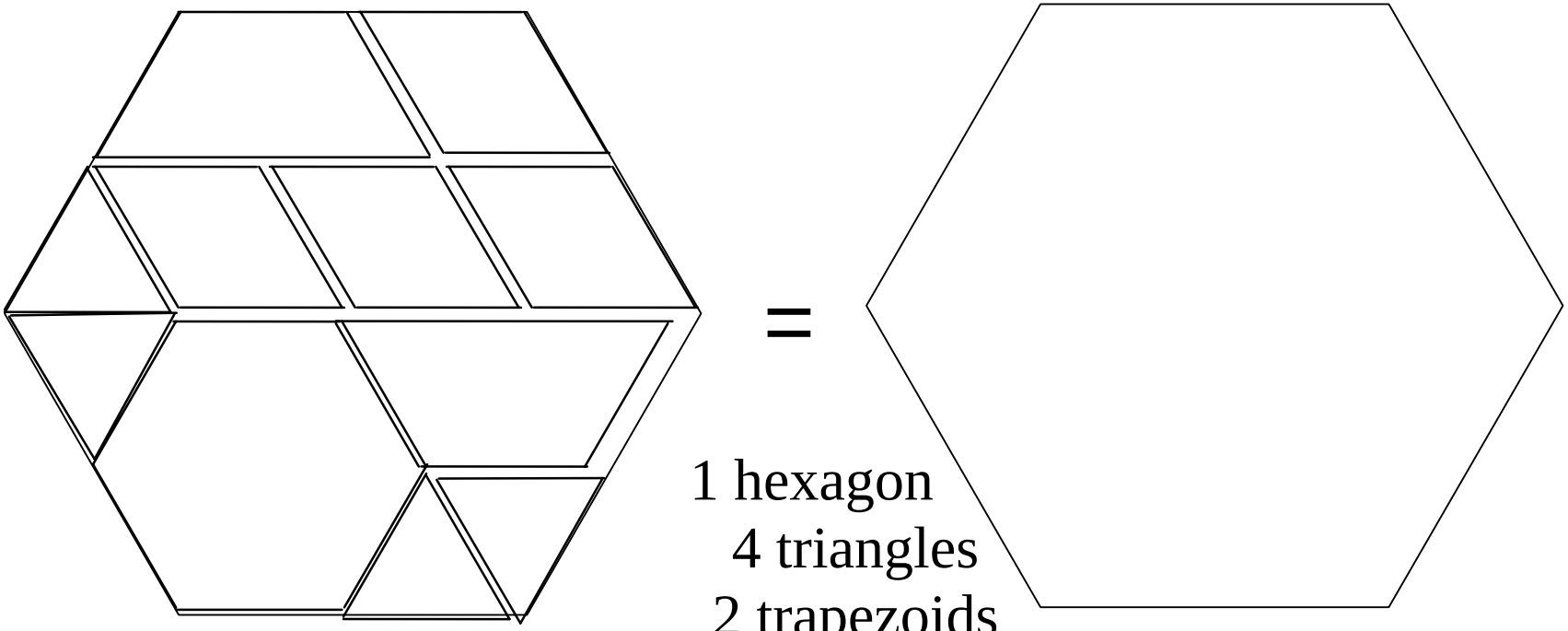
NAME _____

Work Space	Equation
	Use at least 2 parallelograms
	Use at least 1 triangle
	Use at least 1 parallelogram
	Use at least 1 trapezoid

Name _____

Modeled on the overhead

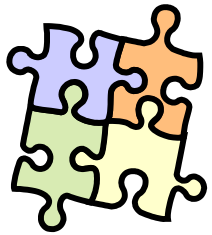
Hexagon Puzzle



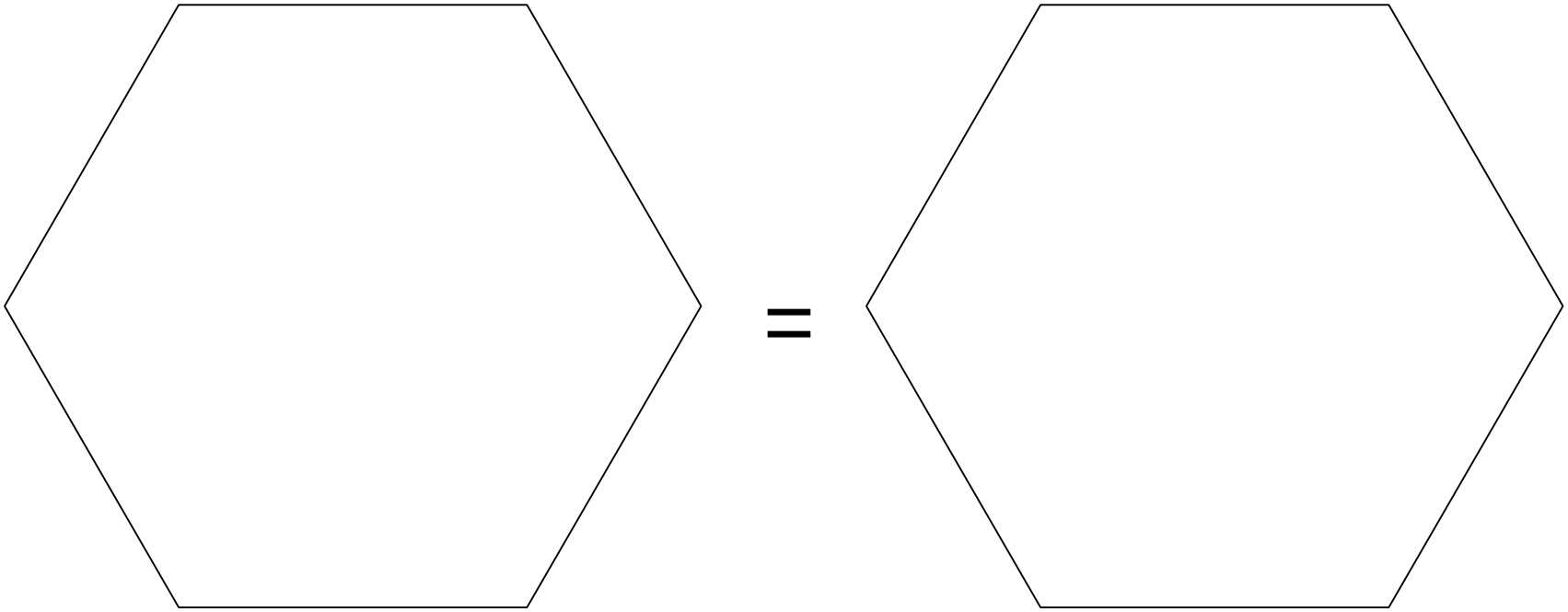
1 hexagon
4 triangles
2 trapezoids
4 parallelograms

H = h + 4t + 2z + 4p

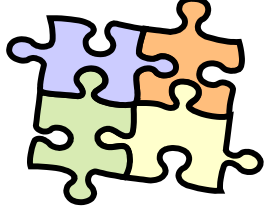
Name _____



Hexagon Puzzle



Student Resource Sheet # 4



Name _____

1. Find a different way to create H. Write the equation below use the same variables as be
(h = hexagon, z = trapezoid, p = parallelogram, t = triangle)
and H as the large hexagon on the paper.

_____

2. Find another way to create H using a different combination of pattern blocks. Write this new equation on the line below.

_____

3. Take the equation from #1 and from #2 and write them showing their equivalence.

_____ = _____

4. How do you know that they are equal?

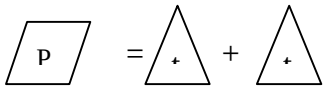
_____
_____
_____
_____
_____

Name_____ Date_____

Summative Assessment

Directions – Choose the best answer to complete each question.

1. Choose the correct equation for the following picture.



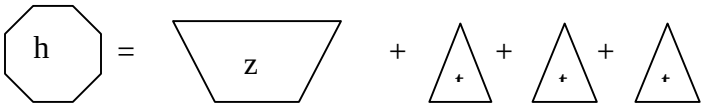
a. h = t + t

b. p = t + t

c. p = t

d. t = p + t

2. Choose the correct equation for the following picture.



a. z + h = t + t

b. h = z + z

c. h = 3t + z

d. h = 3z + t

3. Which equation is balanced?

a. p = 4t

b. h = 4t + z

c. z = 3t

d. t = h

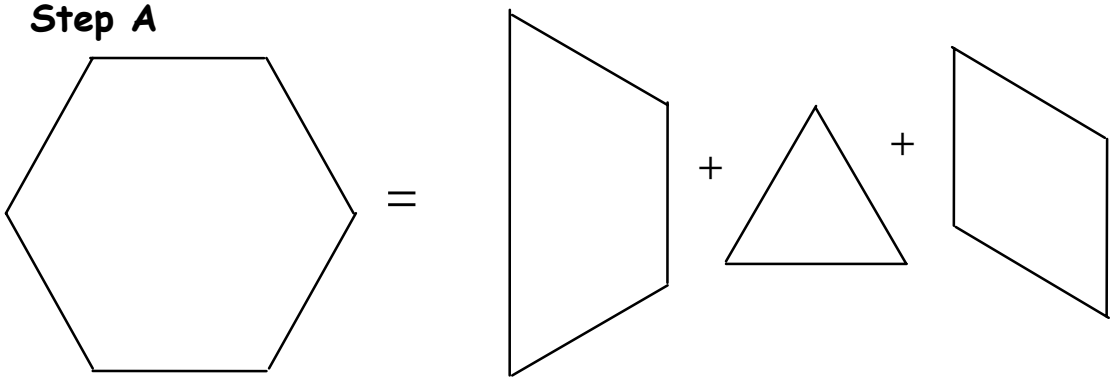
4. A variable is _____

5. What is a balanced equation? _____

Brief Constructed Response

We have spent 3 days working on writing equations with variables. Below is an equation using pattern blocks. Using the same variables from the past days, write the equation shown.

Step A



_____

Step B

Use what you know about balanced equations to explain why your answer is correct. Use new vocabulary in your explanation.

_____

_____

_____

_____

_____

_____

Have pattern blocks available for students to use during the assessment.

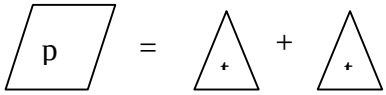
Teacher Resource Sheet # 2

Name_____ Date_____

Summative Assessment

Directions – choose the best answer to complete each question.

1. Choose the correct equation for the following picture.



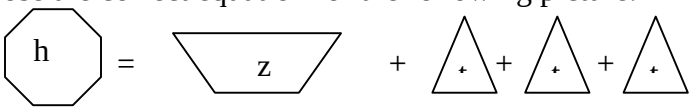
a. h = t + t

c. p = t

<u>b. p = t + t</u>

d. t = p + t

2. Choose the correct equation for the following picture.



a. z + h = t + t

<u>c. h = 3t + z</u>

b. h = z + z

d. h = 3z + t

3. Which equation is balanced?

a. p = 4t

<u>c. z = 3t</u>

b. h = 4t + z

d. t = h

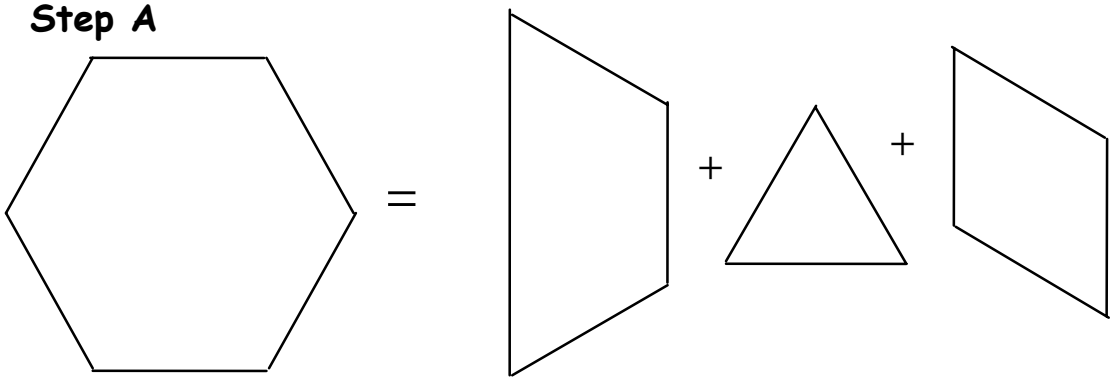
4. A variable is **<u>a letter that stands for a picture or number.</u>**

5. What is a balanced equation? **<u>A balance equation is equal on both sides.</u>**

Brief Constructed Response

We have spent 3 days working on writing equations with variables. Below is an equation shown using pattern blocks. Using the same variables from the past days, write the equation shown.

Step A



h = z + t + p

Step B

Use what you know about balanced equations to explain why your answer is correct. Use new vocabulary in your explanation.

<u>Accept answers that include pictures showing overlapping figures. As well as words that describe putting blocks on top of the other to check that they are the same size.</u>