

Title: March of the Dividing Ant  
***Divisibility Rules; Factorization***

**Brief Overview:**

Students will explore their understanding of the divisibility rules of 2, 3, 5, 6, 9, and 10 in order to deconstruct whole numbers between 0-1000. Divisibility rules can be used as shortcuts to long division. Students who know these rules have an easier time with factorization, simplification of fractions, common denominators, and prime numbers, in addition to having an easier time with division.

**NCTM Content Standard/National Science Education Standard:**

Multiplicative Reasoning  
Computational Fluency

**Grade/Level:**

4-5

**Duration/Length:**

3-4 days, approx. 60 minutes per day

**Student Outcomes:**

Students will:

- ✦ Identify and use divisibility rules of 2, 3, 5, 6, 9, and 10 in order to determine relationships between factors and multiples.
- ✦ Develop number sense in order to become more fluent with multiples.
- ✦ Use rules of divisibility in order to shorten the process of long division.

**Materials and Resources:**

**Lesson One:**

Resource pages:

1. "Don't Leave Out Joe!" (Student Resource Sheet #1)
2. Hundred chart (Student Resource Sheet #2)
3. Journal page 1 (Student Resource Sheet #3)
4. Random number page (Student Resource Sheet #4)
5. Is it Divisible? (Student Resource Sheet #5)

Other resources:

- \* A Remainder of One, a book by Elinor J. Pinczes
- \* 25 snap cubes of one color, 10 snap cubes of a second color, 5 snap cubes of a third color for each student
- \* Envelopes for each group or team containing a random set of numbers.

## **Lesson Two:**

Resource pages:

6. Hundred chart (Student Resource Sheet #6)
7. “What’s the Rule” overhead transparency (Teacher Resource Sheet #1)
8. “Break It Down” strategy journal page 2 and answer key (Student Resource Sheet #7 and Teacher Resource Sheet #2)
9. Divisibility Rule shortcut journal page 3 (Student Resource Sheet #8)

Other resources:

- \* 33 snap cubes of one color, 15 snap cubes of a second color, and 11 snap cubes of a third color
- \* One Hundred Hungry Ants, a book by Elinor J. Pinczes
- \* [www.primarygames.co.uk](http://www.primarygames.co.uk)

## **Lesson Three:**

Resource pages:

10. “Try It Yourself,” (Student Resource Sheet #9)
11. “Find Me A Number” number cards (Student Resource Sheet #10)
12. Journal Page 4 and Journal Page Answer Key (Student Resource Sheet #11 and Teacher Resource Sheet #3)
13. Summative Assessment (Student Resource Sheet #12)
14. Anchor paper (Teacher Resource Sheet #4)
15. Rubric (Teacher Resource Sheet #5)

Other resources:

- \* Calculator
- \* 24 Centimeter cubes

## Development/Procedures:

**Lesson 1** ..... **2's, 5's and 10's**  
**Pre-assessment** – Give each student a box of raisins and the worksheet “Don’t Leave Out Joe!” ([Student Resource #1](#)) Students should work independently for no more than five minutes to recall what they know about even and odd numbers to attempt to arrange their raisins evenly in rows and columns. When they come to an arrangement, they should record their answers on the worksheet. (Some students may not be able to do this depending on the number of raisins they have in their box. The frustration level is important for new concept attainment.)

**Launch** – Discuss each student’s data and results. Conduct a gallery walk\* so each student can see the solutions created by other students. At the conclusion of the gallery walk, introduce the book, [A Remainder of One](#), by Elinor J. Pinczes. Discuss how the main character solves the problem of being a remainder.

\*Gallery Walk- Students display their work on their desks or around the room and then walk around to read and look at the classmates’ products.

**Teacher Facilitation** – Using a hundreds chart ([Student Resource #2](#)) and snap cubes, lead the students to place a cube on each number counting by two’s. Introduce the vocabulary words, *multiple* and *divisible*. Students should write down all the even numbers from 0-50 in a column in their journals ([Student Resource #3](#)). Ask students to repeat the process with multiples of five, stacking the new snap cubes on top of the old ones if the two numbers overlap. Finally, the students should repeat the process again with multiples of ten, continuing to stack the new overlapping numbers and writing the new numbers in columns in their journals. Discuss the properties of the columns and brainstorm the patterns that identify how a number fits into each column. For example, each number in the *Multiples of 10* column ends in zero. Therefore, any number that ends in zero would fit into that column.

**Student Application** – Each student team will receive an envelope containing several numbers ([Student Resource #4](#)). The students will complete “Is It Divisible?” ([Student Resource #5](#)) identifying the properties of each number and stating whether it is evenly divisible by 2, 5, or 10. They should not use long division, but be able to identify the correct properties by sight.

**Embedded Assessment** – On the bottom of “Is It Divisible?” students will write a rule for divisibility by 2, 5, and 10. Criteria are listed on the sheet.

**Reteaching/Extension** – For those who are experiencing difficulty grasping the concept, employ peer tutoring and use a different manipulative, i.e. centimeter cubes, toothpicks, buttons, or macaroni noodles in order to provide individual reinforcement.

## **Lesson 2 .....3's, 6's and 9s**

**Pre-assessment**– *Using a hundreds chart (Student Resource #6) and snap cubes, lead the students to place a cube on each multiple of three, reviewing the vocabulary words from yesterday. Students should repeat the pattern with multiples of nine. They should simply observe that many of the numbers overlap and form patterns on the chart.*

**Launch** – Remind students of how they figured out the problems of the day before. Indicate that today they will be challenged to find a slightly different method to discover a new divisibility rule. On the overhead, use the “What’s the Rule?” transparency (Teacher Resource #1) with 2 columns labeled Yes and No. By placing numbers which are multiples of 3 in the Yes column, and numbers which are not multiples of three in the No column, the teacher leads the students to look for relationships among the Yes column numbers. For example, place the number 18 in the Yes column and the number 14 in the No column. After waiting for student predictions, place the number 6 in the Yes column, and 10 in the No column. Continue the process with the numbers 12, 30, 33, and 27 in the Yes column, and 11, 23, 5 and 29 in the No column.

**Teacher Facilitation** –Using the numbers in the Yes column, the teacher directs students to break the digits apart and find their sum. For example, the number 18 can be broken into the digits 1 and 8. When they are added they equal 9 that is a multiple of three. For another example, the number 24 can be broken into the digits 2 and 4. When they are added they equal 6, which is another multiple of three. Repeat the same process with the other numbers from the Yes column. Elicit a generalization from the students, which states that any number whose digits add up to 3, 6, or 9 fits the divisibility rule for 3.

**Student Application** – Students will work in pairs to complete “Break It Down” (Student Resource #7). Using a number cube to generate 3- digit numbers they will test the divisibility rule of three. Answers can be found on Teacher Resource #2.

**Embedded Assessment** –Students will create a divisibility rule shortcut page (Student Resource #8) in their journals summarizing the rules of 2, 5, 10, 3, 6, and 9.

**Reteaching/Extension** – Students will explore the games “Paint 100,” “Connect 4,” and “Multiple Wipe-out” with this concept on [www.primarygames.co.uk](http://www.primarygames.co.uk) .

### Lesson 3 Factorization

**Pre-assessment-** Hand out the “Try It Yourself” worksheet ([Student Resource #9](#)). Students will decide on which divisibility rule to use in order to break down a larger number.

**Launch-** “Find Me A Number.” Place large sized number signs around the room ([Student Resource #10](#)) and asks the students to a find a number that is divisible by 5. The teacher gives the students 10 seconds to find a number that fits the pattern. After 10 seconds, the teacher should check to see that all students understand the concept.

**Teacher Facilitation-**Hand out bags of 24 centimeter cubes to each student. Instruct students to arrange their cubes in an array of 3 rows of 8 cubes. Students should draw the array in their journal ([Student Resource #11](#)) and write the number sentence that describes it.  $24 = 3 \times 8$ . Answers can be found on [Teacher Resource #3](#).

```
o o o o o o o o
o o o o o o o o
o o o o o o o o
```

Then separate the three sets of eight, and arrange each one into its own rectangle. Have the students copy the array and write the number sentence again.

$$24 = 3 \times 4 \times 2$$

```
o o o o
o o o o
```

```
o o o o
o o o o
```

```
o o o o
o o o o
```

Then each set of four can be rearranged as a 2 by 2, and the number sentence that describes it is  $3 \times 2 \times 2 \times 2$ .

```
o o           o o           o o
o o           o o           o o
```

```
o o           o o           o o
o o           o o           o o
```

It should be emphasized that even though we broke the numbers into the smallest possible units, we still have 24 cubes and when we multiply them we haven’t changed the total. Students should be encouraged to experiment with other numbers in order to see how finding a simpler number can often make a complicated problem easier.

**Student Application-** The students pair up and use calculators to compete against each other in “The Lowest Number Game.” The students take turns rolling a number cube to find a two-or three-digit number. Each student enters the number

into his or her calculator. Then they count 1, 2, 3, Go! and use the divisibility rules of 2, 3, 5, 6, 9, and 10 to divide the number into the lowest possible form.

Students should begin by dividing a single time. For example, the student rolls 432. Student #1 divides by 2, because the number ends with 2. Student #2 divides by 3, because  $4+3+2=9$  which is divisible by 3. Student #2 wins the game because he/she will result in a lower number than student #1.

***Embedded Assessment-*** Walk around and verbally asks the students to explain their strategy for the game. Students will be evaluated based on the following criteria:

- \* Students are able to verbalize which divisibility rule works the best for a given number.
- \* Students are able to use the terms divisible and multiple in a sentence.
- \* Students are able to recognize factors of easily divisible numbers.

***Re-teaching/Extension-*** As they advance the students will notice that they can often get a smaller number if they divide a second time or third time. For example,  $402$  divided by  $3 = 134$ , which is still divisible by 2, for a lowest number of  $67$  that is prime.

### **Summative Assessment:**

The students will complete a brief constructed response ([Student Resource #12](#)) that is intended to connect a real world situation to the application of divisibility rules. In part A, students will draw a picture of a seating arrangement. In part B, they will be asked to explain their answer. The anchor papers ([Teacher Resource #4](#)) and rubric ([Teacher Resource #5](#)) follow the assessment.

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Name \_\_\_\_\_ Date \_\_\_\_\_

## Don't Leave Out Joe!

Solve the problem using pictures, symbols, and/or numbers.



You have an army of ants (raisins) and they need to form even rows for their march. You may group them in any configuration you desire, as long as the rows are even and straight.

Now describe your solution using words.

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# HUNDRED CHART

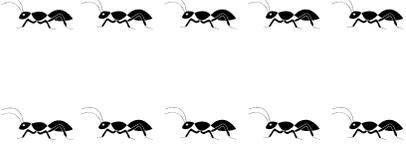
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Name \_\_\_\_\_

Date \_\_\_\_\_

**Journal Page 1**

Write the numbers that fit each category. Circle the numbers that fit into more than one category.

		
<p>Multiples of 2</p>	<p>Multiples of 5</p>	<p>Multiples of 10</p>
Empty space for student input	Empty space for student input	Empty space for student input

**Random Number Page**

Cut apart the numbers and put them into envelopes for the students to do activity 4.

**67      35      36      29      14**

**18      21      26      78      44**

**58      54      41      23      92**

**63      87      32      48      57**

**15      65      72      84      95**

Name \_\_\_\_\_

Date \_\_\_\_\_

### IS IT DIVISIBLE?

Pull each number out of the envelope one by one and write it in the first row of the chart. Decide if the number has the properties listed. Complete the table.

Number →						
<i>Is the number even?</i>						
<i>Is the number odd?</i>						
<i>Is the number divisible by 2?</i>						
<i>Is the number divisible by 5?</i>						
<i>Is the number divisible by 10?</i>						



Using the table, write a divisibility rule for 2, 5, and 10.

Hint: A number is divisible by \_\_\_\_\_ if...

2 \_\_\_\_\_

\_\_\_\_\_

5 \_\_\_\_\_

\_\_\_\_\_

10 \_\_\_\_\_

\_\_\_\_\_

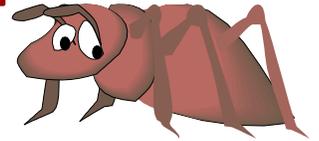
**Criteria:**

- ❑ Students can explain the divisibility rules of 2, 5, and 10 using words, pictures, and/or numbers.
- ❑ Student can generalize to other numbers.
- ❑ Student shows understanding of the word *multiple*.

# HUNDRED CHART

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

# What's the Rule?



YES

NO

YES	NO







Name \_\_\_\_\_

Date \_\_\_\_\_

# Divisibility Rule Shortcuts

1. A number is divisible by 2 if \_\_\_\_\_

\_\_\_\_\_

2. A number is divisible by 3 if \_\_\_\_\_

\_\_\_\_\_

3. A number is divisible by 5 if \_\_\_\_\_

\_\_\_\_\_

4. A number is divisible by 6 if \_\_\_\_\_

\_\_\_\_\_

5. A number is divisible by 9 if \_\_\_\_\_

\_\_\_\_\_

6. A number is divisible by 10 if \_\_\_\_\_

\_\_\_\_\_





Name \_\_\_\_\_

Date \_\_\_\_\_

# Divisibility Rule Shortcuts

1. A number is divisible by 2 if the ones place digit is a multiple of 2 or ends with 0, 2, 4, 6, or 8.
2. A number is divisible by 3 if the digits of the dividend, when added, equal a multiple of 3 (3, 6, or 9).
3. A number is divisible by 5 if the ones place digit is a 0 (zero) or a 5 (five).
4. A number is divisible by 6 if the dividend is divisible by 2 AND 3.
5. A number is divisible by 9 if the digits of the dividend, when added, equal a multiple of 9.
6. A number is divisible by 10 if the dividend ends in a zero (0); the ones place digit is a zero (0).

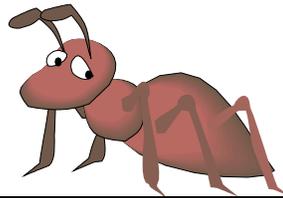


Name \_\_\_\_\_

Date \_\_\_\_\_

## TRY IT YOURSELF

Given the number 7,653,298 decide which divisibility rule you would use to break it down into even groups.



Describe your solution using words.

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Name \_\_\_\_\_ Date \_\_\_\_\_

## Find Me A Number

Directions: Cut out each number and place around the room. Tell students that given the following numbers decide which one of the divisibility rules they could use to break it down into evenly divisible groups.

1,000,486

99,025,485

3,157,470

65,123,487

201,923,015

6,357,369

56,008,935

4,569,234



Name \_\_\_\_\_

Date \_\_\_\_\_

### Journal Page 4

When you use divisibility rules to break a number down it is called factoring. Use your centimeter cubes to represent the number 24 in an array. Draw the array in the box:

Write a matching number sentence here:

Now, break your 24 cubes into 3 arrays. Draw the arrays in the box:

Write a matching number sentence here:

Now, break the 3 arrays into 6 arrays. Draw the arrays in the box:

Write a matching number sentence here:



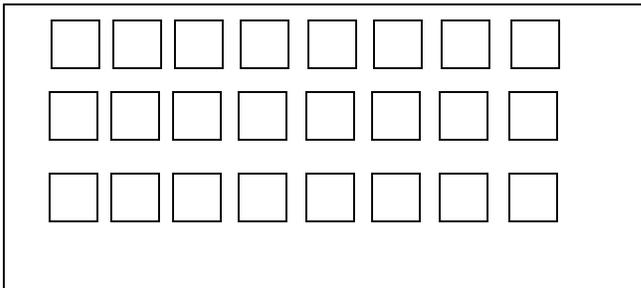


Name \_\_\_\_\_

Date \_\_\_\_\_

### Journal Page 4

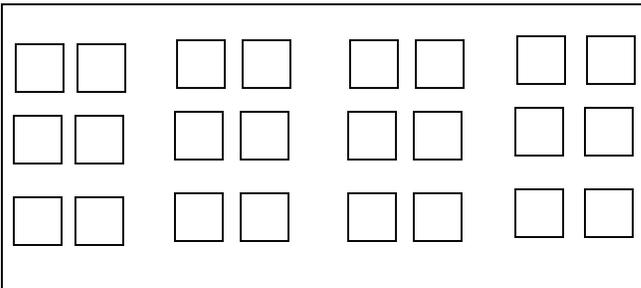
When you use divisibility rules to break a number down it is called factoring. Use your centimeter cubes to represent the number 24 in an array. Draw the array in the box:



Write a matching number sentence here:

$$24 = 8 \times 3$$

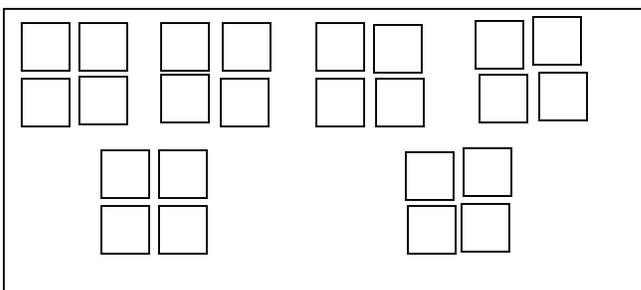
Now, break your 24 cubes into 3 arrays. Draw the arrays in the box:



Write a matching number sentence here:

$$24 = 4 \times 2 \times 3$$

Now, break the 3 arrays into 6 arrays. Draw the arrays in the box:



Write a matching number sentence here:

$$24 = 2 \times 2 \times 2 \times 3$$



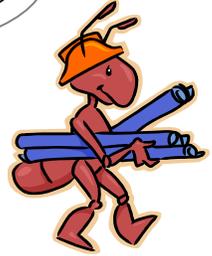
Student Summative Assessment

Name \_\_\_\_\_

Date \_\_\_\_\_

Divisibility Brief Constructed Response

The Ant Family is having a reunion! Pretend you are the picnic planner for the Ant family and have to make a seating chart for the family reunion picnic. There will be a group of 108 guests. Using what you know about divisibility rules, show your seating arrangement for the Ant family. Remember Joe wouldn't want you to leave anyone out!



Part A Use what you know about divisibility rules to divide 108 into even groups.

Part B Use pictures, words, symbols, and/or numbers to explain how you got your answer.

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Student Summative Assessment/Anchor Page

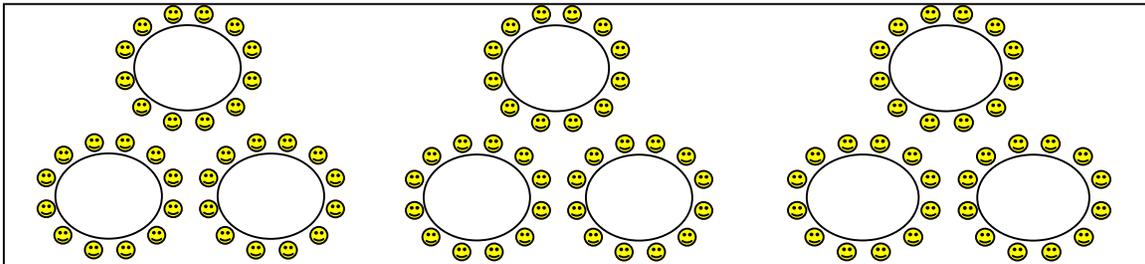
Name \_\_\_\_\_

Date \_\_\_\_\_

## Divisibility Brief Constructed Response

The Ant Family is having a reunion! Pretend you are the picnic planner for the Ant family and have to make a seating chart for the family reunion picnic. There will be a group of 108 guests. Using what you know about divisibility rules, show your seating arrangement for the Ant family. Remember Joe wouldn't want you to leave anyone out!

Part A Use what you know about divisibility rules to divide 108 into even groups.



I know that 108 is divisible by 2, 3, 6, and 9. 108 is divisible by 2 because I looked at the ones column and I saw an 8 that is an even number. I added the digits  $1 + 0 + 8$  and got the sum of 9 which means it's divisible by 3 and 9. I know that if a number is divisible by 2 and 3, then it is divisible by 6. I arranged the guests at nine tables of twelve because 2 tables of 54 is too large. I do not have any leftovers and Joe would be happy!

**MSA Mathematics BCR Rubric  
Grades 3 through 8**

**1 The response demonstrates a complete understanding and analysis of a problem.**

- Application of a reasonable strategy in the context of the problem is indicated.
- Explanation<sup>1</sup> of and/or justification<sup>2</sup> for the mathematical process(es) used to solve a problem are clear, developed, and logical.
- Connections and/or extensions made within mathematics or outside of mathematics are clear.
- Supportive information and/or numbers are provided as appropriate.<sup>3</sup>

**1 The response demonstrates a minimal understanding and analysis of a problem.**

- Partial application of a strategy in the context of the problem is indicated.
- Explanation<sup>1</sup> of and/or justification<sup>2</sup> for the mathematical process(es) used to solve a problem is partially developed, logically flawed, or missing.
- Connections and/or extensions made within mathematics or outside of mathematics are partial or overly general, or flawed.
- Supportive information and/or numbers may or may not be provided as appropriate.<sup>3</sup>

**0 The response is completely incorrect, irrelevant to the problem, or missing.<sup>4</sup>**

**Notes:**

<sup>1</sup> **Explanation** refers to students' ability to communicate **how** they arrived at the solution for an item using the language of mathematics.

<sup>2</sup> **Justification** refers to students' ability to support the reasoning used to solve a problem, or to demonstrate **why** the solution is correct using mathematical concepts and principles.

<sup>3</sup> Students need to complete rubric criteria for *explanation*, *justification*, *connections* and/or *extensions* as cued for in a given problem.

<sup>4</sup> Merely an exact copy or paraphrase of the problem will receive a score of "0".