Title: BASE-ic Space Travel

Brief Overview:

This unit introduces the concepts of bases and exponents (or powers) in order to gain a deeper understanding of place value. Students will assume the role of intergalactic space travelers. Traveling in crews of four, they will journey to galaxies that use number systems other than base 10, including base 5 and base 2. To facilitate this travel, they must purchase fuel by learning about these bases, compare them to one another, and convert numbers between bases.

NCTM Content Standard/National Science Education Standard

Understand numbers, ways of representing numbers, relationships among numbers, and number systems:

- Understand the place value structure of the base-ten number system and be able to represent and compare whole numbers and decimals
- Recognize equivalent representations for the same number and generate them by decomposing and composing numbers

Compute fluently and make reasonable estimates:

- Select appropriate methods and tools for computing with whole numbers from among mental computation, estimation, calculators, and paper and pencil according to the context and nature of the computation and use the selected method or tools

Grade/Level:

Grade 5 (Accelerated Program)

Duration/Length:

3 days (60 minutes per day)

Student Outcomes:

Students will:

- Distinguish between base and exponent (or power)
- Understand that in the base 10 system, each place value increases by the power of 10
- Represent numbers using base subscript and exponent superscript
- Convert base 10 numbers to base 5 and numbers in base 5 to base 10
- Explore conversions between bases other than base 5 and base 10

Materials and Resources:

Day 1

- Spacecraft Design (Student Resource 1)
- Base 10 materials (cubes, flats, units) for each group to have at least one of each
- Board/Overhead Example (Teacher Resource 1)
- Place Value Pocket Chart Directions (Teacher Resource 2A-B) You might want to make a pocket chart to use as a visual reference for students.
- Colored construction paper
- Glue sticks
- Scissors
- Markers
- 3" x 5" Index cards – 10 per student (You may want to make a set of digit cards and trim to 2" x 5".)
- I Have/Who Has with Exponents (Teacher Resource 3) Cut apart each card to make 1 set per group of 12 students.
- Boarding Pass Day 1 exit ticket (Student Resource 2)
- Boarding Pass Day 1 Key (Teacher Resource 4)

Day 2

- Quintopian Creature (Teacher Resource 5)
- 40 colored chips per group (10 each of 4 different colors: white, red, blue, yellow)
- Additional Place Value Pocket Charts (Teacher Resource 7)
- Board/Overhead Example (Teacher Resource 6A-C)
- Other base 5 information can be found at:
    - http://www.beaconlearningcenter.com/WebLessons/ChipCounting/default.htm
    - http://www.europa.com/~paulg/mathmodels/base5intro.html
    - http://www.ecawa.asn.au/home/jfuller/binary/binary3.htm#
- Galaxy Quintopia (Student Resource 3A-B)
- Galaxy Quintopia Key (Teacher Resource 8A-B)
- Boarding Pass Day 2 exit ticket (Student Resource 4)
- Boarding Pass Day 2 Key (Teacher Resource 9)

Day 3

- Biaculean Creature (Teacher Resource 10)
- Venn Diagram (Student Resource 5)
- Venn Diagram Key (Teacher Resource 11)
- Additional Place Value Pocket Charts (Teacher Resource 6 from Day 2) Make an eight-column chart by taping two charts together and labeling with base 2 notation.
- Board/Overhead Example (Teacher Resource 12A-D)
- Galaxy Biaculus (Student Resource 6A-C)
- Galaxy Biaculus Key (Teacher Resource 13)

- Quintopian Practice (Student Resource 7)
- Quitopian Practice Key (Teacher Resource 14)
- Ticket Home Summative Assessment (Student Resource 8A-B)
- Ticket Home Key (Teacher Resource 15A-B)

Development/Procedures:

Day 1

- Pre-assessment
  On an overhead or chalkboard, write a 3-digit number (i.e. 743) and have students identify the place value and the digit value. *Ex: What place is the 4 in? The 7 has what value?*

  Have students verbally give their answers and explain how they know. Repeat with other three- or four-digit numbers as necessary.

- Engagement
  Ask: What do you think about the possibility of life on other planets? Have students share responses.

  Say: Today we are going to start on a journey to visit other beings, on other planets, in other galaxies! Let's pretend that you and your group members are a space crew. You will travel together in an imaginary space craft. Starting today, we will begin our three-day voyage to other galaxies to observe the life forms and number systems there.

  Divide students into groups of four.
  Say: With your group, decide on a creative name for your spacecraft. Have one group member write that name on your spaceship illustration sheet (Student Resource 1) and the names of all group members as well. Decorate as you see fit!

- Exploration
  Ask students on what number our own number system is based? (10)
  Distribute base 10 materials to each group. Tell students that you want them to use the base 10 materials to identify how each place is similar and different (ones, tens, hundreds, thousands). Possible observations may include that as you move to the left, a 0 is added to each place or that each place is multiplied by 10.

  Lead students to also recognize that in the base 10 materials, the dimensions increase by 1 (units have no side length of 10, rods have 1 side length of 10, flats are square with 2 side lengths of 10, and cubes have 3 side lengths of 10.

- Explanation
  Explain that our number system is based on 10 and is called the base 10 system. Use an overhead or chalkboard to explain that each place can be expressed as a power of ten. See Board/Overhead Example for teaching directions (Teacher Resource 1).

  Have students follow along with you as you give the directions on the board. This could be having students record notes in their math journals and write down two important facts learned on a sticky note, etc.

- Application
  Tell students that they are about to make a place value pocket chart for base 10. Distribute to each student a sheet of construction paper, a glue stick, scissors, and a marker. Guide them through making their chart using yours as an example (see Teacher Resource 2A-B). Have them label each pocket with the correct place in base 10 notation. Discuss each place to reinforce new vocabulary: base, exponent, power. Ask: What is the value of any number raised to the zero power? (1)

  Distribute index cards to each student.
  Have them trim the cards to about 2" x 5". Use your example as a reference.
  Have them write one digit on each card for digits, 0-9.
  Give examples of 3-digit and 4-digit numbers and have students place the corresponding digits in their pocket charts, with students justifying their answers.

  Ask groups to identify the exponential expression corresponding to each digit's place (i.e. in 2,453, the 2 is in the 10 to the third power place.) Continue in this way, asking questions about identifying the value of digits, bases and exponents or powers.

  Tell students that they are about to play a game using their new knowledge of bases and powers. Divide the students into groups of 12. Distribute one set of I Have/Who Has cards made from Teacher Resource 3 to each group. (If there are more cards than students, give some students a second card.) To begin play, each group selects one student to begin by reading their card. Students should listen for their number and respond by reading their card. Play ends when each person has read their card. Guide students as they play, giving hints where needed.

  Students having trouble remembering their facts may benefit from the reteaching activity below.

- Differentiation

  - Reteach
    Have students create cards with an exponential expression (i.e. 4^2) on one side and the numerical value (i.e. 16) on the other side. They can use these to play a matching game of Concentration.

  - Enrich
    Give students an additional sheet of construction paper. They should make another chart for the next four places in base 10. They should identify the places independently (i.e. 10^4, 10^5, 10^6, 10^7) and tape these to their first chart.

- Assessment
  Distribute Boarding Pass #1 exit ticket (Student Resource 2) to board the spacecraft and travel to Galaxy Quintopia. Say: It's time to travel! Let's blast off! Tomorrow we will land at our first destination, Galaxy Quintopia.

  Read over these exit tickets to determine levels of attainment and adjust next day's instruction. Answer key can be found on Teacher Resource 4.

Day 2

- Engagement
  The class is about to enter their first galaxy: Quintopia. Say: Space crews, prepare for entry…5…4…3...2...1! We're here. We are now in the galaxy of Quintopia.

  Ask students what they think is different about this galaxy. Show them the picture of the Quintopian (Teacher Resource 4). Ask: What do you know about the prefix, quint? Lead them to the idea of quint meaning 5. Ask: How do you think the number 5 will impact our journey? Tell them that the Quintopians are different creatures that have 5 antennae. Ask: What number system do you think they use? (5) How do you think the base five system *is different than base 10?*

- Exploration
  Tell students that the Quintopians count everything in groups of five and use the base 5 system. That means that they don't use the numerals 5, 6, 7, 8, or 9!

  Display the base 10 pocket chart from yesterday. Say: Yesterday, we used base 10 materials. What do you think base 5 materials would look like? (Wait for responses.) Today we will create our own base 5 materials using chips.

Distribute about 40 chips (10 each of white, red, blue, yellow) to each group. Tell them that each color should represent a different numerical value for the base 5 system. They should explore using the chips. You can provide them with a hint. Say: You may want to think about the base 10 pocket charts you made yesterday. Why do you want 4 different colors? Lead them to understand that the 4 colors should represent the first 4 places of the new system.

Tell them that the first place is always units or ones and choose white. The next place is "fives", or 5 to the first power and choose red. Ask what the next place should be. Guide them to 5 to the second power or "twenty-fives" and choose blue. Ask what the last place should be. Students should recognize the pattern and respond with "one hundred twenty-fives" for 5 to the third power and choose yellow.

Now, tell students that you will be using the chips to create a base 5 number. Direct students to make piles of 4 blue chips, 2 red chips, and 4 white chips. Explain that in the base 5 system this number is: 424 in base 5. Ask students if they can tell what that number is in the base 10 system. (Answer: 114 in base 10. $4 \times 25 = 100$, $2 \times 5 = 10$, $4 \times 1 = 4$; $100+10+4=114$.)

A model of this process can be found at http://www.beaconlearningcenter.com/WebLessons/ChipCounting/default.htm

- Explanation
  Use an overhead or chalkboard to explain that each place in the base five system can be expressed as a power of five. See Board/Overhead Example for teaching directions (Teacher Resource 6A-B).

  Have students follow along with you as you give directions on the board. This could be having students record notes math journals, write down 2 important facts learned on a sticky note, etc.

- Application
  Tell students that they will have to convert Earth's base 10 numbers to base 5 to survive on Quintopia. They will make a place value pocket chart for base 5 to help them.

  Distribute materials to make another place value chart for base 5. This time it should be labeled as directed on Additional Place Value Pocket Charts (Teacher Resource 7). You might want to make another chart with the base 5 labeling and display.

Guide them through making their chart using yours as an example. Discuss each place to reinforce new vocabulary: base, exponent, power. Ask: What is the value of any number raised to the zero power? (1) Using their digit cards from the day before, write a 3-digit base 10 number and have students convert it to base 5. Do one example with the whole group and then write another number to be converted independently.

Now explain how to convert base 5 numbers to base 10. Convert 234 in base 5 to base 10. Have students place their 2, 3 and 4 digit cards in their base 5 pocket charts.

- Ask: What is the value of the 2? (2 x 25 = 50) Record the number 50 on the board.
- Ask: What is the value of the 3? (3 x 5 = 15) Record the number 15 under the 50 on the board.
- Ask: What is the value of the 4? (4 x 1 = 4) Record the number 4 under the 15 and add all three addends. The sum should be 69; the answer is 69 in base 10.
- Repeat conversions as necessary.

Distribute Galaxy Quintopia (Student Resource 3A-B). Circulate as students work in groups to make the conversions.

Students having trouble may benefit from the reteaching activity below. Answer key can be found on Teacher Resource 8A-B.

- Differentiation
  - Reteach
    Have students do the base five counting activity on website:
    http://www.ecawa.asn.au/home/jfuller/binary/binary3.htm#
    Have students practice in pairs converting 2-digit base 10 numbers to base 5 using pocket charts.

  - Enrich
    Have students answer the following: Why will you never reach the number 6 in base 5? What are the first 20 counting numbers in base 5? Have students create their own 4-digit base 10 numbers and convert them to base 5.

- Assessment
  Distribute Boarding Pass #2 exit ticket (Student Resource 4) to board the spacecraft and travel to Galaxy Biaculus. Say: It's time to travel! Let's blast off! Tomorrow we will land at our second destination, Galaxy Biaculus.

  Read over these exit tickets to determine levels of attainment and adjust next day's instruction. Answer key can be found on Teacher Resource 9.

Day 3

- Engagement
  The class is about to enter their second galaxy: Biaculus.
  Say: Space crews, prepare for entry…5…4…3...2...1! We're here. We are now in the galaxy of Biaculus. Ask students what they think is different about this galaxy. Show them the picture of the Biaculean (Teacher Resource 10).

  Lead them to the meaning of the prefix, bi being 2. Ask: How do you think the number 2 will impact our journey? Tell them that the Biaculeans are different creatures that have 2 antennae. Ask: What number system do you think they use? (base 2 or binary) How do you think the base 2 system is different than base 10? How is it different than base 5?

  Tell students that the Biaculeans count everything in groups of two and use the base 2 system. That means that they only use numerals 0 and 1!

- Exploration
  Say: Yesterday, we used base 10 and base 5. Today we will compare base 5 to base 10. Distribute Venn Diagram (Student Resource 5). Display both pocket charts and have students complete the diagram comparing the base 5 and base 10 systems.

  Students share their thoughts. Record on board (see Teacher Resource 11 for the answer key) and discuss.

- Explanation
  Use an overhead or chalkboard to explain that each place in the base two system can be expressed as a power of two. See Board/Overhead Example for teaching directions (Teacher Resource 12A-D). Repeat with additional examples as necessary.

  Have students follow along with you as you give directions on the board. This could be having students record notes in math journals or write down two important facts learned on a sticky note, etc.

  Ask students why so many places are needed when converting to base 2. Lead them to understand that the lower the base the higher the number of digits in the number, since there are a limited number of digits available for use.

  Ask which conversion seems easier and have students share their answers.

- Application

  Tell students that they will have to convert Quintopia's base 5 numbers and Biaculus' base 2 numbers back to base 10 for survival and safe travel home. They will make a place value pocket chart for base 2 to help them.

  Distribute materials to make another place value chart for base 2. They will make two and tape them together for an 8-column chart. This time it should be labeled as directed on Additional Place Value Pocket Charts (Teacher Resource 7). Guide them through making their chart using yours as an example. Discuss each place to reinforce new vocabulary: base, exponent, power. Ask: What is the value of any number raised to the zero power? (1) Using their digit cards from the day before, write a 3-digit base 5 number and have students convert it to base 10, then to base 2. Do one example with the whole group and then write another number to be converted independently.

  Now explain how to convert base 2 numbers to base 10. Convert 1110 in base 2 to base 10. Have students place their digit cards in their base 2 pocket charts. They may have to write additional ones and zeros on the backs of other digit cards.
  - Ask: What is the value of the first 1? (1 x 8 = 8) Record the number 8 on the board.
  - Ask: What is the value of the second 1? (1 x 4 = 4) Record the number 4 under the 8 on the board.
  - Ask: What is the value of the third 1? (1 x 2 = 2) Record the number 2 under the 4.
  - Ask: What is the value of the 0? (1 x 0 = 0) Add all three addends. The sum should be 14; the answer is 14 in base 2.

  Repeat conversions with larger numbers.

  Distribute Galaxy Biaculus (Student Resource 6A-B). Circulate as students work in groups to make the conversions.

  Students having trouble may benefit from the reteaching activity below. Answer key can be found on Teacher Resource 13A-B.

- Differentiation

  - Reteach
    Students can review conversions step by step in Quintopian Practice (Student Resource 7).
  - Enrich
    Have students convert base 10 to other bases, including base 3, 4, or 6.

Summative Assessment:

Distribute A Ticket Home Summative Assessment (Student Resource 8A-B) to board the spacecraft and travel back home to the Milky Way. Say: It's time to travel! You need to purchase fuel to get to your home base! In order to do that, you must make conversions from base 10 to base 5 to base 2 and back to base 10. Good luck on your mission.

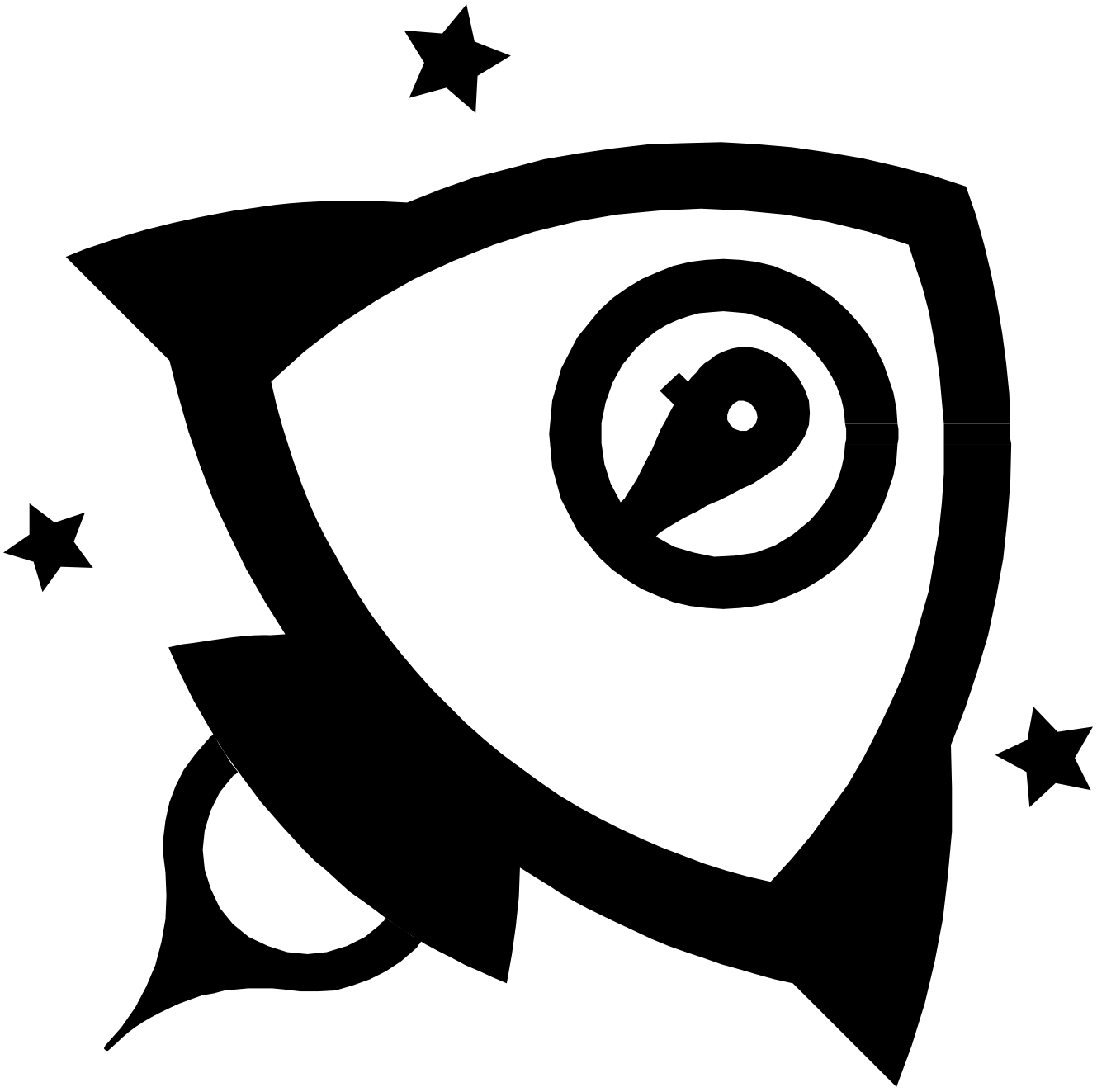
Graded assessments (see Teacher Resource 15A-B) will determine the level of performance.

Authors:

Marian Dingle
Guilford Elementary School
Howard County Public Schools

Cheryl D. Madden
Eastwood Center
Baltimore County Public Schools

Name of Spacecraft_____



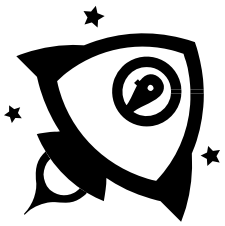
Crewmembers _____

_____

_____

_____

Student Resource 2



Boarding Pass #1 – Day 1

Complete this boarding pass in order to board your spacecraft and travel to our first destination:

Use the number **6,475** to answer the following questions:

1. What is the value of the digit 4? _____
2. In what place is the 4? _____
3. Write that place in exponential form. _____

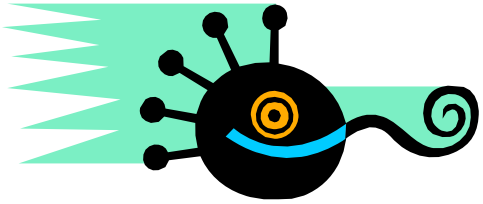
4. What is the value of the digit 7? _____
5. In what place is the 7? _____
6. Write that place in exponential form. _____

7. In what place is the digit 6? _____
8. Write that place in exponential form. _____

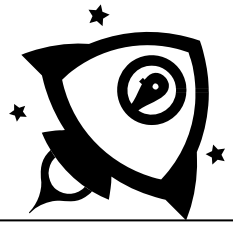
Use the expression $\mathbf{6^3}$ to answer the following questions:

9. Which digit is the base? _____
10. Which digit is the exponent? _____

It's time to travel! Let's blast off! Tomorrow we will land at our first destination, Galaxy Quintopia!



Galaxy Quintopia



Today our spacecraft arrived in the Galaxy of Quintopia. The inhabitants of Quintopia use the Base 5 place value system. You and your spacecrew need to translate Quintopian numbers into Planet Earth's Base 10 system! Look at the following example:

$5^3 = 125$	$5^2 = 25$	$5^1 = 5$	$5^0 = 1$
	3	4	3
	3 x 25 = 75	4 x 5 = 20	3 x 1 = 3

So, $343_5 = 75 + 20 + 3 = 98_{10}$

Now you try! Use the Base 5 Place Value Chart shown above to help you.

1. 124_5 = _____ x 25 + _____ x 5 + _____ x 1
   _____ + _____ + _____ = _____$_{10}$

2. 4220_5 = _____ x 125 + _____ x 25 + _____ x 5 + _____ x 1
   _____ + _____ + _____ + _____ = _____$_{10}$

3. 312_5 = _____ x 25 + _____ x 5 + _____ x 1
   _____ + _____ + _____ = _____$_{10}$

4. 123_5 = _____$_{10}$

5. 114_5 = _____$_{10}$

6. 231_5 = _____$_{10}$

Now try converting from Base 10 into Base 5! Look at this example:

172_{10} = _____$_5$

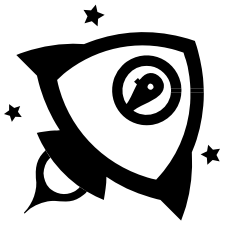
Think…
There is one group of 125 in 172. 172 – 125 = 47.
There is one group of 25 in 47. 47 – 25 = 22.
There are four groups of 5 in 22. 22 – 20 = 2.
There are two groups of 1 in 2.

172_{10} = __1___ x 125 + __1__ x 25 + ___4__ x 5 + __2___ x 1 = **1142_5**

7. 163_{10} = _____ x 125 + _____ x 25 + _____ x 5 + _____ x 1
_____ _____ _____ _____ = _____$_5$

8. 61_{10} = _____ x 25 + _____ x 5 + _____ x 1
_____ _____ _____ = _____$_5$

9. 268_{10} = _____ x 125 + _____ x 25 + _____ x 5 + _____ x 1
_____ _____ _____ _____ = _____$_5$



Boarding Pass #2 – Day 2

Complete this boarding pass in order to board your spacecraft and travel to our next destination. Use the place value chart to help you. Be sure to show your work!

$5^3 = 125$	$5^2 = 25$	$5^1 = 5$	$5^0 = 1$
	3	4	3
	3 x 25 = 75	4 x 5 = 20	3 x 1 = 3

Convert this Base 5 number to Base 10.

4. 3241_5 = _____ x 125 + _____ x 25 + _____ x 5 + _____ x 1
_____ + _____ + _____ + _____ = _____$_{10}$

Convert this Base 10 number to Base 5.

5. 146_{10} = _____ x 125 + _____ x 25 + _____ x 5 + _____ x 1
_____ _____ _____ _____ = _____$_5$

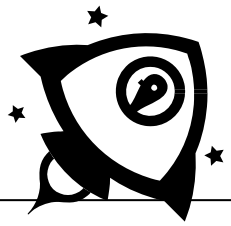
It's time to travel! Let's blast off! Tomorrow we will land at our next destination, Galaxy Biaculus!

Base 5

Base 10

Student Resource 5

Galaxy Biaculus



Today our spacecraft arrived in the Galaxy of Biaculus. The inhabitants of Biaculus use the Base 2 place value system. Since we just came from Galaxy Quintopia, you and your space crew need to translate Quintopian base 5 numbers into Biaculean base 2 numbers!

Example: Convert $\mathbf{1023}_5$ to base 2.

5^3	5^2	5^1	5^0
125	25	5	1
1 x 125 = 125	0 x 25 = 0	2 x 5 = 10	3 x 1 = 3
1	0	2	3

First convert to base 10:

$1 \times 125 + 0 \times 25 + 2 \times 5 + 3 \times 1 = 3$

$= 125 + 10 + 3 = 138_{10}$

Now convert to base 2.

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
128	64	32	16	8	4	2	1
1x128=128	0x64=0	0x32=0	0 x 16 = 0	1 x 8 = 8	0 x 4 =0	1 x 2 = 2	0x 1 = 0
1	0	0	0	1	0	1	0

Think…

There is one group of 128 in 138. 138 – 128 = 10.

There are zero groups of 64, zero groups of 32, and zero groups of 16 in 10.

There is one group of 8 in 10. 10 – 8 = 2.

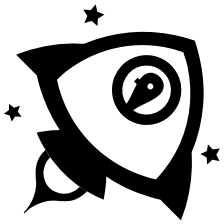
There are zero groups of 4 in 2, one group of 2 in 2, and zero groups of 1.

Answer: 10001010_2

Now you try, using the above charts to help you.

Galaxy Biaculus

(page 2)



Convert these numbers from base 5 to base 2.

1. 421_5 = _____ x 25 + _____ x 5 + _____ x 1

_____ + _____ + _____ = _____$_{10}$

_____$_{10}$ = _____ x 128 + _____ x 64 + _____ x 32 +

_____ x 16 + _____ x 8 + _____ x 4 +

_____ x 2 + _____1

___ ___ ___ ___ ___ ___ ___ ___ = _____$_2$

2. 2340_5 = _____ x 125 + _____ x 25 + _____ x 5 +

_____ x 1

_____ + _____ + _____ + _____ = _____$_{10}$

_____$_{10}$ = _____ x 128 + _____ x 64 + _____ x 32 +

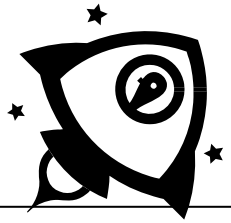
_____ x 16 + _____ x 8 + _____ x 4 +

_____ x 2 + _____1

___ ___ ___ ___ ___ ___ ___ ___ = _____$_2$

Galaxy Biaculus

(page 3)



Almost home! In order to get there, you and your space crew should practice converting Biaculean numbers back into Earth's base 10 numbers!

Look at the following example:

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
128	64	32	16	8	4	2	1
			1 x 16 = 16	1 x 8 = 8	0 x 4 =0	1 x 2 = 2	1 x 1 = 1
			1	1	0	1	1

So, $11011_2 = 16 + 8 + 0 + 2 + 1 = 27_{10}$

Now you try! Use the Base 2 Place Value Chart shown above to help you.

6. 101101_2 = _____ x 128 + _____ x 64 + _____ x 32 + _____ x 16 + _____ x 8 +_____ x 4 + _____ x 2_____ x 1

_____ + _____ + _____ +_____ + _____ + _____

= _____$_{10}$

7. 11101110_2 = _____ x 128 + _____ x 64 + _____ x 32 + _____ x 16 + _____ x 8 +_____ x 4 + _____ x 2_____ x 1

_____ + _____ + _____ +_____ + _____ + _____

= _____$_{10}$

CHALLENGE: Can you convert 11001100_2 to a base 5 number?

Quintopian Practice

Use the chart to convert 4321 in base 5 to base 10.

	Base 5 Places			
	125	25	5	1
Step 1	4	3	2	1
Step 2	4 x 125 = 1000	3 x 25 =75	2 x 5 = 10	1 x 1 = 1
Step 3	1000 +	75 +	10 +	1
Step 4	1086_{10}			

1. Write the digits in the chart.
2. Multiply the digit by the place.
3. Add the values of each place.
4. The sum is the number in base 10.

Now, use the chart to help convert from base 5 to base 10.

1. 3044_5

	Base 5 Places			
	125	25	5	1
Step 1				
Step 2				
Step 3				
Step 4				

2. 1241_5

	Base 5 Places			
	125	25	5	1
Step 1				
Step 2				
Step 3				
Step 4				

A Ticket Home – Day 3

In order to train future space cadets in BASE-ic space travel, please complete the following to show all that you have learned. Good luck on your mission.

1. The following number is written in base 10. What does the digit 4 represent? $\mathbf{2\underline{4}50_{10}}$

   Ⓐ 4

   Ⓑ 40

   Ⓒ 400

   Ⓓ 4,000

2. In the base 10 number below, the 3 is in what place?

   $\mathbf{\underline{3},972_{10}}$

   Ⓐ ones

   Ⓑ tens

   Ⓒ hundreds

   Ⓓ thousands

3. In the following number, the digit 3 is called the:

   $\mathbf{2^3}$

   Ⓐ base

   Ⓑ exponent

   Ⓒ square

   Ⓓ cube

4. The number 100 can be represented by which expression?

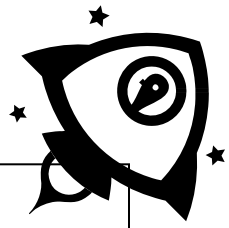
   Ⓐ 10^0

   Ⓑ 10^1

   Ⓒ 10^2

   Ⓓ 10^3

A Ticket Home – Day 3



Brief Constructed Response

Complete this boarding pass in order to board your spacecraft and travel back to our home "base," Planet Earth:

Part A

Convert the following base 5 number to base 10:

143_5 = _____$_{10}$

Part B

Use what you know about place value to explain why your answer is correct. Use numbers and/or words in your explanation.

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

Let's go home!!

Board/Overhead Example – Day 1

Write a 4-digit number on the board, as below:

3562

Ask students to identify places and record underneath each digit.

3	5	6	2
thousand	hundred	ten	one

Then write the numeral of each place.

3	5	6	2
thousand	hundred	ten	one
1000	100	10	1

Ask students to help you represent each number using the number 10. You will eventually have this:

3	5	6	2
thousand	hundred	ten	one
1000	100	10	1
10 x 10 x 10	10 x 10	10	(leave blank)

Explain that there is another way to represent numbers that are multiplied by themselves.
Define **base** as a number that is multiplied by itself and **exponent** or **power** as the number of times it is multiplied. It is written like this:

$$\text{base}^{\text{exponent}} \text{ or } \text{base}^{\text{power}}$$

Ask the class these questions:

How many times is the 10 multiplied in the thousands place? (3)
How many times is the 10 multiplied in the hundreds place? (2)
How many times is the 10 multiplied in the tens place? (1)
How many times is the 10 multiplied in the ones place? (0)

Since 10 multiplied by itself 0 times in the ones place, it is written as 10^0.
Add that any base raised to the zero power is one.

Board/Overhead Example – Day 1

Add each exponential expression underneath each place, so it looks like this:

3	5	6	2
thousand	hundred	ten	one
1000	100	10	1
10 x 10 x 10	10 x 10	10	(leave blank)
10^3	10^2	10^1	10^0

Ask: *How do we write the ones place?* (10 to the zero power.)
How do we write the tens place? (10 to the first power.)
How do we write the hundreds place? (10 to the second power.)
How do we write the thousands place? (10 to the third power.)

Place Value Pocket Chart Directions

Student Materials:

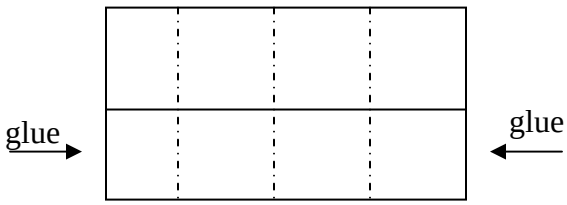
- one piece of 9 x 12 construction paper
- glue stick
- marker
- index cards
- scissors

Directions:

1. Holding the construction paper horizontally, fold approximately one-half of the bottom portion upward.

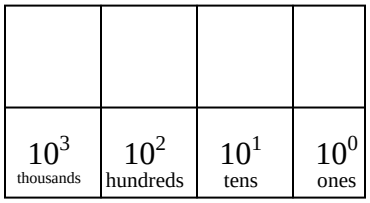
2. Crease the fold so that it lies flat and forms a pocket at the bottom.

3. Fold the paper horizontally in half, then in half again, so that three creases are formed, dividing the sheet into fourths. Unfold. Glue pockets at outer edges.



4. Use a marker to draw lines on the place value pocket chart over each vertical crease.

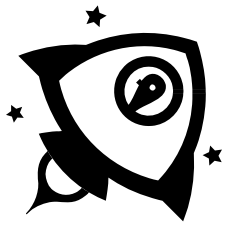
5. Label the bottom of each pocket with the appropriate base and power and corresponding word name. (for Base 10 charts: 10^0 ones; 10^1 tens; 10^2 hundreds; 10^3 thousands; for Base 5 charts: 5^0 ones; 5^1 fives; 5^2 twenty-fives; 5^3 one hundred twenty-fives etc.)



6. Have students trim 10 index cards to approximately 2 x 5 (trim off about one inch so that the cards fit easily into each pocket). Have students label each card with a digit from 0 through 9. The cards will be inserted into the pockets to demonstrate place value of digits in various bases.

I Have Who Has with Exponential Numbers

I have 100. Who has 2^2?	I have 4. Who has 3^2?	I have 9. Who has 4^2?
I have 16. Who has 5^2?	I have 25. Who has 6^2?	I have 36. Who has 9^2?
I have 81. Who has 7^2?	I have 49. Who has 8^2?	I have 64. Who has 2^3?
I have 8. Who has 3^3?	I have 27. Who has 5^3?	I have 125. Who has 10^2?



Boarding Pass #1 – Day 1 Answer Key

Complete this boarding pass in order to board your spacecraft and travel to our first destination:

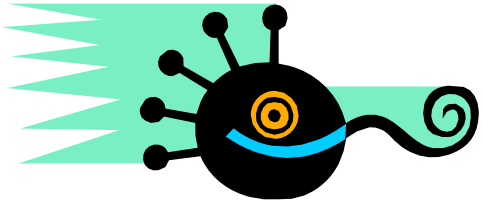
Use the number **6,475** to answer the following questions:

1. What is the value of the digit 4? **400**
2. In what place is the 4? **hundreds**
3. Write that place in exponential form. **10^2**
4. What is the value of the digit 7? **70**
5. In what place is the 7? **tens**
6. Write that place in exponential form. **10^1**
7. In what place is the digit 6? **thousands**
8. Write that place in exponential form. **10^3**

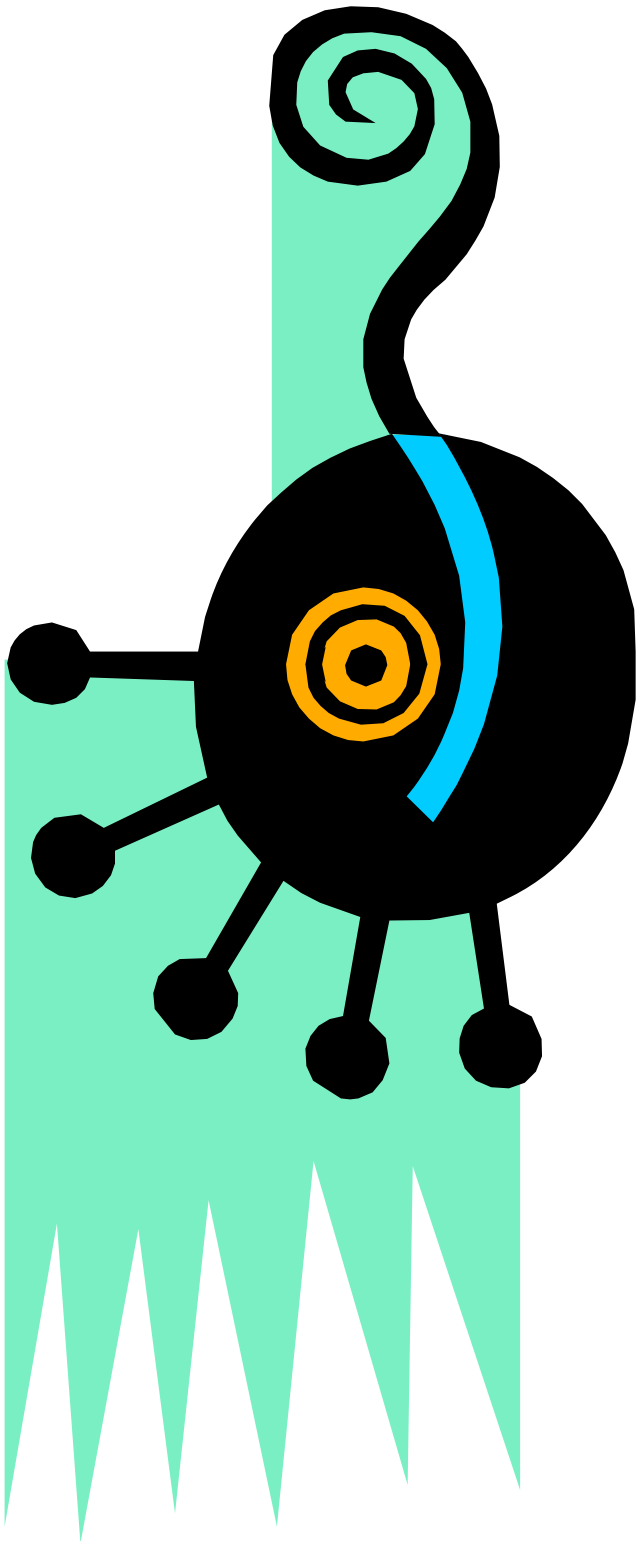
Use the expression **6^3** to answer the following questions:

9. Which digit is the base? **6**
10. Which digit is the exponent? **3**

It's time to travel! Let's blast off! Tomorrow we will land at our first destination, Galaxy Quintopia!



Quintopian Creature



Board/Overhead Example – Day 2

Write a 2-digit number on the board, as below:

72_{10}

Explain: The number reads, seventy-two, base 10. The base is written as a subscript. Since we have always dealt with base 10 numbers, we have dropped the base notation until now.

Now, write the first four places for the base 5 system. Remember, each digit increases by a power of 5, and that the digit to the right is always the unit or one, which is five to the zero power.

_____	_____	_____	_____
			one
			1

Now, explain that the next place has a value of 5, which is 5 to the first power.

_____	_____	_____	_____
		five	one
		5	1

The next place has a value of 25, which is five to the second power. Similarly, the next place has a value of 125, which is five to the third power.

_____	_____	_____	_____
one hundred twenty-five	twenty-five	five	one
125	25	5	1
5 x 5 x 5	5 x 5	5	-
5^3	5^2	5^1	5^0

Base System Conversions:

Our next challenge is to convert our base 10 number to base 5.

Step 1:
What is the highest place in base 5 that is less than or equal to 72? (25)
How many groups of 25 are in 72? (2)

Board/Overhead Example – Day 2

So, we write a 2 in the 25 place.

_____	2	_____	_____
one hundred twenty-five	twenty-five	five	one
125	25	5	1
5^3	5^2	5^1	5^0

Since we have 2 groups of 25 which makes 50, subtract 50 from 72 to get 22.

Step 2:

Now go to the next place to the right, which is the 5 place.

How many groups of 5 are in 22? (4)
So, we write a 4 in the 5 place.

_____	2	4	_____
one hundred twenty-five	twenty-five	five	one
125	25	5	1
5^3	5^2	5^1	5^0

Since we have 2 groups of 5 which makes 20, subtract 20 from 22 to get 2.

Step 3:

Now go to the next place to the right, which is the 1 place.

How many groups of 1 are in 2? (2)
So, we write a 2 in the 1 place.

_____	2	4	2
one hundred twenty-five	twenty-five	five	one
125	25	5	1
5^3	5^2	5^1	5^0

So, $72_{10} = 242_5$

Additional Place Value Pocket Charts

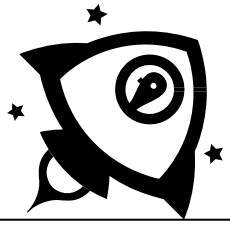
Base Five

5^3 one hundred twenty-fives	5^2 twenty-fives	5^1 fives	5^0 ones

Base Two

2^7 one hundred twenty-eighths	2^6 sixty-fourths	2^5 thirty-seconds	2^4 sixteens	2^3 eights	2^2 fours	2^1 twos	2^0 ones

Galaxy Quintopia Answer Key



Today our spacecraft arrived in the Galaxy of Quintopia. The inhabitants of Quintopia use the Base 5 place value system. You and your spacecrew need to translate Quintopian numbers into Planet Earth's Base 10 system! Look at the following example:

$5_3 = 125$	$5_2 = 25$	$5_1 = 5$	$5_0 = 1$
	3	4	3
	3 x 25 = 75	4 x 5 = 20	3 x 1 = 3

So, $343_5 = 75 + 20 + 3 = 98_{10}$

Now you try! Use the Base 5 Place Value Chart shown above to help you.

1. $124_5 =$ **1** x 25 + **2** x 5 + **4**x 1
   25 + **10**+ **4** = $\mathbf{39}_{10}$

2. $4220_5 =$ **4** x 125 + **2** x 25 + **2** x 5 + **0**x 1
   500 + **50** + **10** + **0** = $\mathbf{560}_{10}$

3. $312_5 =$ **3** x 25 + **1** x 5 + **2** x 1
   75 + **5** + **2** = $\mathbf{82}_{10}$

4. $123_5 =$ $\mathbf{38}_{10}$

5. $114_5 =$ $\mathbf{34}_{10}$

6. $231_5 =$ $\mathbf{66}_{10}$

Now try converting from Base 10 into Base 5! Look at this example:

$$\mathbf{172_{10}} = \_\_\_\_\__5$$

Think…
There is one group of 125 in 172. 172 – 125 = 47.
There is one group of 25 in 47. 47 – 25 = 22.
There are four groups of 5 in 22. 22 – 20 = 2.
There are two groups of 1 in 2.

$\mathbf{172_{10}}$ = __**1**___ x 125 + __**1**__ x 25 + ___**4**__ x 5 + __**2**___ x 1 = $\mathbf{1142_5}$

7. 163_{10} = **1** x 125 + **1** x 25 + **2** x 5 + **3** x 1
1 1 2 3= $\mathbf{1123_5}$

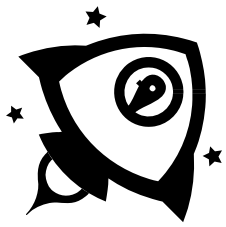
163
-125
38
-25
13
-10
3

8. 61_{10} = **2** x 25 + **2** x 5 + **1** x 1
2 2 1 = $\mathbf{221_5}$

61
-50
11
-10
1

9. 268_{10} = **2** x 125 + **0** x 25 +**3** x 5 + **3** x 1
2 0 3 3 = $\mathbf{2033_5}$

268
-250
18
-15
3



Boarding Pass #2 – Day 2

Complete this boarding pass in order to board your spacecraft and travel to our next destination. Use the place value chart to help you. Be sure to show your work!

$5^3 = 125$	$5^2 = 25$	$5^1 = 5$	$5^0 = 1$
	3	4	3
	3 x 25 = 75	4 x 5 = 20	3 x 1 = 3

Convert this Base 5 number to Base 10.

1. 3241_5 = **3** x 125 + **2**x 25 + **4**x 5 + **1** x 1
   375 + **50** + **20** + **1** = **446**$_{10}$

Convert this Base 10 number to Base 5.

2. 146_{10} = **1** x 125 + **0** x 25 + **4** x 5 + **1** x 1
   1 0 4 1 = _____$_5$

```
 146
-125
  21
 -20
   1
```

It's time to travel! Let's blast off! Tomorrow we will land at our next destination, Galaxy Biaculus!

Biaculean Creature

Venn Diagram Answer Key

Base 5

Base 10

Sample Answers:
- *five digits, 0 – 5*
- *places are ones, fives, twenty-fives, one hundred twenty-fives, etc.*

Sample Answers:
- *digits 0 – 5*
- *places for different values*
- *values grow exponentially*

Sample Answers:
- *ten digits, 0 – 9*
- *places are ones, tens, hundreds, thousands, etc.*

Board/Overhead Example – Day 3

Write a 3-digit base 5 number on the board, as below:

333_5

Convert base 5 to base 2 in two parts. First, convert base 5 to base 10, then base 10 to base 2.

Write the digits in their base 5 places. Remember, each digit increases by a power of 5, and that the digit to the right is always the unit or one, which is five to the zero power.

3	3	3
twenty-five	five	one
25	5	1

Convert to base 10:

3 x 25 = 75
3 x 5 = 15
3 x 1 = 3

Add: 75 + 15 + 3 = 93

Answer: 93_{10}

Now convert this number to base 2 by writing the place values for base 2. You'll need more places because the denominations increase at a slower rate. Start with the one place and add each place to the left by multiplying by 2, one at a time (as in Day 1 and Day 2).

sixty-four	thirty-two	sixteen	eight	four	two	one
64	32	16	8	4	2	1
2x2x2x2x2x2	2x2x2x2x2	2x2x2x2	2x2x2	2x2	2	
2^6	2^5	2^4	2^3	2^2	2^1	2^0

Step 1:

What is the highest place in base 2 that is less than or equal to 93? (64)
How many groups of 64 are in 93? (1)

Board/Overhead Example – Day 3

So, we write a 1 in the 64 place.

sixty-four	thirty-two	sixteen	eight	four	two	one
1						
64	32	16	8	4	2	1
2^6	2^5	2^4	2^3	2^2	2^1	2^0

Since we have 1 group of 64 which makes 64, subtract 64 from 93 to get 29.

Step 2:

Now go the next place to the right which is the 32 place.

How many groups of 32 are in 29? (0)
So, we write a 0 in the 32 place.

sixty-four	thirty-two	sixteen	eight	four	two	one
1	0					
64	32	16	8	4	2	1
2^6	2^5	2^4	2^3	2^2	2^1	2^0

Since we have 0 groups of 32 which makes 0, subtract 0 from 29 to get 29.

Step 3:

Now go the next place to the right which is the 16 place.

How many groups of 16 are in 29? (1)
So, we write a 1 in the 16 place.

sixty-four	thirty-two	sixteen	eight	four	two	one
1	0	1				
64	32	16	8	4	2	1
2^6	2^5	2^4	2^3	2^2	2^1	2^0

Since we have 1 group of 16 which makes 16, subtract 16 from 29 to get 13.

Step 4:

Now go the next place to the right which is the 8 place.

Board/Overhead Example – Day 3

How many groups of 8 are in 13? (1)
So, we write a 1 in the 8 place.

1	0	1	1			
sixty-four	thirty-two	sixteen	eight	four	two	one
64	32	16	8	4	2	1
2^6	2^5	2^4	2^3	2^2	2^1	2^0

Since we have 1 group of 8 which makes 8, subtract 8 from 13 to get 5.

Step 5:

Now go the next place to the right which is the 4 place.

How many groups of 4 are in 5? (1)
So, we write a 1 in the 4 place.

1	0	1	1	1		
sixty-four	thirty-two	sixteen	eight	four	two	one
64	32	16	8	4	2	1
2^6	2^5	2^4	2^3	2^2	2^1	2^0

Since we have 1 group of 4 which makes 4, subtract 4 from 5 to get 1.

Step 6:

Now go the next place to the right which is the 2 place.

How many groups of 2 are in 1? (0)
So, we write a 0 in the 2 place.

1	0	1	1	1	0	
sixty-four	thirty-two	sixteen	eight	four	two	one
64	32	16	8	4	2	1
2^6	2^5	2^4	2^3	2^2	2^1	2^0

Since we have 0 groups of 4 which makes 0, subtract 0 from 1 to get 1.

Board/Overhead Example – Day 3

Step 7:

Now go the next place to the right which is the 1 place.

How many groups of 1 are in 1? (1)
So, we write a 1 in the 1 place.

1	0	1	1	1	0	1
sixty-four	thirty-two	sixteen	eight	four	two	one
64	32	16	8	4	2	1
2^6	2^5	2^4	2^3	2^2	2^1	2^0

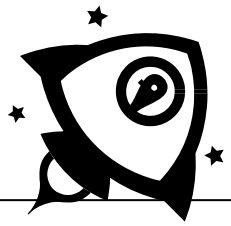
So,

$$333_5 = 93_{10} = 1011101_2$$

Observation:

The process of conversion to base 2 may seem tedious, but when deciding how many groups are in a given place, the choices are either 0 or 1. Thus, the mental math is simpler than in other base conversions.

Galaxy Biaculus Answer Key



Today our spacecraft arrived in the Galaxy of Biaculus. The inhabitants of Biaculus use the Base 2 place value system. Since we just came from Galaxy Quintopia, you and your space crew need to translate Quintopian base 5 numbers into Biaculean base 2 numbers!

Example: Convert **1023_5** to base 2.

5^3	5^2	5^1	5^0
125	25	5	1
1 x 125 = 125	0 x 25 = 0	2 x 5 = 10	3 x 1 = 3
1	0	2	3

First convert to base 10:

1 x 125 + 0 x 25 + 2 x 5 + 3 x 1 = 3

= 125 + 10 + 3 = 138_{10}

Now convert to base 2.

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
128	64	32	16	8	4	2	1
1x128=128	0x64=0	0x32=0	0 x 16 = 0	1 x 8 = 8	0 x 4 =0	1 x 2 = 2	0x 1 = 0
1	0	0	0	1	0	1	0

Think…

There is one group of 128 in 138. 138 – 128 = 10.

There are zero groups of 64, zero groups of 32, and zero groups of 16 in 10.

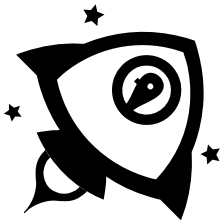
There is one group of 8 in 10. 10 – 8 = 2.

There are zero groups of 4 in 2, one group of 2 in 2, and zero groups of 1.

Answer: 10001010_2

Now you try, using the above charts to help you.

Galaxy Biaculus Answer Key



Convert these numbers from base 5 to base 2.

1. 421_5 = **4** x 25 + **2** x 5 + **1** x 1
   100 + **10** + **5** = **115**$_{10}$

115$_{10}$ = **1** x 64 + **1** x 32 +
1 x 16 + **0** x 8 + **0** x 4 +
1 x 2 + **1** x 1

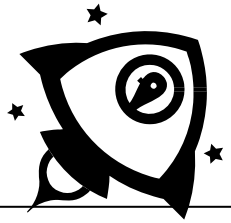
1 1 1 0 0 1 1 = **1110011**$_2$

2. 2340_5 = **2** x 125 + **3** x 25 + **4** x 5 + **0** x 1
   150 + **75** + **20** + **0** = **245**$_{10}$

245$_{10}$ = **1** x 128 + **1** x 64 + **1** x 32 +
1 x 16 + **0** x 8 + **1** x 4 +
0 x 2 + **1** x 1

1 1 1 1 0 1 0 1 = **11110101**$_2$

Galaxy Biaculus Answer Key



Almost home! In order to get there, you and your space crew should practice converting Biaculean numbers back into Earth's base 10 numbers!

Look at the following example:

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
128	64	32	16	8	4	2	1
			1 x 16 = 16	1 x 8 = 8	0 x 4 =0	1 x 2 = 2	1 x 1 = 1
			1	1	0	1	1

So, $11011_2 = 16 + 8 + 0 + 2 + 1 = 27_{10}$

Now you try! Use the Base 2 Place Value Chart shown above to help you.

$1.101101_2 =$ (**1** x 32) + (**0** x 16) + (**1** x 8) + (**1** x 4) + (**0** x 2)+ (**1** x 1)

32 + 0 + 8 + 4 + 0 + 1 = 45_{10}

8. 11101110_2 = **1** x 128 + **1** x 64 + **1** x 32 + **0** x 16 +
1 x 8 + **1** x 4 + **1** x 2+ **0** x 1
128 + 64 + 32 + 0 + 8 + 4 + 2 + 0 = 238_{10}

CHALLENGE: Can you convert 11001100_2 to a base 5 number?

Quintopian Practice Answer Key

Use the chart to convert 4321 in base 5 to base 10.

	Base 5 Places			
	125	25	5	1
Step 1	4	3	2	1
Step 2	4 x 125 = 1000	3 x 25 =75	2 x 5 = 10	1 x 1 = 1
Step 3	1000 + 75 + 10 + 1			
Step 4	1086_{10}			

1. Write the digits in the chart.
2. Multiply the digit by the place.
3. Add the values of each place.
4. The sum is the number in base 10.

Now, use the chart to help convert from base 5 to base 10.

1. 3044_5

	Base 5 Places			
	125	25	5	1
Step 1	**3**	**0**	**4**	**4**
Step 2	**3 x 125**	**0 x 25**	**4 x 5**	**4 x 1**
Step 3	**375 + 0 + 20 + 4**			
Step 4	$\mathbf{399_{10}}$			

2. 1241_5

	Base 5 Places			
	125	25	5	1
Step 1	**1**	**2**	**4**	**1**
Step 2	**1 x 125**	**2 x 25**	**4 x 5**	**1 x 1**
Step 3	**125 + 50 + 20 + 1**			
Step 4	$\mathbf{196_{10}}$			

A Ticket Home – Day 3 Answer Key

In order to train future space cadets in BASE-ic space travel, please complete the following to show all that you have learned. Good luck on your mission.

3. The following number is written in base 10. What does the digit 4 represent? **$2\underline{4}50_{10}$**

   Ⓐ 4

   Ⓑ 40

   Ⓒ 400

   Ⓓ 4,000

4. In the base 10 number below, the 3 is in what place?

   $\underline{3},972_{10}$

   Ⓐ ones

   Ⓑ tens

   Ⓒ hundreds

   Ⓓ thousands

3. In the following number, the digit 3 is called the:

   2^3

   Ⓐ base

   Ⓑ exponent

   Ⓒ square

   Ⓓ cube

4. The number 100 can be represented by which expression?

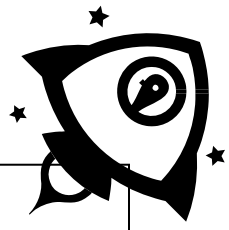
   Ⓐ 10^0

   Ⓑ 10^1

   Ⓒ 10^2

Ⓓ 10^3

A Ticket Home – Day 3 Answer Key



Brief Constructed Response

Complete this boarding pass in order to board your spacecraft and travel back to our home "base," Planet Earth:

Part A

Convert the following base 5 number to base 10:

143_5 = **48_{10}**

Part B

Use what you know about place value to explain why your answer is correct. Use numbers and/or words in your explanation.

To convert from Base 5 to Base 10, it is important to understand the place value of each digit. In Base 5, the digit farthest to the right has a value of one, so you multiply that digit by one. The next digit represents five, so you multiply that digit by five. The next place has a value of 25, so you multiply that digit by twenty-five. You continue in this manner, and each place in Base 5 is worth five times more than the previous place. After the value of each digit is determined, the values are added to calculate the Base 10 equivalent. See the chart below:

	Base 5 Places			
	125	25	5	1
Step 1	0	1	4	3
Step 2	0 x 125 = 0	1 x 25 =25	4 x 5 = 20	3 x 1 = 3
Step 3	0 +	25 +	20 +	3
Step 4	48_{10}			

Let's go home!!